Localization and Bearing Estimation via Structured Sparsity Models

Marco F. Duarte
Department of Electrical and Computer Engineering

Sparsity-Based Localization

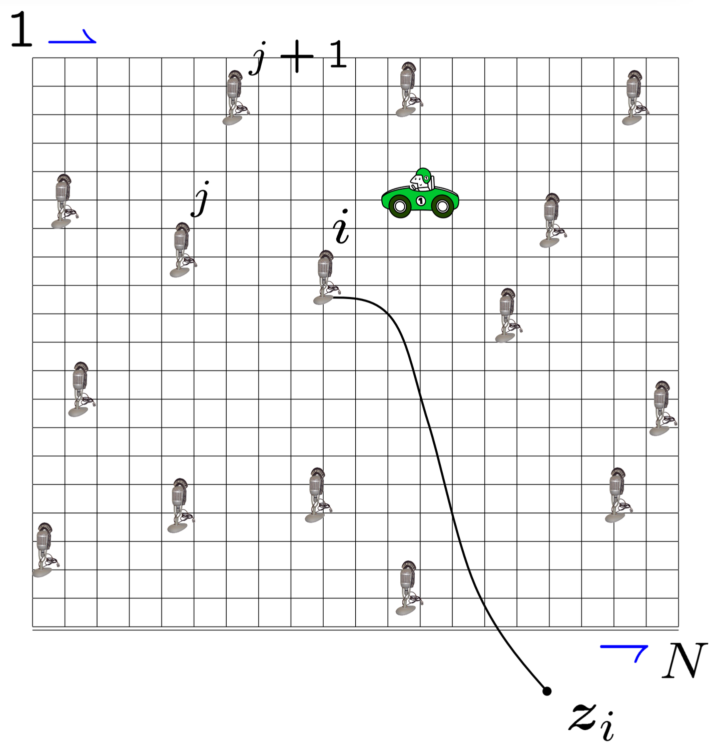
- Build N-sample grid $\{z_i\}_{i=1}^N$ to ***discretize*** localization space (field, range of orientations...)
- A ***small number of targets*** K is assumed to be present at locations within grid
- Build ***localization dictionary*** by collecting observations at P sensors for a target located at each of the grid points z_i
- Recorded observations can be expressed as product of localization dictionary and ***sparse location vector*** c:

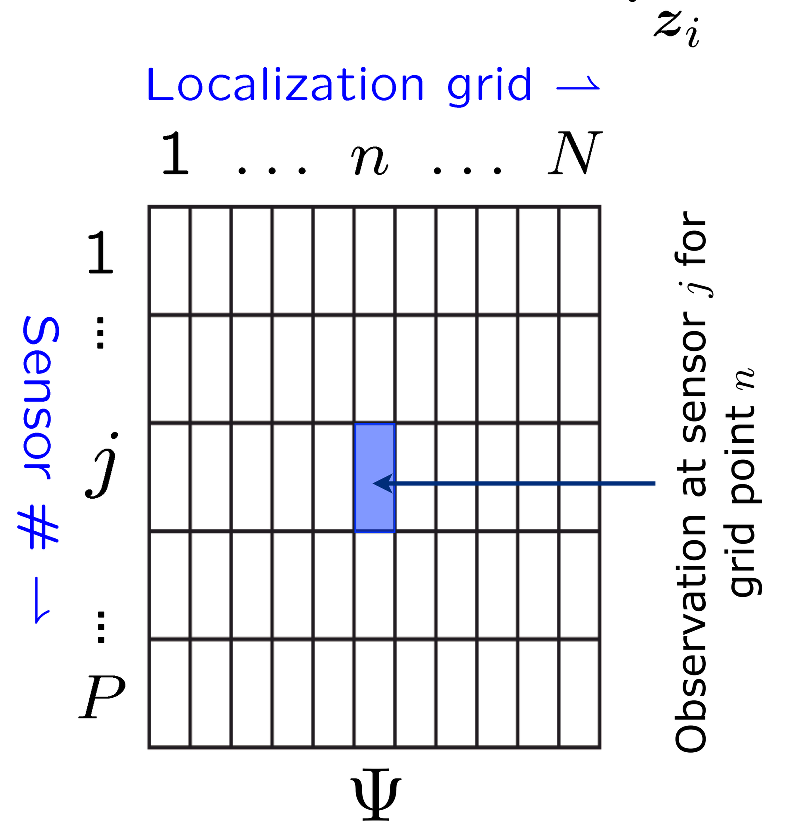
$$x = \Psi c, \ \ c \in \Sigma_K$$

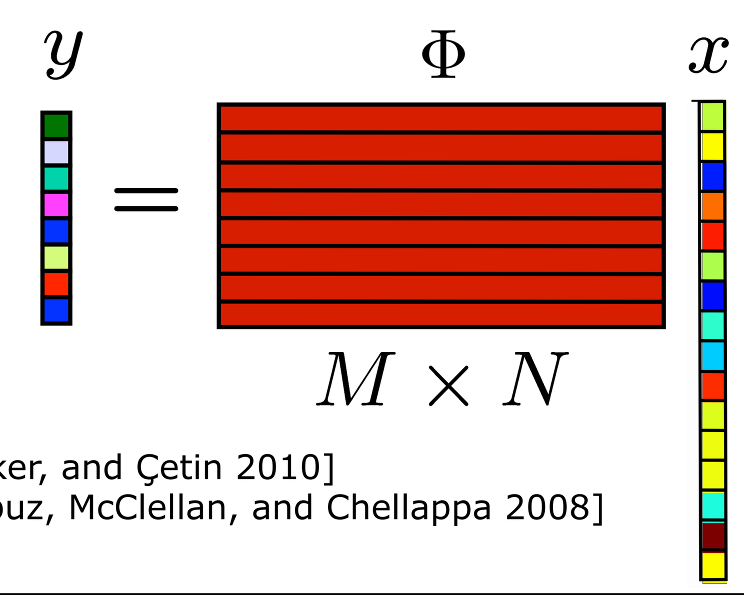
- Sparse vector c encodes locations in its ***support*** (indices of nonzero entries)
- Use ***sparse approximation*** to obtain target locations from collected observations (e.g., IHT, CoSaMP, ...)
- Can be combined with ***compressive sensing*** to reduce dimensionality of acquired data:

$$y = \Phi x = \Phi \Psi c$$

Requires $M = \mathcal{O}(K \log(N/K))$ random measurements







[Gorodnitsky and Rao 1997][Potter, Ertin, Parker, and Çetin 2010]
[Cevher, Duarte, Baraniuk 2008][Cevher, Gurbuz, McClellan, and Chellappa 2008]
Many others...

Issues with Sparsity

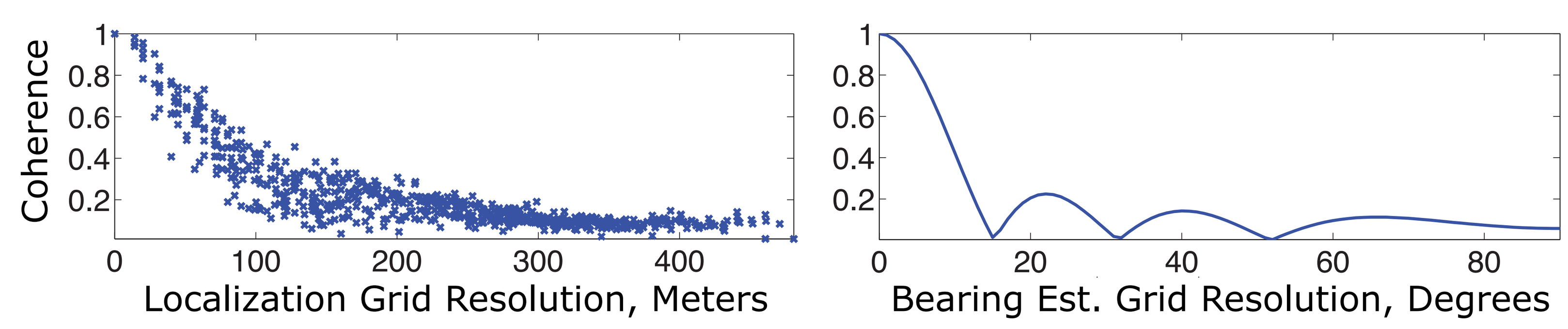
- ***Coherence*** measures similarity between columns of sparsity dictionary: for $\Psi = [\psi_1 \ \psi_2 \ \dots \ \psi_N]$,

$$\mu(\Psi) = \max_{1 \le i,j \le N, i \ne j} \frac{|\langle \psi_i, \psi_j \rangle|}{\|\psi_i\|_2 \|\psi_j\|_2}$$

- Connection between dictionary coherence and performance of sparse approximation: *one can recover a K-sparse vector c from $x = \Psi c$ if*

$$K \le \frac{1}{2}\left(1 + \frac{1}{\mu(\Psi)}\right)$$ [Donoho and Elad 2003]

- Discretization requires that actual target locations are within (or sufficiently close to) grid samples, which ***requires fine resolution*** in dictionary
- As dictionary grid resolution increases, coherence ***approaches maximum value*** of 1



Resolution ***tradeoff***:

- increased resolution ***enlarges*** set of configurations that provide $x \approx \Psi c$ for a K-sparse vector c
- increased resolution ***worsens*** coherence, ***limits*** values of K for which the K-sparse vector c can be recovered

- Connection between coherence in localization dictionary and ***resolution/ambiguity issues*** present in standard localization/bearing estimation algorithms
- These issues are ***exacerbated*** with compressive sensing

Structured Sparsity Models

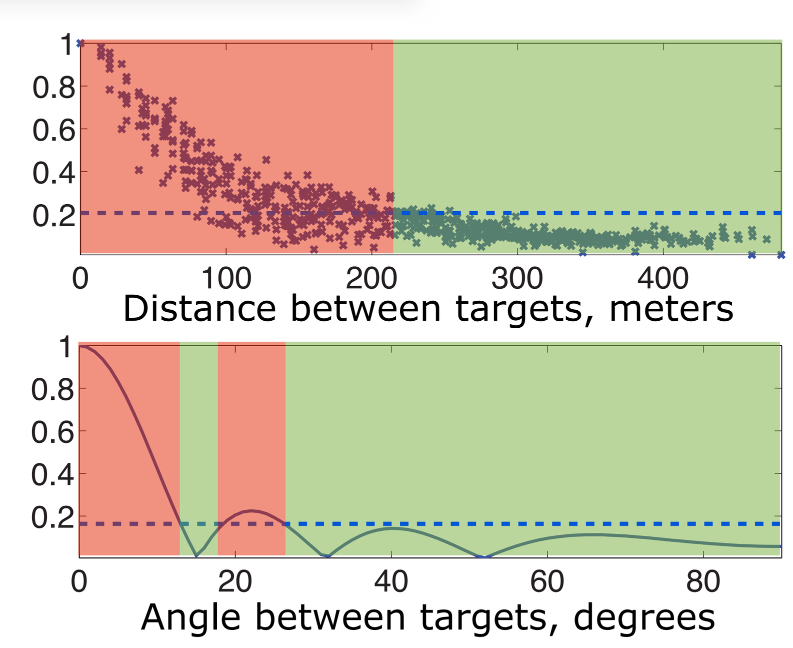
- A structured sparsity model $\mathcal{M}_K \subseteq \Sigma_K$ ***restricts*** possible locations of nonzero coefficients of $c \in \Sigma_K$ so that they exhibit some desirable additional structure
- Structure can be enforced during sparse approximation (and compressive sensing recovery) by replacing usual thresholding steps by ***structured thresholding***:

$$\mathbb{M}_K(c) = \arg \min_{\bar{c} \in \mathcal{M}_K} \|c - \hat{c}\|_2$$

- Example: Structured Iterative Hard Thresholding (IHT)

$$\hat{c}_t = \mathbb{M}_K(\hat{c}_{t-1} + \Psi^T \Phi^T (y - \Phi \Psi c))$$

- The ***stronger*** the structure assumed, the ***larger the reductions*** in the number of measurements required for signal recovery in compressive sensing
- We restrict set of nonzeros to include ***only indices that correspond to incoherent columns*** of localization dictionary:



$$x \in \mathcal{M}_K \text{ if } x = \sum_{i=1}^{K} c_i \Psi_{n_i}, \frac{|\langle \psi_{n_i}, \psi_{n_j} \rangle|}{\|\psi_{n_i}\|_2 \|\psi_{n_j}\|_2} \le \mu_0$$
$$\text{for all } i \ne j, n_i \in \{1, \dots, N\}, \ i,j = 1, \dots, K.$$

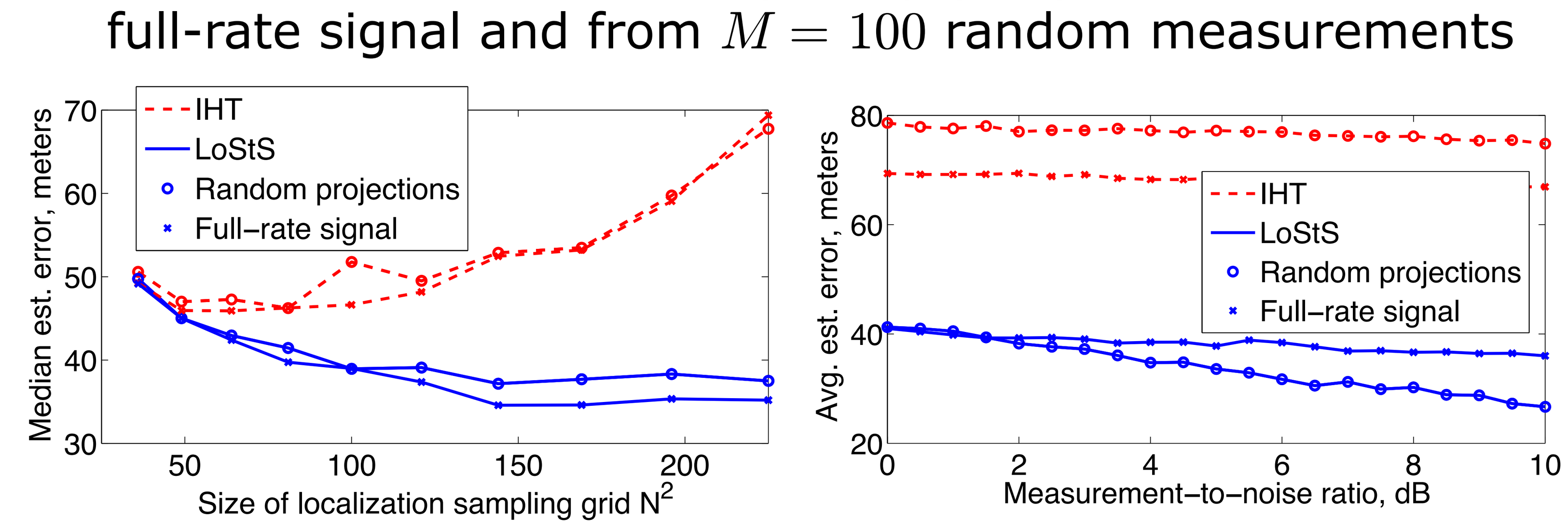
- This coherence-inhibiting structured sparsity model shows a ***connection*** between localization resolution and sparse approximation
- For this model, structured thresholding can be performed via ***linear programming*** or approximated via ***heuristics***

[Baraniuk, Cevher, Duarte, and Hegde 2010][Duarte and Baraniuk 2011]

Numerical Results

Localization via Structured Sparsity (LoStS) Simulation:

- Randomly deployed $K = 5$ sources and $P = 20$ microphones
- Regular grid of $\sqrt{N} \times \sqrt{N}$ locations in 340×340 m field
- Sources transmit MSK-modulated binary sequence
- Localization via standard sparsity (IHT) and LoStS from full-rate signal and from $M = 100$ random measurements



Bearing Estimation via Structured Sparsity (BEStS) Simulation:

- Randomly deployed $K = 5$ far-field sources
- Linear array of $P = 10$ microphone sensors, 25 cm spacing
- Regular grid of N bearings/angles in range [0,360]
- Sources transmit beacon at 500 kHz frequency
- Bearing estimation via IHT and BEStS from full-rate signal and from $M = 100$ random measurements