COMPRESSIVE PARAMETER ESTIMATION VIA APPROXIMATE MESSAGE PASSING

Shermin Hamzehei and Marco F. Duarte

Dept. of Electrical and Computer Engineering, University of Massachusetts, Amherst, MA 01003

ABSTRACT

The literature on compressive parameter estimation has been mostly focused on the use of sparsity dictionaries that encode a sampling of the parameter space; these dictionaries, however, suffer from coherence issues that must be controlled for successful estimation. We propose the use of statistical parameter estimation methods within the approximate message passing (AMP) algorithm for signal recovery. Our proposed work leverages the recently highlighted connection between statistical denoising methods and the thresholding step commonly used during recovery. As an example, we consider line spectral estimation by leveraging the well-known Root MUSIC algorithm. Numerical experiments show significant improvements in estimation performance.

Index Terms— Compressive sensing, frequency-sparse signals, spectral estimation, Approximate Message Passing (AMP), Multiple Signal Classification (MUSIC)

1. INTRODUCTION

Compressive sensing (CS) describes a scheme for signal acquisition that replaces the standard band-limitedness assumption of uniform sampling with a sparsity assumption in order to reduce the number of measurements [1, 2]. Here, sparsity refers to the property that the signal has a compact representation in an appropriate basis or frame expansion. By leveraging the sparsity model, it is possible to recover the original signal from a number of measurements that correspond to inner products of the signal with a set of measurement vectors. Commonly, these vectors have entries that are drawn from a subgaussian random distribution, as this construction has been shown to be suitable with very high probability. In certain cases, the measurement vectors are instead designed so that they are incoherent with the sparsity basis or frame [3].

While most of the literature in CS has focused on the recovery of the observed signal from its measurements, there has been growing interest in the past few years on the application of CS concepts to parameter estimation problems. In particular, several researchers have made efforts on the design of measurement schemes and estimation algorithms that allow for accurate line spectral estimation from a small number samples or measurements [4–12]. The line spectral estimation problem deals with signals that are linear combinations of a small number of complex exponentials, and refers to the estimation of the complex exponentials' frequencies (and in some cases, their magnitudes) from the observations obtained.

Many contributions in this regard have focused on the design of sparsity bases or frames that provide exact or approximate sparse representations for the signals of interest and whose elements are complex exponentials for a discrete set of frequency values. However, due to the link between the resolution provided by the dictionary and its coherence, most work in this vein involves modifications to existing CS recovery algorithms that control the coherence among the complex exponentials present in the solution [4, 5, 7]. More recently, it has been shown that for certain choices of sampling patterns (either low-rate regular sampling or high-rate random subsampling), one can recover a frequency-sparse signal from a small set of samples when the frequencies involved are sufficiently spaced apart by solving a clever reformulation of the problem as a semidefinite program [9–12]. While such optimization-based methods can achieve high precision, they also have a high computational cost.

In this paper, we leverage recent variations to the well-known approximate message passing (AMP) algorithm [13] for CS recovery that allow for it to be extended to compressive parameter estimation. The AMP algorithm is endowed with performance guarantees due to the fact that it involves the computation of a signal proxy whose probability distribution matches that of the original signal observed under additive white Gaussian noise (AWGN), thanks to the use of a so-called Onsager correction term. AMP then leverages the soft thresholding operation, which is commonly used in CS recovery algorithms and can be shown to provide an optimal denoiser for sparse vectors [14]. In fact, recent results have shown that AMP can be adapted to recover signals under arbitrary models simply by replacing the thresholding step by a denoising step where, once again, the denoiser is designed to act on AWGN-polluted observations of signals from the model under consideration [15–17]. Following this program, we propose the use of statistical parameter estimation algorithms that are designed specifically for AWGN observations. Our proposed method replaces the soft thresholding step by a custom "denoiser" consisting of a statistical parameter estimation algorithm followed by a signal synthesis module based on the frequency estimates. While we focus on

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line spectral estimation and the Multiple Signal Classification (MUSIC) statistical estimation algorithm in the developments in this paper, our formulation is generic: it can be applied to any parameter estimation problem as long as there exists a statistical parameter estimation method designed for AWGN observations that can be leveraged to provide a corresponding compressive parameter estimation scheme.

Our paper is organized as follows. Section 2 provides additional background on CS, line spectral estimation, and recent efforts combining these two areas. Section 3 presents in detail our proposed merging of AMP and statistical line spectral estimation. Section 4 provides initial experimental evidence of the improvements afforded by our proposed approach. Finally, Section 5 provides conclusions and directions for future work.

2. BACKGROUND AND RELATED WORK

Compressive Sensing: Consider a signal $x \in \mathbb{C}^N$ with at most K nonzero elements (i.e., $\|x\|_0 \leq K$) and a measurement matrix $A \in \mathbb{C}^{M \times N}$ with i.i.d. Gaussian elements, which is then column-normalized. The measurement vector $y = Ax \in \mathbb{C}^M$ represents a compressive sensing (CS) acquisition of the signal when $M \ll N$. The overall goal of CS is to recover x from its measurements y given knowledge of the measurement matrix A.

As it turns out, one can successfully recover the signal from the CS measurements by searching for the sparsest signal x such that $y = Ax$, as long as enough measurements are taken. This can be done in a computationally feasible fashion via linear programming, and this framework can be extended to include other optimization programs when the measurements are noisy or quantized. An often faster alternative is provided by a family of greedy algorithms, which iteratively look for increasingly more accurate approximations to the search's solution [18]. For example, iterative signal recovery using hard thresholding or soft thresholding proceeds as follows with $x^0 = 0$ and $z^0 = 0$, at iteration t we set

$$x^{t+1} = \eta_t(A^H z^t + x^t),$$
$$z^t = y - Ax^t,$$

where $x^t \in \mathbb{C}^N$ and $z^t \in \mathbb{C}^M$ denote the signal estimation and residual at iteration t, respectively, and $\eta_t(.)$ are arbitrary thresholding functions.

Approximate Message Passing: Notably, Donoho et al. suggested a modification in the traditional iterative algorithm based on the theory of belief propagation in graphical models, adding an "Onsager" correction term to the algorithm [13]. The resulting first-order approximate message passing algorithm (AMP) proceeds as follows:

$$x^{t+1} = \eta_t(A^H z^t + x^t),$$
$$z^t = y - Ax^t + \frac{1}{\delta} z^{t-1} \langle \eta'_{t-1}(A^H z^{t-1} + x^{t-1}) \rangle,$$

where $\eta'_t(s) = \frac{\partial}{\partial s}\eta_t(s)$ is the entry-wise derivative of the function $\eta_t(.)$, δ is a fixed constant, and for a vector $\mathbf{u} = \begin{bmatrix} u(1) \ldots u(N) \end{bmatrix}$ we denote $\langle \mathbf{u} \rangle = \frac{1}{N}\sum_{i=1}^N u(i)$. It can be shown that the Onsager term improves the sparsity-undersampling tradeoff (cf. Section 5), yielding performance that approximates that of optimization-based algorithms [13].

Line Spectral Estimation: Rather than studying sparse time-domain signals, we focus on frequency-sparse signals instead. Succinctly, these signals have a number of component frequencies much smaller than their length. For these signals, CS can be implemented using an appropriate basis, such as the discrete Fourier transform (DFT) basis; CS then attempts to find the sinusoid frequencies f_k present in the signal in addition to their corresponding amplitudes a_k (for $k = 1, \ldots, K$), so that the sparse signal can be represented as

$$x[n] = \sum_{k=1}^K a_k e^{-j2\pi f_k n/N}, \ n = 1, \ldots, N. \qquad (1)$$

The process of estimating the frequencies f_k and amplitudes a_k from the signal samples $x[i]$, known in the literature as line spectral estimation, has a much longer history than CS. There are many well-known line spectral estimation algorithms [19, 20], including periodogram-based methods, Thomson's multi-taper method, eigenanalysis-based methods, etc. We will focus on eigenanalysis-based methods, taking the multiple signal classification (MUSIC) [20] algorithm as a specific example.

For the K-sparse signal x from (1), we consider observations $s = x + n$, where $n \sim \mathcal{N}(0, \sigma_n^2 I)$ denotes a white Gaussian noise vector. One can rewrite (1) as $x = \Gamma a$, where Γ is a matrix with columns $\Gamma = \begin{bmatrix} e(f_1) \ldots e(f_K) \end{bmatrix}$ defined as

$$e(f) = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & e^{\frac{j2\pi f}{N}} & e^{\frac{j2\pi 2f}{N}} & \ldots & e^{\frac{j2\pi(N-1)f}{N}} \end{bmatrix}^T,$$

and the vector of coefficients $\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \ldots & a_K \end{bmatrix}^T$. Therefore, $s = \Gamma a + n$. The autocorrelation matrix for s is defined as

$$R_{ss} = \mathbb{E}[ss^H] = R_{xx} + R_{nn} = \Gamma \bar{A}^2 \Gamma^H + \sigma_n^2 I \qquad (2)$$

where as $\bar{A} = \text{diag}(a)$ denotes the matrix diagonalization of the vector a. Since $\text{rank}(\Gamma \bar{A}^2 \Gamma^H) = K$, it is easy to see that R_{xx} has k nonzero eigenvalues $\{\tilde{\lambda}_i\}_{i=1}^K$ (sorted by magnitude), with all other eigenvalues equal to zero. Consequently, for the sorted eigenvalues $\{\lambda_i\}_{i=1}^N$ of R_{ss}, we have

$$\lambda_i = \begin{cases} \tilde{\lambda}_i + \sigma_n^2, & i \leq K, \\ \sigma_n^2, & K < i \leq N. \end{cases}$$

Defining G as the matrix containing the column eigenvectors for the $N - K$ smallest eigenvalues of R_{ss}, we have that $R_{ss}G = \sigma_n^2 G$, as the corresponding eigenvalues are all σ_n^2; thus, plugging in (2), we have that $\Gamma \bar{A}^2 \Gamma^H G + \sigma_n^2 G = \sigma_n^2 G$,

which in turn implies that $\Gamma^H G = 0$. Thus, the frequencies $f \in \{f_k\}_{k=1}^K$ are the only solutions to $e(f)^H G G^H e(f) = 0$.

In order to determine the component frequencies, MUSIC searches for the peaks of the pseudospectrum function

$$P_{MUSIC}(f) = \frac{1}{e(f)^H G G^H e(f)}, \; f \in [0, N).$$

In practice, MUSIC and other eigenanalysis-based methods operate on the sampled autocorrelation matrix $\widehat{R_{xx}} = \frac{1}{p} \sum_{i=1}^p \hat{x}_i \hat{x}_i^T$, where $\hat{x}_i = \begin{bmatrix} x[i] & \dots & x[i+W-1] \end{bmatrix}^T$ denotes the ith window from x of length W (an algorithm parameter), and $p = N - W$ denotes the number of windows present in x. Note that the window size should follow $W \in [K, N]$ [20].

3. INTEGRATING LINE SPECTRAL ESTIMATION AND APPROXIMATE MESSAGE PASSING

Recently, Donoho et al. have shown that the standard soft thresholding function $\eta(\cdot)$ can be replaced by a signal-suitable denoising operator within the AMP algorithm to iteratively recover the signal from its measurements [13]. As motivation, they have shown that in each iteration of AMP, the proxy $A^H z^t + x^t \approx x + n$, where n is i.i.d. additive white Gaussian noise (AWGN) and the approximation is in terms of probability distributions [13, 15]. However, their work and later derivations by other researchers [16, 17] has focused on signal recovery and denoising methods.

Our premise in this work is that the AMP extensions described above can be taken further into compressive parameter estimation simply by leveraging statistical estimation algorithms in place of the denoising methods mentioned above. Once estimation is performed, it is simple to obtain a signal estimate x^t for the following iterations by using a generative signal model that is based on the parameter values, such as (1). Thus, in a nutshell, we propose a family of compressive parameter estimation algorithms in which the denoising step is replaced by the concatenation of a parameter estimation step and a signal synthesis step.

Nonetheless, one quickly notices that the denoising term also appears during the computation of the Onsager term at each iteration. Luckily, recent work by Metzler et al. [17] has provided a numerically simple algorithm for the estimation of the Onsager term using Monte Carlo iterations: for an arbitrary denoiser $D(x)$ acting on x, and using an i.i.d. random vector $\mathbf{b} \sim \mathcal{N}(0, I)$, the divergence of the denoiser can be approximated as

$$D'(x) = \lim_{\epsilon \to 0} \mathbb{E}_b \left\{ b^* \left(\frac{D(x+\epsilon b) - D(x)}{\epsilon} \right) \right\}$$
$$\approx \mathbb{E} \left(\frac{1}{\epsilon} b^* (D(x+\epsilon b) - D(x)) \right).$$

Generating L i.i.d. $\mathcal{N}(0, \sigma^2 I)$ vectors $b_1, \dots, b_L$ with sufficiently small variance $\sigma^2 = \epsilon/N$ and computing L point es-

timates of the divergence $D'(x, b_i) = b_i^* (D(x+\epsilon b_i) - D(x))$, the divergence can be estimated as $D'(x) \approx \frac{1}{L} \sum_{i=1}^L D'(x, b_i)$. According to the weak law of large numbers, this estimate converges to the real value as $L \to \infty$. The resulting recovery algorithm is termed D-AMP in [17].

To showcase our framework, we introduce AMP+MUSIC, a (D-)AMP-based compressive line spectral estimation algorithm based on Root MUSIC, a variant of MUSIC. The "denoiser" $\hat{x} = D_{\text{MUSIC}}(x)$ used for this setup is described as follows:

$$\{\hat{f}_k, \hat{a}_k\}_{k=1}^K = \text{MUSIC}(x, K),$$
$$\hat{x}[i] = \sum_{k=1}^K \hat{a}_k e^{-j2\pi \hat{f}_k i/N}$$

for $i = 1, \dots, N$. Here $\text{MUSIC}(x, k)$ denotes the application of the Root MUSIC algorithm to the vector x, which returns the frequencies $\{\hat{f}_m\}_{m=1}^k$; the amplitude estimates $\{\hat{a}_m\}_{m=1}^k$ can then be obtained, for example, by constructing the matrix Γ corresponding to these estimated frequencies and computing $\hat{a} = \Gamma^\dagger x$, where $\Gamma^\dagger$ denotes the pseudoinverse of Γ.

We wrap our presentation by discussing practical aspects of our approach. The values of L and ϵ in D-AMP are not specified in the algorithm construction. Nonetheless, [17] discusses the robustness of the method to the choice of these parameters due to the high-dimensional nature of the data; in fact, that paper sets $L = 1$ and $\epsilon = \|x\|_\infty/1000$. Our experimental results are consistent with this hypothesis, leading us to set $L = 2$ and $\epsilon = 1$ for all subsequent experiments. Furthermore, the convergence criterion for AMP is often unspecified; in practice, the accuracy of the estimates increases with further iterations, and the algorithm is often either executed for a fixed number of iterations or until subsequent estimates have a negligible distance from one another. In our experiments, we occasionally observed oscillatory non-monotonic behavior in the performance of the algorithm through the iterations of AMP; we have found that checking the accuracy of each iteration's estimate against the measurements by evaluating the measurement residual norm $\|y - Ax^t\|_2$ provides the algorithm with consistent accurate estimation. Finally, the computational complexity of the algorithm will depend on (i) the choice of statistical estimation algorithm (Root MUSIC plus a pseudoinverse in our case, with complexity $\mathcal{O}(N^3)$) and (ii) the number of Monte Carlo iterations to estimate the Onsager term; the remaining steps of the D-AMP algorithm have complexity $\mathcal{O}(MN) \sim \mathcal{O}(N^2)$.

4. NUMERICAL EXPERIMENTS

In this section, we present numerical simulations that test the performance of several line spectral estimation algorithms that work from CS measurements. We consider frequency-sparse signals of the form (1) of length $N = 512$ and measurement matrices with variance $\sigma^2 = 1/M$, where M is the number of rows of the measurement matrix. We measure

the frequency estimation error by computing the cost of the Hungarian matching between the vectors containing the frequency values and their estimates. In our experiments, we compare the performance of AMP+MUSIC to that of several alternative baselines: CS recovery followed by standard line spectral estimation (AMP $\to$ MUSIC and ℓ_1-min. $\to$ MUSIC); IHT + MUSIC, which is akin to AMP+MUSIC without the Onsager correction term, as proposed in [4]; and band-exclusion interpolated subspace pursuit (BISP) [5], a coherence-controlling sparsity-based algorithm. The iterative algorithms are run for 20 iterations.

Our first experiment generates a phase transition plot for spectral estimation, inspired by the recovery-based counterparts from [15, 21]. The phase transition plot of a given recovery algorithm finds the maximum value of the normalized sparsity $\rho = K/M$, as a function of the normalized measurement rate $\delta = M/N$, for which the algorithm successfully recovers a sparse signal at least 50% of the time for a set of signals drawn at random from a uniform distribution over K-sparse signals. The plot is usually interpreted as showing the division between the (δ, ρ) region for which the probability of success goes to one as $N \to \infty$ (below the curve) from the (δ, ρ) region for which the probability of success goes to zero as $N \to \infty$ (above the curve).

For our algorithm's phase transition plot, we define success as having an average frequency estimation error (over the k frequencies) of up to 1 Hz. For each value of the (δ, ρ) duplet, we execute 100 trials with randomly drawn frequencies (uniformly at random in $[0, N)$, with arbitrary resolution), amplitudes (uniformly at random in $[0, 1]$), and measurement matrices. Figure 1 shows the line spectral estimation phase transition for our proposed AMP+MUSIC algorithm and the aforementioned baselines, where the AMP+MUSIC algorithm achieves noticeably better performance, i.e., much higher ρ for each value of δ. This result implies that the combination of the use of statistical estimation and the use of the Onsager correction term provides gains in compressive parameter estimation that are as significant as those achieved in signal recovery by the denoiser-based (D-)AMP.

Our second experiment compares the performance of the different algorithms among randomly drawn signals under the same probability model as the first experiment. In this case, we fix the number of frequencies $K = 8$ and evaluate the average frequency estimation error as a function of the number of measurements n over the same 100 trials for each of the compressive parameter estimation algorithms. Figure 2 shows once again that the performance of AMP+MUSIC is significantly improved over those of its baseline counterparts.

5. CONCLUSIONS AND FUTURE WORK

We have introduced a new scheme for compressive parameter estimation that leverages existing statistical parameter estimation algorithms within the approximate message passing framework. In particular, we have focused on the example
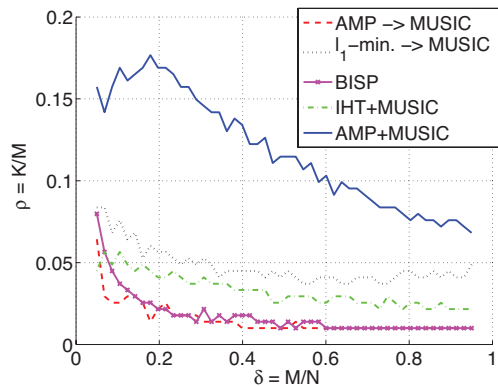

Fig. 1. Phase transition plot for compressive line spectral estimation. The performance of AMP+MUSIC is significantly better than that of all baseline counterparts.
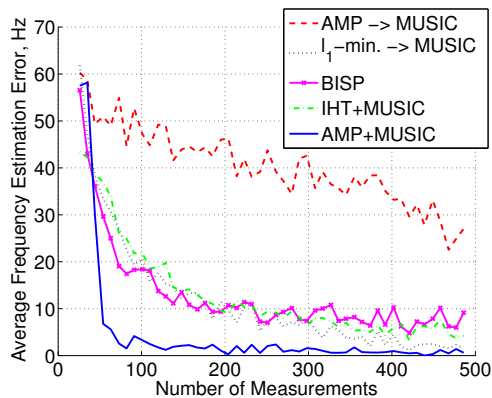

Fig. 2. Average frequency estimation error for several compressive line spectral estimation algorithms. Once again, the performance of AMP+MUSIC is significantly better than that of its baseline counterparts.

of line spectral estimation by leveraging the MUSIC algorithm. Our formulation is feasible thanks to a numerical estimation method for AMP's Onsager correction term that leverages Monte Carlo approximation. Our experimental results showcase the considerable improvements in estimation performance when measured both via a phase transition curve and via the average frequency estimation error, showing that AMP+MUSIC can potentially enable significantly higher CS compression while achieving accurate frequency estimation.

While we have focused on a single parameter estimation problem, our broader compressive parameter estimation scheme could be used in many other applications for which statistical methods are well established. Examples of such compressive parameter estimation applications include time delay estimation [22], localization [23, 24], direction of arrival estimation [25, 26], etc. We also plan to adapt the analysis framework for AMP, which employs a so-called state evolution formalism [15, 17], to our proposed framework.

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