Technical Report
Nonpoint Source Pollution Management Models for Regional Groundwater Quality Control: with Western Cape Cod Case Study

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FOR REGIONAL GROUND WATER QUALITY CONTROL:
WITH WESTERN CAPE COD CASE STUDY

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ABSTRACT

NONPOINT SOURCE POLLUTION MANAGEMENT MODELS
FOR REGIONAL GROUND WATER QUALITY CONTROL:
WITH WESTERN CAPE COD CASE STUDY

Kirk Hatfield and Richard R. Moss

Nonpoint source pollution threatens the quality of ground water nationwide. Common strategies to protect ground water quality include density limits on distributed sources, effluent quality standards for sources, and regulations on land use activities. Ground water management models have been developed to facilitate the formulation and evaluation of strategies to protect ground waters from nonpoint source pollution. Decision makers can use these models to obtain estimates of regional subsurface wasteload allocations that can in turn be used to develop desirable distributed source densities (e.g., septic tank densities) or identify land use patterns that will maintain acceptable subsurface water quality.

This report describes the development and application of several such ground water management models. These management models are constructed as linear programming optimization models. Equations from a finite difference, steady-state, two-dimensional horizontal, unconfined, advective contaminant transport model are used as part of each optimization problem constraint set. The management models were applied over the western portion of the Sole Source Aquifer of Cape Cod, Massachusetts. The modeling approach requires data normally available through state geological surveys, regional planning commissions, and the census bureau. The optimal regional nonpoint source ground water wasteload allocations are generated from this data as are resultant contaminant distributions, boundaries of critical recharge areas, and the associated water quality tradeoffs for changes in existing and proposed land use management schemes. The optimal wasteload allocations were converted to estimates of distributed source densities and land use development patterns. The results of these ground water quality management models, used in conjunction with field and planning information, provide valuable insight into the linkage between potential development scenarios and future ground water quality, thereby providing essential feedback in the planning of regional land use activities to assure consideration of ground water resource protection.
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The purpose of this report is to demonstrate how a class of mathematical models, known as ground water management models, can be used to evaluate the long term ground water impacts of alternative development patterns. In particular, the models predict the ground water contamination resulting from the nonpoint source pollution associated with these land surface activities. Examples of nonpoint source pollution are lawn fertilizers, large scale use of septic tank-leaching field systems, and distributed leakage from sewer pipes. All of these activities result in nitrate pollution of the groundwater.

The long term impacts of nonpoint source pollution are significant because the ground water systems receiving contamination
from the various land use activities are at the same time serving as sources of drinking water, either for on-lot wells or for centralized municipal systems. Thus current decisions on land surface activities must take into account the long term impacts of such activities on ground water. If the ground water resources are expected to continue to function as drinking water sources, then some management of the land surface activities may be necessary to ensure that desired ground water quality standards are met over the long run. This is a particularly difficult problem on two accounts:

1) management decisions must be made now even though the total impacts of these decisions (in terms of nonpoint source pollution of the aquifer) will not be fully evidenced for decades due to the long response time of the ground water,

2) managing the polluting activities requires that a feedback loop be added to the usual cause and effect analysis framework so that the land use activities being managed can be limited to levels which will not violate desired future water quality levels.

This report distinguishes between ground water management models and ground water simulation models. Ground water simulation models represent the changes over time and space of ground water flows or chemical composition for an aquifer under a given set of conditions (e.g., recharge, pollutant inputs, etc.). Ground water simulation
models can be used to address management issues by running multiple simulations with trial and error combinations of pollutant inputs (in this case land use patterns) and choosing the best among the range of land use scenarios evaluated. Such an approach is neither efficient nor does it give any indication of the goodness of the management strategy selected.

Ground water management models, on the other hand, incorporate the criteria for choosing "best" within the model. The ground water management model then implicitly evaluates all possible combinations of land use and generates a solution which is the best combination of the land uses in terms of the criteria and constraints used. The ground water management models developed here are optimization models which have been coupled to the output of a calibrated ground water mass transport model. These ground water management models are efficient tools for generating strategies of coordinating water supply development and subsurface disposal needs in ground water systems which function as both sources of water supply and as recipients of waste waters.

Four ground water management models are developed and applied to western Cape Cod, Massachusetts, to identify these water quality tradeoffs associated with alternate development patterns.
Background

The western Cape Cod region was chosen for the application of these ground water management models for several reasons. Most importantly, the nature of existing and future waste water disposal and water supply provision in this region typify the conflicts that arise when the same medium is used for both functions. That is, the Falmouth area must be very careful that future development does not result in excessive pollution of the subsurface aquifer upon which they rely for their water supply (both private and public). A major management issue in guiding future development is where to supply municipal sewers and where to provide municipal water (and conversely where to rely on on-lot waste water disposal and where to rely on private wells).

The second major reason for applying the ground water management models to western Cape Cod is that the Cape Cod aquifer has been relatively well studied. Thus the data exist to develop and calibrate the hydraulic and mass transport models upon which the ground water management model is based.

The ground water management models are regional water quality models. The region was represented by a discretization into elements based on a 2 km grid spacing. All attributes of each element are
lumped, or averaged, over the 2 km by 2 km square of land represented by that element.

Data were developed for pollutant inputs (nitrate nitrogen) based on per capita loadings from commercial and domestic on-site disposal systems, or sewers, and lawn fertilizers. In addition, factors were developed to describe recharge to the aquifer depending on the source of drinking water (on-site wells or municipal wells) and the method of waste disposal (on-site disposal or sewers).

The models then traced out future development scenarios by adding population to each element in accordance with pre-specified rules (objectives) and constraints. Note that adding population also adds pollutants according to the per capita pollutant loading and recharge rates.

A typical optimization might be to maximize the population of the region modeled subject to the constraint that no element's nitrate nitrogen concentration could exceed 5.0 mg/L. The ground water management model solution would then be the total regional population, the population of each element (i.e., how the total regional population is distributed), and the (average) nitrate concentration in each element. Another variation could be to require that 40,000 persons be located in the region and use the ground water management model to allocate those inhabitants over the model's
elements so that the cumulative nitrate concentration is minimized. In the applications described in the report the constraints imposed on the models were fine-tuned to recognize existing facilities such as municipal water supply wells, waste water treatment plants, sewer and water service areas, nondevelopable land, and existing population.

The Ground Water Management Models

The objective of these management models is to relate regional nonpoint source pollutant loadings to regional water quality. Thus the models' strength is in explaining the relative impacts of alternative development patterns (i.e., alternative placement and strength of sources of pollutants) and at evaluating areally averaged water quality (assuming steady state conditions are achieved). The concentration at a specific well in a given element may be greater or less than the predicted concentration depending on the location of the well relative to the local sources of pollution. The model's predicted concentration indicates that if a given well in an element has a pollutant concentration greater than that predicted, then there must be another well (i.e., alternate well location or screen depth) within that element which will produce water with a concentration less than that predicted. (This is because the regional contaminant predictions represent average elemental contaminant levels.)
Model I

The first ground water management model projects the long term nitrate nitrogen concentration from the 1980 development pattern in Bourne and Falmouth. The projections are based on nitrogen loadings from septic systems, lawn fertilizers, leaky sewers, background loads, and subsurface recharge from municipal sewage treatment plants.

The results of model I are presented in Figure ES-1 (Figure 24 in the text). The model predicts the steady state concentration of nitrate nitrogen in each element due to existing 1980 development. These values were interpolated to produce the contours of equal nitrate concentration drawn in the figure. For instance, high nitrate concentrations are observable down gradient from the Otis Waste Water Treatment Plant and along the south coast (due to existing heavy development there). The nitrate nitrogen in elements containing municipal wells was predicted to be below 1.5 mg/L.

Model I was also used to assess the impact of additional development in each of the model's elements on overall water quality. These solutions showed that certain regions of the area studied (especially areas occupied by the military reservation) are more critical than others to the preservation of good regional water quality.
The second ground water management model is designed to look at the impact of surrounding development on nitrate concentrations at specific points. Model II was used to examine the water quality changes at Falmouth's Long Pond municipal water supply wells that would result from incremental development in the surrounding elements.

The iso-water quality impact plots developed from Model II can be used to identify critical recharge zones: areas most important to the long term preservation of ground water quality at target elements. (See Figure ES-2. This is Figure 26 in the text.) In this case the elements containing Falmouth's municipal water supply (Long Pond) were chosen as the target elements. As expected, the implication of the Model II results is that activities situated close to the pond have the greatest impact on water quality at the pond. The isopleths give an indication of the extent of areas having significant water quality impact (note the increased areal extent of the contours in the upgradient direction). In addition, the isopleths define the relative significance of separate zones within the recharge area around Long Pond which are critical to the preservation of water quality at the pond. The water quality impacts of placing sources within the region containing a municipal water supply can be evaluated in terms of the approximate water quality impacts on target elements. The actual long term water quality at
municipal wells could be more or less than the elemental nitrate concentration shown depending on the actual positions of the wells relative to the pollutant sources (i.e., the model's resolution is limited by the 2 km grid spacing used).

Model III
Model III was developed to determine the maximum feasible development, given restrictions on available resources (i.e., land and water), imposed water quality standards, and specified land use density regulations. The pattern of surface activities at optimality incorporate the present pattern of land use and development.

Model results include: 1) contour plots of the steady-state regional ground water nitrate distribution from the maximum potential residential/commercial development; 2) maximum feasible population predictions for each element, and for each land use type in each element; 3) maps illustrating optimal locations of development; and 4) maps showing which numerical elements have land use densities approaching zoning restrictions and which elements have predicted nitrate concentrations on the verge of violating specified standards.

The application of Model III to Falmouth, Massachusetts illustrated the successful identification of optimal residential/commercial development patterns which incorporate existing development, accommodate maximum population growth, preserve
water quality within standards, satisfy source density regulations, and operate within available resource limits. Optimal growth patterns varied with regard to type, location and density of the land use activities developed. Under conditions where global water quality constraints were nonbinding development approached maximum feasible uniformity. Alternatively, when global water quality constraints were effectively constraining development, they operated to restrict development in the interior while growth along the coast reached maximum allowable levels.

The nondegradation constraints defined zones where additional development was unacceptable. These constraints were a dominant factor in the design of optimal development patterns. Outside this 'zero growth zone' expanding development is determined by global water quality constraints and source density constraints.

Relaxing the global water quality standard increased the real assimilative capacity of the aquifer and, as a result, land use activity expanded to fill the increased capacity. For a given set of nondegradation constraints and a given source density limit there is a minimum global standard above which the optimal development pattern is no longer defined by binding global water quality constraints. For Falmouth this level was 8 mg/l nitrate nitrogen under a maximum source density limit of 500 houses/km², and approximately 5 mg/L for a density of 200 houses/km².
There exists a land use density for a specified ground water quality standard above which the development changes from as uniform as possible to a nonuniform pattern where development opportunity is determined by global and land use density constraints. Use of stringent density constraints yields lower regional contaminant concentrations, more uniform development opportunities, but lower maximum feasible growth. Higher density limits generate, reduced average water quality, nonuniform development opportunity, but higher feasible population growth.

Model IV

The fourth ground water management model is formulated to ascertain patterns to expand development (i.e., population), such that the resultant ground water quality impacts are minimized. The optimum pattern and combination of surface activities incorporates the present pattern of land use development and is identified from a specified population projection, stated development restrictions around municipal water supplies, given restrictions on available resources (i.e., land and water), imposed water quality standards, and specified housing density regulations. The specified population projection is the anticipated development level at some future time; it represents the minimum amount of growth which must be included in the study region.
Model results include: 1) contour plots of the minimum feasible steady-state regional ground water nitrate distribution from the projected residential/commercial development; 2) the optimal population predictions for each element and for each land use; 3) maps illustrating optimum locations for growth; and 4) maps showing the degree of development and the long term status of water quality in each element.

Application of Model IV to Falmouth, Massachusetts demonstrated the identification of feasible development scenarios which can accommodate specified population increases with minimal additional ground water degradation. The feasible development scenarios prevent additional development in elements containing municipal supplies, but allow development elsewhere as long as ground water quality remains within global and nondegradation standards, present development is left intact, and all development densities fall within zoning limits.

The optimal pattern of growth which leads to lower changes in average ground water quality is one that concentrates sources near the discharge areas. The water quality advantages of coastal development (over interior growth) were elucidated in the results from Model I. In the several model runs under small population projections development was curtailed primarily by density constraints. For higher development projections the minimum ground water impact pattern for residential/commercial growth was determined

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by global water quality, nondegradation and land use density constraints.

Comparisons were made of average nitrate nitrogen levels obtained from Models III and IV for equivalent regional populations but different prevailing source density regulation. The results of the comparisons give two members of the set of feasible development patterns which can accommodate the same population but have differing impacts on ground water quality. Used in combination Models III and IV could identify many feasible development scenarios.

CONCLUSIONS

1) The models developed in this work characterize where and to what extent future nonpoint source ground water nitrate pollution should be controlled in a study area in order to preserve regional ground water quality at specified levels.

2) Models III and IV identified regions where meeting land use density limits and water quality standards would be difficult if the optimum development pattern was pursued. Postoptimal analysis revealed the development and water quality tradeoffs of relaxing land use density limits and water quality standards.
3) The management model perceives differences between land use development alternatives as differences in unit mass loadings. Under water quality limited conditions the model selects for the land surface activities which generate the lowest contaminant loads.

4) Where nondegradation conditions exist within the system of regional ground water flow, feasible development patterns included areas where potential growth was precluded. These zero-growth areas often extended upgradient from the protected waters.

5) Use of stringent density constraints can yield lower regional contaminant concentrations, more uniform development opportunities, but lower maximum feasible growth. Higher density limits generate higher average contaminant levels, nonuniform development opportunity, and higher feasible population growth.

6) For a given population, different combinations of water quality standards and land use density restriction can lead to different development patterns which in turn effect different regional ground water quality impacts.
7) For a given population, uniform development brought about from stringent land use density limits can lead to higher regional contaminant levels than nonuniform development with relaxed source density limits.

8) Development in discharge zones is preferable over growth in recharge zones both from the perspective of maximum achievable growth and from the perspective of preserving ground water resources.

RECOMMENDATIONS

1) Coupled with thorough geologic work, the above models could be applied to better define the relative significance of ground water protection efforts in separate zones within recharge areas around water supplies.

2) Clearly there are many feasible development scenarios for a given population. Elucidating the noninferior set of development patterns could be achieved by further work with these models with specifications of different development and water quality objectives.
3) More land use activities could be included in the model. This would test the optimality of development scenarios identified with simple models which may not incorporate all the complexities of the planning process.

4) Rarely do multiple land use activities produce one contaminant that affects ground water quality. Future research with these models could address ground water protection from nonpoint source pollution involving multiple contaminants (i.e., multi-objective analysis).
CHAPTER 1

INTRODUCTION

The purpose of this report is to demonstrate how a class of mathematical models, known as ground water management models, can be used to evaluate the long term ground water impacts of alternative development patterns. In particular, the models predict the ground water contamination resulting from the nonpoint source pollution associated with these land surface activities. Examples of nonpoint source pollution are lawn fertilizers, large scale use of septic tank-leaching field systems, and distributed leakage from sewer pipes. All of these activities result in nitrate nitrogen pollution of the ground water.

The long term impacts of nonpoint source pollution are significant because the ground water systems receiving contamination from the various land use activities are at the same time serving as sources of drinking water, either for on-lot wells or for centralized municipal systems. Thus current decisions on land surface activities must take into account the long term impacts of such activities on ground water. If the ground water resources are expected to continue to function as drinking water sources, then some management of the land surface activities may be necessary to ensure that desired ground water quality standards are met over the long run. This is a particularly difficult problem on two accounts:

1) management decisions must be made now even though the total impacts of these decisions (in terms of nonpoint source pollution of the aquifer) will not be fully evidenced for decades due to the long response time of the ground water,
2) managing the polluting activities requires that a feedback loop be added to the usual cause and effect analysis framework so that the land use activities being managed can be limited to levels which will not violate desired future water quality levels.

These difficulties can be overcome with existing simulation models (Robson and Saulnier, 1981, Konikow and Bredehoeft, 1974, Gelhar and Wilson, 1974, Mercado, 1976), but to do so would require multiple simulations with trial and error combinations of various land use patterns until a satisfactory land use management strategy is found. Such an approach is neither efficient nor does it give any indication of the goodness of the management strategy selected.
This report presents several steady-state regional ground water quality management models. These management models are optimization models, variations on a technique known as linear programming. A later section of this report will explain the management information that linear programming results provide, as well as the limitations on the interpretation of such results.

1.1. NONPOINT SOURCE POLLUTION OF GROUND WATER

The integrity of our ground water resources is threatened on a local and regional scale by numerous point and nonpoint sources delivering an array of organic, inorganic, and biological substances to the subsurface. Unlike surface water pollution, subsurface contamination is more persistent, complex, and expensive to reverse; consequently, ground water pollution may exact lasting restrictions on water resource availability where comparable contamination of surface waters may not.

As demands for ground water increase so have the threats placed on supplies from the continual expansion of urban and other land uses. Considering the dependence on subsurface waters and the persistence of ground water contamination, nonpoint source pollution is particularly nefarious because it endangers enormous reservoirs of water and because it may take decades to see the total impact of nonpoint source pollution from existing activities, let alone from future development. Nonpoint source pollution (which includes areally distributed point sources) from septic tanks, buried pipelines and storage tanks, various agricultural activities, and highway deicing salts is creating regional ground water quality problems across the United States (Miller, DeLuca, and Tessier, 1974, and U. S. Environmental Protection Agency, 1984). The resultant ground water contamination is characterized by small contaminant concentration gradients and homogeneity of pollutant levels over large areas (Gormly and Spalding, 1979, and Robertson, 1979).

On-site waste disposal systems (septic tanks, cesspools, etc.) have been found to be the most frequent cause of ground water contamination in the United States (Perkins, 1984). Of the 27 counties across the United States which have over 50,000 on-site domestic waste disposal systems, 21 are in the eastern U.S., and 8 are in Massachusetts and Connecticut alone (U.S. Environmental Protection Agency, 1977). Furthermore, these systems are currently installed in approximately 25 percent of all newly constructed houses (Canter and Knox, 1985).

Agricultural activities are also significant contributors of nonpoint source ground water pollution. Poultry farms and intensive crop production have contaminated coastal wells in Sussex County, Delaware, where 32 percent of 210 wells sampled have nitrate levels
above the U.S. Environmental Protection Agency's drinking water standard (Ritter and Chinnsid, 1984). On Long Island, New York, the combined usage of lawn and garden fertilizers and septic systems has brought increases in nitrate over the last 30 years (Flipse et al., 1984). Despite the threat posed by nonpoint source pollution, few regulations have been implemented on the local, state, or federal level to protect ground water quality from such diffuse, areally distributed sources (Devine and Ballard, 1983).

1.2. PROTECTION OF GROUND WATER FROM NONPOINT SOURCE POLLUTION

Protection of underlying aquifers against nonpoint source pollution is intimately tied to the control of overlying land use activities. Fortunately, regulating land use activities is an expedient approach to controlling nonpoint source pollution, because most states and local governments have the necessary authority and institutional structures to readily invoke land use controls. Barnstable, Massachusetts, Dade County, Florida, and the State of Connecticut are among several local, county, and state governments which have incorporated land use controls as components of larger ground water protection strategies. The focus of their strategies has been to manage land use activities in critical recharge areas through local zoning ordinances, the purchase of land or development rights, state-wide ground water classification systems, source performance standards, and source design standards. Various land use controls have used permitting systems to prohibit the placement of selected nonpoint sources in sensitive recharge areas or to regulate contaminant loading through source density restrictions or effluent quality requirements.

A strategy for protecting ground water from nonpoint source pollution must deliver protection where protection is most needed and to a degree which ensures the long term preservation of ground water availability. Protection requires knowledge of the boundaries of critical recharge areas and the relationships defining the impacts of land use activities on local and distant ground water quality. Unless it is determined where and to what extent ground water protection is needed, the effectiveness of existing and proposed regulations remains uncertain.

1.3. RESEARCH OBJECTIVES

Nonpoint source pollution ground water quality management models can facilitate formulation, evaluation, application, and justification of ground water protection strategies incorporating land use controls and water quality goals. To this end the overall objectives of this research were:
1) To develop regional ground water quality management models to:
- evaluate the long term water quality impacts of nonpoint source pollution,
- delimit boundaries of critical recharge zones,
- estimate regional nonpoint source ground water wasteload allocations,
- determine the optimum pattern of land use development over a region given a development objective and the constraints to be imposed on that development (for instance, a model that will facilitate maximum growth potential but minimize deleterious ground water quality impacts).

2) To calibrate the management models to a region now experiencing a developing nonpoint source pollution problem.

3) To generate insight into how existing hydrologic and distributed anthropogenic stresses affect regional ground water quality.

The material presented in the following chapters is arranged to present a lucid picture of the development and application of ground water quality management models. The general field of ground water quality management modeling is reviewed in Chapter 2. Chapter 3 presents a conceptual picture of an aquifer receiving sustained nonpoint source contamination. The nature of information available through management models is discussed in terms of a two dimensional horizontal aquifer. The general components of a management model constructed as a linear program are presented. Chapter 4 describes the ground water flow and contaminant transport models which form the basis for the ground water quality management models discussed in Chapters 5 - 8.

Chapters 5 through 8 are devoted to the application and construction of three management models. The ground water fluid velocity field must be defined before a ground water management model is constructed and applied for a study area. In Chapter 5 the validated ground water flow model is calibrated for a defined study area on Cape Cod, Massachusetts. All applications of the ground water quality management models are made within the hydrologically defined area of Cape Cod. The first two management models are applied in Chapter 6. The first model elucidates areas overlying an aquifer which are salient to maintaining regional ground water quality goals. A different application is made with the second model to identify critical ground water recharge zones surrounding municipal water supplies. In Chapter 7 a third management model is constructed and applied over the town of Falmouth, Massachusetts. This determines patterns of maximum residential/commercial development for a population committed to the long term application
of lawn fertilizers and use of septic systems for domestic/commercial waste disposal. Ultimate development is limited by requirements to maintain ground water nitrate concentrations within specified water quality standards and housing development within zoning restrictions. The construction and application of the last management model is accomplished in Chapter 8. This model identifies residential/commercial development patterns that effect a minimum regional impact on ground water quality. Finally, Chapter 9 reviews results and presents research conclusions and recommendations for further work in this area.
Concerns over the impacts of ground water pollution have stimulated considerable research to characterize and control the fate and transport of dissolved contaminants. Most investigators have focused on the local impacts of highly visible point sources (e.g., hazardous waste sites), while few have researched the less obvious regional ground water degradation from distributed and nonpoint sources of pollution. Ground water quality simulation models have been used in most efforts to characterize regional and local subsurface contamination. A few investigators have used simulation models or ground water quality management models (simulation models coupled to optimization models) to investigate strategies of controlling the extent and rate of ground water pollution.

Ground water quality simulation models approximate changes in water quality through the separate mathematical description of fluid flow and solute transport. A deterministic ground water quality simulation model is usually composed of a solute transport model coupled to a ground water flow model. The models are employed sequentially. Initially a flow model is implemented to reproduce an observed phreatic surface. Next Darcy's equation is used to translate the phreatic surface into a fluid velocity field. Finally a dissolved contaminant mass transport model is exercised over the defined fluid velocity field to estimate solute concentrations in ground water.

Two common types of deterministic mathematical ground water quality simulation models are empirical lumped parameter models and conceptual distributed parameter models. Lumped parameter modeling treats the aquifer as a single cell or compartment: spatial variations in physical, chemical and biological characteristics of the ground water system are ignored. These models yield a representative regional average contaminant concentration (with the mass transport model) and water table elevation (with the hydraulic model) for the entire aquifer system being modeled. Steady-state or temporal variations in regional average ground water quality or phreatic surface elevation are investigated.

The most common approach to regional ground water quality modeling has been through the use of distributed parameter models.
Unlike lumped parameter models, distributed parameter models attempt to reproduce observed spatial and transient variations in the phreatic surface elevation and contaminant concentrations. Distributed parameter models incorporate the physical, chemical, and biological mechanisms which induce spatial variations in fluid flow and solute transport. As such, data on the locations and magnitudes of ground water recharge and discharge, the locations and magnitudes of contaminant loading, the location of boundary conditions (e.g., impermeable boundaries), the spacewise variation of aquifer characteristics (e.g., porosity and permeability), and contaminant reactions are incorporated in the model. Selection of a lumped parameter or a distributed parameter model depends primarily on the goals of the modeling effort; however, the amount of data available and the complexity of the ground water system determines whether a chosen model can be calibrated and implemented in the field (Balek, 1983).

Predictions of regional aquifer responses to contaminant loadings have been attempted with both lumped and distributed parameter simulation models. These models have contributed significantly to the characterization of regional ground water contamination from nonpoint source pollution. In addition, these models have served directly or indirectly as components of more complex attempts to explore strategies of managing nonpoint source pollution. Examples of both the distributed and the lumped parameter ground water models are discussed in detail in the first half of the literature review.

The second half of the literature review presents several ground water quality management models. These ground water quality management models are much more efficient at evaluating ground water quality management schemes than simple simulation models. Several transient and steady-state models will be reviewed which have yielded information useful to the development of ground water quality management models for nonpoint source pollution.

2.1. LUMPED PARAMETER MODELING OF REGIONAL GROUND WATER QUALITY

One of the earliest attempts to evaluate the ground water quality impacts of nonpoint source pollution was through lumped parameter models. Here the aquifer was treated as a completely mixed compartment. Sources of pollution were assumed uniformly distributed over the region being modeled. The relative importance of contaminant sources in recharge areas versus sources near discharge areas was ignored because sources were combined and assumed to pose a uniform hazard to the subsurface environment.

Lumped parameter models are simple and easy to calibrate with a limited amount of water quality and hydrologic data; however, too few
or too many samples from highly contaminated areas may bias the representativeness of estimates of the mean regional contaminant concentrations. As tools for evaluating point source ground water pollution problems, lumped parameter models are inappropriate because major point sources can constitute local rather than regional water quality threats; hence, including major point sources can artificially elevate estimated regional contaminant levels for a system modeled as completely mixed. As a result, application of lumped parameter models to nonpoint source pollution problems has been limited to estimating transient changes in regional ground water quality.

Gelhar and Wilson (1974) proposed a lumped parameter model suitable for an aquifer bounded by a ground water divide at one end and a stream at the other. For their model they assumed a phreatic aquifer could be described as a completely mixed linear reservoir. Recharge from precipitation flowed in a path perpendicular to the stream. Contaminant concentrations were constant throughout the ground water system. Gelhar and Wilson used their model to study the transient effects of highway deicing salts on ground water in Massachusetts. Several highway deicing policies, involving different salt applications, were reviewed. Their work clearly demonstrated that years could elapse before continuous source loading would be reflected in changes in ambient ground water quality; consequently, Gelhar and Wilson concluded that ground water quality monitoring alone would not adequately reflect the true magnitude of developing ground water contamination from land use activities.

Mercado (1976) developed a lumped parameter model of a coastal aquifer underlying 87 square kilometers of agricultural land. The whole aquifer was represented mathematically as a completely mixed compartment. Contaminant concentrations at natural points of ground water discharge and at pumping wells were the same for the entire system. The model consisted of a simple water balance equation and an equation expressing the conservation of solute mass. Mercado studied the regional chloride and nitrate pollution from irrigation, fertilizers, and land application of treated waste water. The model was calibrated to reproduce an historical water quality record, and then used to evaluate 13 alternative ground water protection measures.

Gelhar and Wilson, and Mercado, found that their lumped parameter models could deliver useful information on temporal changes in regional ground water quality if sources of comparable magnitude were uniformly distributed. In addition, these investigators showed that lumped parameter models could be used to facilitate the evaluation of land use management strategies for the protection of ground water quality from nonpoint source pollution. However, spatial heterogeneities in the intensity of nonpoint source pollution exist in the field. In such cases lumped parameter models may result
in erroneous conclusions. This is because the lack of spatial resolution may preclude determination of where and to what extent ground water protection should be implemented. Mercado, and Gelhar and Wilson, employed their models to investigate the merits of alternative regional nonpoint source ground water protection strategies. However, spatial variations of nonpoint source pollution intensity were surpressed, leaving open the possibility that their models could have obscured the true merits of some of the strategies.

2.2. DISTRIBUTED PARAMETER MODELING OF REGIONAL GROUND WATER QUALITY

Unlike lumped parameter models, which yield steady-state or transient changes in the regional average phreatic surface elevation and contaminant concentration, distributed parameter models attempt to reproduce observed spatial and temporal or just spatial changes. These models are conceptual in that they are predicated on differential equations descriptive of the conservation of mass, energy, and momentum. Distributed parameter models are used if a more detailed or more realistic depiction of the ground water system is desired. As in the lumped parameter models, the components of distributed parameter models are mathematical descriptions of ground water flow and contaminant transport. Because distributed parameter models attempt to reproduce that which is observed in both space and time, the equations for solute transport and fluid flow are considerably more complex than the simple mass balance equations used in the lumped parameter models.

2.2.1. Governing Equations of Distributed Parameter Ground Water Flow and Mass Transport Models

In most cases the aquifers being modeled are characterized by natural spatial heterogeneities in the physical and chemical characteristics (e.g., porosity, permeability, chemical adsorptive capacity) of the porous medium which determine the ease of fluid flow and contaminant movement in different directions through the solid matrix. An isotropic medium allows fluid to flow with equal ease in all directions, while an anisotropic medium exhibits directional variability in the properties of the solid matrix to transmit water. If vertical movement of water is minor, equation (1) describes the transient, two-dimensional, areal flow of a homogeneous fluid through an unconfined, horizontal, nonhomogeneous, anisotropic aquifer (Pinder and Bredehoef, 1968).

\[
\frac{\partial}{\partial x_1} \left( K_{i,j} \frac{\partial h}{\partial x_1} \right) = S_y \frac{\partial h}{\partial t} + Q_{out} - Q_{in} \quad i,j=1,2
\]
where $K_{i,j}$ = vertically averaged hydraulic conductivity tensor which is a physical parameter indicating the ease with which water passes through porous material in the direction $i,j$ (L/t),

$S_y$ = vertically averaged specific yield (dimensionless), which physically corresponds to the percent of saturated porosity which drains under the force of gravity,

$h$ = hydraulic head (L),

$B$ = saturated thickness which may equal $h$ for an unconfined aquifer with a horizontal impermeable bottom boundary (L),

$Q_{in}$ = volumetric flux of recharge per unit surface area of aquifer (L/t),

$Q_{out}$ = volumetric flux of withdrawal per unit surface area of aquifer (L/t),

$x_i, x_j$ = horizontal coordinate axis $i,j$ (L),

$L$ = length,

$t$ = time.

The unconfined ground water flow equation is solved analytically or numerically to reproduce observed areal and transient changes in the water table elevation. Several numerical models have been developed by Prickett and Lonnquist (1971), Trescott, Pinder, and Larson (1976), and others. Assuming the flow equation has been solved by whatever method is deemed appropriate, Darcy's equation is used to translate spatial variations in the phreatic surface elevations, $h$, into a fluid velocity field for two-dimensional, horizontal ground water flow. Darcy's equation is written as:

$$v_i = K_{i,j} \left( \frac{\partial h}{\partial x_j} \right) \quad i,j=1,2$$

(2)

where $v_i$ = the vertically averaged specific discharge (L/t) or the mass average flux of fluid flow in the $i$ direction, and $K_{i,j}$, $h$, $i$, $j$, and $x_j$ are defined above.

Once the velocity field is defined a third and final equation is solved to estimate solute concentrations in space and time. Equation (3) describes transient, two-dimensional areal, advective-dispersive transport of a miscible contaminant through an unconfined, horizontal, nonhomogeneous, anisotropic aquifer (Konikow and Bredehoeft, 1978).
\[
\frac{\partial}{\partial x_i} \left( B D_{i,j} \frac{\partial C}{\partial x_j} \right) - \frac{\partial}{\partial x_i} \left( B V_{i} C \right) =
\]
\[
  n_{e} \left( \frac{\partial (BC)}{\partial t} \right) - Q_{in}^C + Q_{out}^C + nB_{i}^{s} v_{k} \]

\[i,j = 1,2 \quad (3)\]

where \( C \) = the vertically averaged concentration of dissolved chemical species (M/L^3),
\( D_{i,j} \) = vertically averaged coefficient of hydrodynamic dispersion (L^2/t) which is descriptive of the combined effects of Fickian diffusion and dispersion caused by microscopic variations in fluid velocities within individual pores,
\( n_{e} \) = vertically averaged effective porosity (dimensionless),
\( C^{p} \) = concentration of dissolved chemical at the source (M/L^3),
\( V_{k} \) = chemical, biological, or physical reaction k, negative for the addition of solute and positive for the removal of solute (M/L^3 t),
\( s \) = total number of possible contaminant reactions,
\( x_{i}, x_{j} \) = horizontal coordinate axis i,j (L),
\( L \) = length,
\( t \) = time.
\( v_{i}, h, B, Q_{in}, \) and \( Q_{out} \) are defined above.

The above equation for advective-dispersive contaminant transport has been solved numerically (e.g., Konikow, 1977) and analytically (van Genuchten and Alves, 1982).

The first term in equation (3) approximates contaminant transport due to hydrodynamic dispersion. Dispersion is important wherever steep concentration gradients occur in the ground water system, such as along the edge of a plume. The dispersion coefficients are often described in the literature as functions of the fluid velocity and the longitudinal and lateral dispersivity coefficients (Anderson, 1979 and Mercer and Faust, 1981). Dispersivity coefficients operate as convenient calibration parameters. The effects of dispersive transport are difficult to replicate in simulation without considerable data to calibrate the dispersivity coefficients.

The second term in the solute transport equation describes advective transport. If it is assumed that dispersion can be
ignored, the first term appearing in equation (3) is dropped leaving a simple equation for two-dimensional horizontal advective transport of a miscible contaminant.

2.2.2. Numerical Approximation of the Ground Water Flow and Mass Transport Models

Unless the ground water flow field is simple or predefined, a distributed parameter model for simulating transient or steady-state changes of ground water quality in space contains three governing equations: a ground water flow equation, Darcy's equation to translate the results from the flow model into a constant velocity field, and a contaminant transport equation. Numerical, as opposed to analytical, solutions to the governing equations have permitted modellers to evaluate more complex transient and steady-state ground water quality problems involving multiple sources and boundary conditions in two and three dimensional flow regimes in anisotropic aquifers.

To obtain a numerical solution for a steady-state two-dimensional horizontal ground water flow model, the aquifer is first discretized into elements. Within each element aquifer characteristics are defined and assumed to be spacewise constant. From the partial differential equation for ground water flow an algebraic equation is derived (by way of finite difference approximations of the partials) for each element. Each algebraic equation defines a mathematical relationship describing the water table elevation in the center of each element in terms of the water table elevations in neighboring elements. An aquifer discretized into 100 elements will yield 100 algebraic equations written in terms of 100 variables corresponding to the discrete water table elevation in the center of every element in the aquifer domain. The set of 100 equations which numerically approximate the ground water flow equation over a defined aquifer can be written in the form:

\[
\begin{align*}
    r_{1,1}h_1 + r_{1,2}h_2 + r_{1,3}h_3 + \cdots + r_{1,n}h_n &= Q_{\text{out}1} - Q_{\text{in}1} + R^b_1 \\
    r_{2,1}h_1 + r_{2,2}h_2 + r_{2,3}h_3 + \cdots + r_{2,n}h_n &= Q_{\text{out}2} - Q_{\text{in}2} + R^b_2 \\
    \vdots & \quad \vdots \\
    r_{n,1}h_1 + r_{n,2}h_2 + r_{n,3}h_3 + \cdots + r_{n,n}h_n &= Q_{\text{out}n} - Q_{\text{in}n} + R^b_n
\end{align*}
\]

where: $h_i$ = variable corresponding to the hydraulic head in element $i$, (L).
\[ r_{i,j} \] the combined parameters generated from the algebraic approximation of the governing ground water flow equation at given element \( i \) in terms of neighboring element \( j \), for \( i,j = 1, \ldots, n \), \((1/t)\),

\[ Q_{\text{out}i} \] assumed volumetric flux of withdrawal from element \( i \), \((L/t)\),

\[ Q_{\text{ini}} \] assumed volumetric flux of recharge into element \( i \), \((L/t)\),

\[ R^b_i \] known boundary conditions in element \( i \) (e.g., constant flux conditions), \((L/t)\),

\( n \) number of elements,

Using vector notation:

\[ [R][h] + [I][Q_{\text{out}}] - [I][Q_{\text{in}}] = [R^b] \tag{5} \]

where \( [h] \) = \( n \times 1 \) vector of variables corresponding to the hydraulic heads at every element,

\( [R] \) = \( n \times n \) vector of coefficients generated from the algebraic approximation of the governing flow equation,

\( [I] \) = \( n \times n \) identity matrix,

\( \{Q_{\text{out}}\} \) = \( n \times 1 \) vector of assumed or known pumping or withdrawal rates,

\( \{Q_{\text{in}}\} \) = \( n \times 1 \) vector of assumed or known recharge rates,

\( \{R^b\} \) = \( n \times 1 \) vector of known boundary conditions.

A solution to the expanded steady-state ground water flow model is obtained by a simultaneous solution of the 100 algebraic equations. If a numerical solution were desired for the above problem but for transient conditions, then a solution for the 100 algebraic equations would have to be obtained for each time step in a series of steps taken over the desired time period. In observed aquifer behavior the magnitudes of \( Q_{\text{ini}} \), \( Q_{\text{out}} \), and \( R^b \) would vary in time; in the simulation the values of these terms would be specified and held constant for each time step.

A numerical solution to the steady-state two-dimensional horizontal advective-dispersive contaminant transport equation is obtained in a manner similar to the ground water flow equation, yielding a set of 100 algebraic equations corresponding to the same 100 discrete elements in the hypothetical aquifer.
where $C_i$ = concentration of contaminant at node $i$, (M/L$^3$),
e$_{i,j}$ = the combined parameters generated from the algebraic
approximation of the governing mass transport model
at given element $i$ in terms of neighboring elements
$j$, for $i,j=1,\ldots,n$, (L/t),
$C_i^P$ = concentration of waste injected in element $i$, (M/L$^3$),
b$_i$ = known boundary conditions for element $i$ (e.g.,
contributions of contaminant through natural
sources), (M/L$^2$t), and
$Q_{ini}$ is defined above.

Using vector notation:

$$[e]C = -[I][Q_{ini}C^P] + \{b\}$$

where $[C]$ = n x 1 vector of variables corresponding to the
concentration of contaminant in every element,
$[e]$ = n x n vector of coefficients generated from the
algebraic approximation of the governing mass
transport equation,
$[I]$ = n x n identity matrix,
$[Q_{ini}C^P]$ = n x 1 vector of known waste injection fluxes,
$\{b\}$ = n x 1 vector of known boundary conditions.

Again a solution to the expanded steady-state mass transport model is
obtained by simultaneous solution of the 100 algebraic equations.

The validity of simulation results depends on the extent of
model calibration attainable with existing data. Calibration of
distributed parameter ground water quality models is a process of
adjusting parameters (e.g., porosities, hydraulic conductivities,
storage coefficients, dispersivity coefficients, and reaction
coefficients) and boundary conditions (e.g., constant flux) in the
hydraulic and the contaminant transport models until observed phreatic surface elevations and contaminant concentrations are reproduced in the mathematical simulation. Calibration requires data from transient or steady state conditions of flow and contaminant transport. Simulation results are only as accurate as the data which describes aquifer properties, water table elevations, and contaminant concentrations in the elements of a discretized aquifer (Reddell, 1970). As the scale of modeling increases from local to regional levels, the availability of data to describe ground water quality changes decreases. In regional modeling, the elements of discretization are often increased to accommodate the sparseness of available data; to do otherwise yields detailed simulations which are generally insupportable and perhaps deceptive. Therefore, the resolution of regional simulations should be restricted to the same order of detail exhibited by available data.

2.2.3. Applications of Distributed Parameter Models

Distributed parameter models have been used primarily though not exclusively as simulation models. Many modeling efforts have focused on simulating ground water pollution under transient conditions. Models have been employed to trace the movement of contaminant plumes (Konikow, 1977 and Dasqupta et al., 1981), explain historical changes in ground water quality (Bredehoeft and Pinder, 1973), and predict transient ground water quality impacts of various land use activities (Robson and Saulnier, 1981 and Konikow and Bredehoeft, 1974).

Several examples of ground water simulation modeling can be cited. Most of these studies address ground water problems originating from point source pollution. Only a few have detailed water quality implications of nonpoint source pollution. Robson and Saulnier (1981), used a 3-dimensional distributed parameter advective mass transport model coupled to a 3-dimensional flow model to simulate the potential transient nonpoint source water quality impacts of dewatering operations at a proposed oil shale mine in northwestern Colorado. They predicted that changes in ground water flows induced from mine dewatering operations would alter the chemical quality of ground and surface waters in the area. Konikow and Bredehoeft (1974) studied the effects of irrigation practices and strategies on the distribution of dissolved solids in an alluvial aquifer in Colorado where serious nonpoint source pollution has resulted from a long history of crop irrigation.

Other examples of the use of ground water quality simulation models to characterize or evaluate schemes of controlling nonpoint source pollution can be presented; however, more effective use of simulation models has been with their application in ground water quality management models as direct or indirect optimization models.
2.3. GROUND WATER QUALITY MANAGEMENT MODELS

Ground water quality management models are optimization models which have been coupled to the output of a calibrated mass transport model. These models can operate as efficient tools for generating strategies of coordinating water supply demands and subsurface disposal needs in ground water systems which function as both sources of water supply and receptacles of waste waters. Several models are reviewed below which have potential application in the formulation or evaluation of transient and steady-state schemes of managing point and nonpoint source ground water pollution. Unfortunately, because most applications of management models have been with hypothetical test aquifers, the true utility of these models remains to be demonstrated in the field.

2.3.1. Steady-state Ground Water Management Models for Point Source Pollution

Willis (1976) recognized ground water systems as multipurpose resource systems used conjunctively as sources of potable water and as sites for the treatment and disposal of wastes. He examined a hypothetical regional wastewater treatment system comprised of surface waste water treatment, imported dilution water, and the waste assimilative capacity of the underlying aquifer. Willis formulated a nonlinear mixed integer programming model to select a cost effective combination of unit wastewater treatment processes to produce an injectable effluent which would satisfy water quality constraints at the injection wells and at supply wells. The decision variables were \( Q \) (the flow rate of the treatment plant which included the initial wastewater flows and the dilution water \( D \)), \( D \) (the flow rate of the imported dilution water), plus integer variables \( x_i \) (corresponding to affirmative or negative decisions on available choices of unit treatment options \( i = 1, \ldots, 17 \)). The nonlinear cost objective function incorporated transmission costs of imported dilution water (based on pipe capacity, distance, and method of transmission), and annual treatment plant costs (based on flow). The objective function was minimized subject to linear constraints on flow capacities of treatment plants, nonlinear water quality constraints on allowable contaminant levels at injection wells and water supply wells, and finally a linear constraint limiting construction to one treatment plant.

The water quality constraints were derived from matrix manipulations of a finite difference approximation of steady-state two-dimensional horizontal advective contaminant transport model incorporating first order biochemical reactions and linear adsorption. A steady-state model was used because management decisions could be based upon the ultimate response of the ground
water systems to a policy of continuous contaminant injection and sustained demand for potable water. It was necessary to know the location and rates of ground water recharge and discharge to define in advance the contaminant transport simulation model over a desired constant velocity field (meaning that a steady-state solution to a ground water flow model was obtained external to the mixed-integer programming model so that the darcian velocity coefficients appearing in the solute transport model could be defined). Recalling from equation (7) that a numerical finite difference approximation of the two-dimensional horizontal mass transport equation is a set of algebraic equations,

\[ [e][c^e] = \{f\} \]  \hspace{1cm} (8)

where \([e]\) = \(n \times n\) finite difference coefficient matrix which is derived from the numerical discretization for which spacewise constant darcian velocity coefficients, kinetic reaction parameters, and dispersivity coefficients have been defined. However, if injection rates are unknown, this vector will contain expressions defining the darcian velocity coefficients as functions of the decision variable \(D\) (the dilution water flow rate) and the constant initial flow rate of the waste stream,

\[ \{C^e\} = n \times 1 \] column vector of decision variables corresponding to the concentrations of contaminant \(e=1,2,...,z\), for each element,

\[ \{f\} = n \times 1 \] column vector defining boundary conditions and input fluxes of contaminant \(p\) as a nonlinear function of the integer decision variable \(X_i\), the chosen treatment received before subsurface injection and the decision variable \(D\), dilution water flows,

\[ n \] = the number of elements of discretization,

Willis computed the inverse of the coefficient matrix \([e]^{-1}\) to obtain a new set of algebraic equations,

\[ \{c^e\} = [e]^{-1}\{f\} \]  \hspace{1cm} (9)

This new set of equations yielded a vector of decision variables \(\{c^e\}\) corresponding to the contaminant concentrations in each element expressed as a function of decision variables corresponding to treatment plant design \((X_i)\) and dilution water flows \((D)\). Willis selected from this new set of equations, a small subset of equations to serve as water quality constraints. This subset of equations described the concentration of constituent \(e\) in the elements.
containing water supply wells and injection wells; the remaining equations which described contaminant concentrations in the other elements were ignored.

A solution to the programming problem would yield a cost effective combination of unit processes and dilution water necessary to preserve a minimum level of water quality at injection wells and supply wells. Willis was not able to obtain a solution directly but studied each of the 17 possible wastewater treatment plant designs individually by decomposing the mixed integer model into 17 separate continuous variable optimization problems. Each of the 17 optimization problems was further simplified by ignoring the changes imposed on the subsurface hydraulics through the use of imported dilution water.

Willis' greatest contribution in this work was the development of water quality constraints from the distributed parameter model. Willis was able to obtain a linear approximation of the water quality constraints by ignoring the hydraulic effects of injecting an unknown volume of dilution water.

Gorlick and Remson (1982a) developed an efficient steady state management model for siting point source subsurface waste disposal facilities and determining the associated ground water wasteload allocations. A finite difference approximation of a steady-state two dimensional areal advective-dispersive mass transport model expressed as

\[ [e][C] + [P][M_{in}] = \{b\} \]  

(10)

where:
- \([e]\) = \(n \times n\) matrix of coefficients derived from a known constant velocity field and finite difference approximation of the mass transport equation,
- \([P]\) = \(n \times n\) diagonal matrix with values of one for entries that correspond to the injection sites and values equal to zero for all other entries,
- \([C]\) = \(n \times 1\) column vector of decision variables defining solute concentrations throughout the system,
- \([M_{in}]\) = \(n \times 1\) vector of decision variables defining the contaminant injection fluxes (each flux equal to \(Q_{in}C_{in}\), the solute concentration in the injected waste times the flow rate),
- \([b]\) = \(n \times 1\) right-hand side vector reflecting boundary conditions (i.e., existing disposal fluxes),
- \(n\) = the number of nodes or elements,

was directly embedded as part of the constraint set of a linear programming model. Decision variables corresponded to unknown
contaminant concentrations at each element (C) of a discretized aquifer and the waste disposal fluxes (\(M_{in}\)) occurring in predetermined elements. Because the solute transport model was a component of the linear programming formulation of the ground water management model, a solution to the management model led directly to simultaneous estimates of the maximum disposal fluxes and the associated steady-state response (contaminant concentrations in every element) of the ground water system to the disposal activities.

The management model required a mass transport model and a ground water flow model which were both calibrated to the aquifer being managed. Both management models, however, were tested over hypothetical aquifers; hence, selection of dispersivity coefficients, porosity, transmissivities and boundary conditions used in the hydraulic and the mass transport models was purely arbitrary. Steady-state modeling results were desired because Gorelick and Remson felt such ground water system responses often represent worst case scenarios. Although contaminant reactions were ignored, management model formulations were sufficiently general that reactions could be easily incorporated.

Before the solute transport simulation model was embedded into the optimization model, it was necessary to define darcian velocity coefficients and dispersion coefficients (recall that the dispersion coefficients are a function of velocity components) appearing in the mass transport equation; consequently, information on the location of existing waste disposal sites and water supply wells and their respective injection and withdrawal rates was needed to permit advance definition of the hydraulic regime. Finally data were needed on the concentration of contaminant in wastes at existing and proposed subsurface disposal sites to permit calculation of injection volumes from the optimum values of decision variables representing disposal fluxes.

Two different ground water management problems were presented and investigated using the model. In the first problem, a hypothetical aquifer contained one existing disposal site discharging a 1000 mg/L chloride waste at a rate of 200 l/s and two potential disposal sites upgradient from three water supply wells pumping at known rates. The management problem was to maximize waste loading to the aquifer while maintaining water quality standards (250 mg/L Cl) at water supply wells. The objective function of the first ground water quality management model was,

\[
\text{Maximize } \sum_{i=1}^{3} (\ M_{in})_i \quad \text{ (11)}
\]
where \((M_{in})_{i}\) = decision variable corresponding to the injection flux at proposed injection site \(i\),

The above objective function was subject to constraints generated from the embedded solute transport model: constraints imposing maximum allowable values on decision variables corresponding to solute concentrations at water supply wells, constraints defining existing and permissible disposal activities in the various discrete elements throughout the aquifer, and nonnegativity constraints.

A solution to the first management model led to estimates of the allowable subsurface injection fluxes at each proposed waste disposal site which did not violate ground water quality standards at supply wells. Parametric programming was used to investigate disposal tradeoffs between the existing and the proposed disposal sites. It was found that a slight reduction in the disposal flux at the existing site would permit an overall increase in the total allowable waste load delivered from all three sites.

Gorelick and Remson also addressed a different problem of siting new subsurface waste disposal facilities. The goal was to seek out sites suitable for the disposal of a liquid waste at a known constant flux. In another hypothetical aquifer, 56 potential disposal sites were identified in a delineated zone upgradient from two water supply wells. Any one site was considered suitable if a constant disposal flux of 500 g/s chloride could be delivered without violating a 250 mg/L chloride standard at either of the two water supply wells. Gorelick and Remson constrained the waste discharge fluxes for each of the 56 sites to a value of one. They then maximized contaminant concentrations first at one water supply well and then for the other. The resultant optimum values of the dual variables were interpreted as 'unit source impact multipliers' and used to predict the effect of a per unit change in the disposal flux at any of the 56 potential facility sites on water quality at each of the two supply wells. Six sites were identified where waste disposal could be conducted without endangering the potable water supply.

The ground water quality management models presented by Willis (1976) and Gorelick and Remson (1982a) were solved by different programming techniques seeking to optimize single objective functions subject to constraints on water quality and quantity, and contaminant source loading fluxes. The water quality constraints were derived directly (through an embedding technique) or indirectly (through complex matrix inversions) from steady-state areal finite difference contaminant transport models. The models developed by Gorelick and Remson generated more information and were easier to construct and solve than the model formulated by Willis.
2.3.2. Transient Ground Water Quality Management Models for Point Source Pollution

Several transient ground water quality management models have been developed for optimizing the management of aquifers conjunctively used as sources of potable water and as subsurface waste disposal systems. Most of these models have been used to evaluate the continuous transient disposal at injection wells over specified management time horizons and were derived from transient one or two-dimensional horizontal solute transport models. In these management models the constraints defined ground water quality at specified points in space and time as a function of sustained mass flux loadings or constant waste concentrations at disposal wells for specified management periods. The constraints were composed of influence coefficients descriptive of the unit change in water quality at an observation well resulting from a unit change in disposal flux at each disposal well, and decision variables corresponding to the disposal flux for each injection well at each time increment. Influence coefficients were derived from multiple simulation of the mass transport models; one simulation for each source.

In transient management models, the size of the mathematical problem is a function of the number of disposal wells and observation wells, the size of the area under management, and the length of the planning time horizon. With every model either the number of sources, the size of the aquifer, or the length of the planning horizon were curtailed to keep the size of problem sufficiently small that a solution could be obtained. Willis (1979) had to restrict the planning time horizon to 480 days because the aquifer being managed was 50 km². Gorlick and Remson (1982b) developed a one-dimensional horizontal transient management models for a confined aquifer 5 km long, but they limited their study to the management of 3 sources and 3 water supply wells and a management horizon of 600 days. In a more complex two-dimensional model with seven injection wells and eight observation wells, Gorelick (1982) studied the transient disposal policies for a nine year planning horizon over a small hypothetical aquifer of 2 km².

One model (Louie, Yeh, and Hsu, 1984) was formulated to view ground water quality impacts from multiple sources over a large basin at only specified times and not continuously as in the models above; consequently the complexity of the model does not expand as rapidly with increases in the aquifer managed, the number of injection wells, or the management horizon. The utility of transient management models for evaluating the optimal control of nonpoint source pollution is limited in light of the regional nature of the ground water quality modeling problem, the large number of sources, and the fact that long term analysis is needed since many sources are by
nature semipermanent (e.g., agricultural activities and septic systems).

2.3.3. Steady-state Ground Water Quality Management Models for Nonpoint Source Pollution

No steady-state management models for nonpoint source pollution have been found in the literature; however, several reasons can be stated as to why steady-state modeling is particularly suited for deriving management schemes to protect ground water from nonpoint source pollution. Consider first, that many sources contributing to distributed ground water contamination are semi-permanent or are expected to persist for prolonged but indefinite periods of time (e.g., agricultural activities, on-site septic systems, etc.); consequently, resource protection through transient controls (structural and nonstructural) would not be as reliable or as easy to enforce as long term restrictions on the presence or intensity of various activities in critical recharge areas (Devine and Ballard, 1983). Secondly, contaminated aquifers are slow to show the full impact of continuous discharges and are also slow to recover, both of which necessitate long term water resource planning. Finally, steady-state conditions often represent the worst case pollution scenario, which makes consideration of the long term water quality impacts a conservative approach.

Steady-state management models can readily operate as efficient tools to evaluate the long term water quality impacts of nonpoint source pollution, estimate subsurface waste assimilative capacity, and determine the optimum pattern of long term land use and development that minimizes the degradation of subsurface water quality. The development and demonstration of steady-state models for the management of nonpoint source ground water pollution will be seen in Chapter VI, VII, and VIII.

2.3.4. Transient Ground Water Quality Management Models for Nonpoint Source Pollution

The ground water quality management models discussed thus far have directly or indirectly incorporated the response surface generated from distributed parameter models of ground water flow and contaminant transport. These management models were used to examine local (Gorelick and Remson, 1982) and regional (Willis, 1976) ground water contamination from point source pollution alone and then only through hypothetical case problems. Helweg and Labadie (1976 and 1977) were among the first investigators to develop and apply a distributed parameter ground water quality management model for nonpoint source pollution. Working on a nonpoint source pollution problem in the Bonsall Subbasin of the San Luis Rey River basin in
California, Helweg and Labadie sought to manage ground water salinity levels by controlling the distribution of waters pumped from various wells (sources) possessing different water quality to various irrigated fields (destinations) having different underlying ground water quality.

More recently, Mooseburner and Wood (1980) formulated a ground water quality management model to identify land use patterns which would minimize impacts on ground water quality. Information from the model could be used to design land use regulations to control or prevent ground water degradation. The management model incorporated the response surface of a transient two-dimensional horizontal analytical mass transport model (Cleary, 1978) into a multiobjective goal programming optimization model. Mooseburner and Wood were particularly interested in the impacts of a rapidly growing residential population on ground water quality; consequently, their management model was used to identify patterns of unsewered residential development which would permit the attainment of desired ground water quality goals.

Jackson Township of the New Jersey Pine Barrens was chosen as the study site because 50 percent of the existing homes use septic systems and because the population is expected to quadruple between the years 1970 and 2000. On-site domestic waste disposal systems deliver nitrates to the underlying aquifer. This poses a health hazard to the growing population of Jackson Township which depends on the aquifer as a source of potable water.

Based on present and projected land use patterns, 17 discrete land use sectors were found within the 100 square mile area of Jackson Township. Spaced between the 17 identifiable land use sectors were several equally large parcels of land which were ignored in the investigation because they were sparsely populated or were expected to exhibit little or no population growth.

The intensity of pollution in one sector affects water quality in other sectors. The 17 defined land use sectors, were treated as sources of nitrate, generated from undetermined populations of unsewered residents. A transient two dimensional areal analytical mass transport model (Cleary, 1978) was used to determine "transfer coefficients" $T_{i,j}$ which define the associated change in nitrate concentrations at surveillance point $j$ ($j=1,2,...,n$) resulting from per unit change in the concentration of nitrates from the unsewered population ($X_i$) in land use sector $i$ ($i=1,2,...,n$). The physical location of surveillance points corresponded to the population centers of land use sectors. To use Cleary's analytical solution, pollution from land use sectors was posed as contaminant plumes originating from the population centers of offending land use.
sectors; within the boundaries of offending land use sectors, contaminant was distributed in a Gaussian fashion along the horizontal axis perpendicular to the direction of ground water flow. The analytical mass transport model incorporated first order decay and was calibrated for the area under investigation using a constant and uniform flow field and constant dispersion coefficients.

Decision variables used in the management model were $X_i$ (the unsewered population residing in each land use sector $i$), $d_j^+$ and $d_j^-$ (the positive deviation and the negative deviation of simulated nitrate levels at each surveillance point from desired goals), and $C_j$ (the total nitrate concentration at surveillance point $j$). The objective function was composed from a summation of decision variables corresponding to the positive deviations of simulated nitrate concentrations above surveillance point goals.

$$\text{Min} \sum_{j=1}^{m} d_j^+$$  \hspace{1cm} (12)

Knowing the projected population growth and the desired water quality goals at each surveillance point, the model identified the optimum pattern of unsewered population growth which would minimize the positive deviation of the resultant nitrate levels from the specified goals. The objective function was minimized subject to contaminant mass balance constraints, land use sector population constraints, regional population constraints and water quality constraints.

Two types of mass balance constraints were used to couple the multiobjective goal programming optimization model to the response surface of the solute transport model. Equation (13) represents the first type of constraint in which the total concentration of nitrate at any surveillance point $j$ ($C_j$) was determined using linear superposition and was defined simply as the background concentration ($C_0$) plus the summation of appropriate transfer coefficients ($T_{i,j}^r$) from all other sectors $i$ multiplied by the contributed concentration of nitrates resulting from the unsewered population in each sector. Nitrate concentrations contributed from septic systems in each sector $i$ were expressed as function the decision variable $X_i$, the unsewered population in sector $i$.

$$\sum_{i=1}^{n} X_i F_i^{-1} T_{i,j}^r + C_o = C_j \quad \text{for } j=1,2,...,m;$$  \hspace{1cm} (13)
where $X_i$ = the decision variable for the unsewered population of land use sector $i$; $i=1,2,...n$,

$R$ = nitrate production rate (mg/person-day),

$F_i$ = flow rate of ground water at land use section $i$ (l/day),

$T_{i,j}^r$ = transfer coefficient defining the ratio of the resultant concentration (at year 2000) at surveillance point $j$ to the peak concentration at land use sector $i$,

$C_o$ = background nitrate concentration (mg/L),

$m$ = number of surveillance points,

$n$ = number of land use sectors.

The second type of mass balance constraint (Eq. 14) essentially equates the total nitrate concentration at surveillance point $j$ to a desired water quality goal plus any positive or negative deviations from the goal. A set of these water quality constraints simply establishes absolute limits (standard) on nitrate concentrations at every surveillance point.

$$R \sum_{i=1}^{n} X_i F_i T_{i,j}^r + C_o - d_j^+ + d_j^- = G \quad \text{for } j=1,2,...m; \quad (14)$$

where $d_j^+$ = the decision variable for the positive deviation from the water quality goal at surveillance point $j$; $j=1,2,...m$,

$d_j^-$ = the decision variable for the negative deviation from the water quality goal at surveillance point $j$; $j=1,2,...m$,

$G$ = water quality goal (mg/L).

The results of minimizing the objective function subject to the above constraints showed the optimum pattern of residential development which would ensure minimum total positive deviation from water quality goals for the year 2000. The model attempted to concentrate development in land use sectors near the boundaries where ground waters discharge from Jackson Township into adjacent downgradient municipalities. The tendency of concentrating pollution near sites of ground water discharge was a serious problem inherent in model application. The problem could be handled through either strict water quality standards for waters discharging from one region and entering another or through regional modeling of complete hydrologic units. The best possible residential development plan was not necessarily represented in their problem solution because much of
the vacant land in Jackson Township was not considered available for development and because the region was discretized in rather large and nonuniform elements which could have resulted in a loss of model sensitivity and resolution.

Mooseburner and Wood's model could be applied only where an aquifer was assumed homogeneous and horizontal and where groundwater flows were approximately constant and uniform. A more general formulation would have permitted greater flexibility of application over a broader set of hydrogeologic regimes. In addition, this model (and the other management models which used transfer or influence coefficients to couple the response surface of the groundwater quality simulation model to the optimization model) required individual external groundwater water quality simulations for each contaminant source to identify values of transfer coefficients. For regional groundwater quality management problems involving numerous sources, the number of simulations necessary to define the transfer coefficients would render this management modeling approach cumbersome if not infeasible; hence, this management model design would remain limited to management problems involving a small number of sources.

2.4. CONCLUSIONS

Groundwater quality simulation models tied to optimization models are unequivocally more efficient at identifying plans of optimal groundwater management than simulation models alone. Because of the regional nature of nonpoint source pollution (extended over large areas of an aquifer), the large number of sources, and the long time horizon, transient management models are not suited for evaluating or formulating strategies of managing semipermanent nonpoint source pollution on a regional scale. Steady-state management models, however, appear to be promising tools for ascertaining where and to what extent nonpoint source pollution should be controlled to preserve water resource availability, but no models have been developed. Gorelick and Remson (1982a) have developed an efficient approach of tying the response surface of a steady contaminant transport model to an optimization model. Their approach of embedding the finite difference mass transport model as part of the constraint set of an optimization model is a feasible means of formulating a water quality management model for nonpoint source pollution generated from numerous sources distributed over a large expanse of aquifer. Using the embedding approach in a management model will yield simultaneous estimates of optimal nonpoint source disposal fluxes and the associated steady-state response (contaminant concentrations in every element) of the groundwater system to the disposal activities.
CHAPTER 3

CONCEPTUAL FRAMEWORK, GENERAL FORMULATION, AND APPLICATION OF A NONPOINT SOURCE GROUND WATER QUALITY MANAGEMENT MODEL

This chapter introduces the conceptual framework for viewing an aquifer under sustained nonpoint source pollutant loading. Next, the basic components of management model are presented. Finally, an outline is given for model application in the field.

3.1. Conceptual Framework

In the research presented here ground water flow and ground water quality changes will always be regarded in the conceptual frame of regional changes occurring in a two-dimension horizontal aquifer. To facilitate the modeling of subsurface flow and contaminant transport processes, the aquifer is discretized into elements or nodes. Spatial changes in nonpoint source ground water pollution and ground water quality impacts are approximated over the discretized two-dimensional horizontal aquifer. Contaminant concentrations, hydraulic stresses (i.e., pumping and recharge), and all activities contributing to the pollution of ground waters are perceived piecewise constant within each element. Figure 1 illustrates a hypothetical aquifer discretized into elements which are identified by an i,j coordinate system. The management models are used to simultaneously select total elemental flows of pollutants from various nonpoint source subsurface disposal activities (e.g. septic tank densities), and calculate the consequent steady-state contaminant concentrations in each element, i,j.

In the case of a distributed source such as pollution from on-lot septic tanks discharging into a discretized aquifer, the waste source can be characterized as either the total volume of septic tank effluent entering the aquifer within that element or the density of septic tanks within that element. The two perspectives are equivalent since all septic tanks are assumed to have the same strength and flow rate. In fact, the model works with discharge flows (at constant concentration) and the interpretation of the model is in terms of development density (number of houses per unit area).

The elemental regional nonpoint source ground water wasteload allocations are calculated from the land use activities and the known contaminant concentration in subsurface disposal flows. From the
Figure 1. Two-dimensional and horizontal numerical discretization of a hypothetical, two-dimensional, and horizontal aquifer. Elements are addressed by the $i,j$ coordinate system.
discrete contaminant mass loadings, decision makers can obtain estimates of desirable elemental source densities (i.e., septic tank densities or agricultural land use densities in each element), which are compatible with stated ground water quality goals and land use controls.

3.2. General Management Model Formulation and Components

The various management models presented in later chapters are used to identify patterns of nonpoint source pollution that are compatible with water quality goals and subsurface disposal needs. All of these management models are linear programs. Linear programming formulations can simultaneously locate multiple sources, set each source contaminant flux, and predict ground water impacts. In addition, linear programs provide postoptimal information about waste disposal and water quality tradeoffs associated with relaxing constraints on source densities (land use activities) and water quality standards.

The management models presented in this work usually have these five components:
- Decision Variables
- Objective Function
- Continuity Constraints
- Management Constraints
- Nonnegativity Constraints

3.2.1. Decision Variables

Two categories of decision variables are used in the management models. The first type represents the total flow rates of recharge from three land use activities contributing to nonpoint source pollution of ground waters within a node \( i,j \). These recharge decision variables are represented as \( W_{i,j} \), \( V_{i,j} \), and \( Z_{i,j} \). The recharge flows from each of the three land use activities have known (constant) contaminant concentrations. The other category of decision variables corresponds to the steady-state depth-averaged concentration of dissolved constituents in each element resulting from the nonpoint source pollution. In this work the variable symbol \( C_{i,j} \) is used to represent dissolved contaminant concentrations in node \( i,j \).

3.2.2. Objective Function

The objective function is an algebraic representation of a criterion used to judge the quality of linear program solution. A
solution is a set of values for decision variables used in a management model. To solve a linear program is to identify a set of values for the decision variables which maximizes or minimizes the value of objective function. An example objective function is the summation of decision variables representing subsurface disposal flows.

\[
\text{Max } \sum_{i=1}^{m} \sum_{j=1}^{n} z_{i,j}
\]

where \( n \times m \) equals the number of elements.

### 3.2.3. Continuity Constraints

One continuity constraint is written for each element of a discretized study area. A continuity constraint is an algebraic approximation of the partial differential equation governing contaminant transport in a specified element \( i,j \). Continuity constraints assume the following general form for each element \( i,j \).

(Refer to equations 6 and 7 for the derivation of this formulation.)

\[
G_{i,j} \left( c_{1,1}, c_{1,2}, \ldots, c_{1,m}, c_{2,1}, c_{2,2}, \ldots, c_{n,m}, w_{i,j}, z_{i,j} \right) =
\]

\[
q_{i,j} + s_{i,j} + u_{i,j}
\]

\( G_{i,j} \) is a linear algebraic function composed of dissolved contaminant variables from all elements in the study area, plus variables representing nonpoint source subsurface disposal flow rates in element \( i,j \). Terms on the right hand side are actually constants corresponding to known background contamination from existing sources not subject to control (i.e., contamination from natural recharge, \( q_{i,j} \) and artificial recharge, \( s_{i,j} \) and \( u_{i,j} \)). The continuity constraints tie subsurface disposal variables with variables representing contaminant concentrations. These constraints operate as an expressed approximation of the relationship between subsurface disposal activities and the resultant ground water contamination.

### 3.2.4. Management Constraints

Management constraints place upper and lower boundaries on the numerical values of decision variables. Water quality and source density constraints are two major types of management constraints which appear in the nonpoint source ground water pollution management models. Water quality constraints specify upper limits on values of
decision variables representing dissolved contaminant concentrations. The upper limits for dissolved contaminant concentrations are either specified water quality standards or water quality goals. One water quality constraint is written for each element within a discretized aquifer. Typical water quality constraints assume the form:

$$ C_{i,j} \leq \{\text{water quality standard or goal}\} \forall i \text{ and } j $$

(17)

Water quality constraints ensure that the optimal values selected for the subsurface disposal flows meet water quality standards or goals for those elements.

Source density constraints specify upper and lower limits on subsurface disposal flows. Two types of source density constraints are used. The first type specifies minimum discharge flows in each element that reflect existing nonpoint source pollution activities. A simple form of this constraint is

$$ Z_{i,j} \geq \{\text{present day disposal flows}\} \forall i \text{ and } j $$

(18)

The second form of source density constraint places upper limits on allowable nonpoint source pollution activity in each element, for example:

$$ Z_{i,j} \leq \{\text{upper limit on disposal flows in element } i,j\} \forall i \text{ and } j $$

(19)

Because each source has constant flow and concentration, limiting the magnitude of the elemental source term is equivalent to limiting the number of individual sources in the element (i.e., a development density restriction).

The source density constraints ensure selection of an optimal pattern of nonpoint source pollution activities which recognizes limits on existing and allowable levels of ground water pollution and septic tank density.

3.2.5. Nonnegativity Constraints

Nonnegativity constraints are the final component of a management model. These constraints require that all decision variables must be greater than or equal to zero. Beyond the fact that negative concentration and disposal flows have no physical meaning, it is a restriction of linear programming that decision variables not assume negative values. The nonnegativity constraints are imposed implicitly in the solution algorithm; consequently, these constraints do not appear explicitly in any of the models.
3.3. Construction and Application of a Nonpoint Source Ground Water Quality Management Model

The combined use of numerical simulation models, numerical inverse models and linear programming will allow the construction and application of several nonpoint source ground water pollution management models. Management model construction and application occurs over three phases. The first phase details the nature of the nonpoint source ground water pollution problem and specifies site specific management information needs. In the second phase the combined use of a validated numerical inverse model and a validated numerical ground water flow model are used to define the subsurface fluid velocity field within the boundaries of the study area. Once the ground water flow field is defined, phase three, the creation of the site specific management model, begins. This last phase is initiated with the selection of decision variables. Next, the continuity and management constraints and the objective function are constructed. Finally the linear program is solved and the results are evaluated.
CHAPTER 4

DEVELOPMENT AND VALIDATION OF NUMERICAL MODELS TO DESCRIBE GROUND WATER FLOW

The construction of subsurface water quality management models is predicated on a governing equation for unconfined solute transport coupled to a mathematical description of unconfined ground water flow. The governing equations for unconfined ground water flow and solute transport were introduced in Chapter 2. Hatfield (1987) presents the full development, validation, and calibration of the models described in this chapter.

The ground water flow problem was solved with a finite difference model as presented by Trescott, Pinder, and Larson (1976). For modeling purposes the aquifer was discretized into n*m elements, with one finite difference equation for each element. Thus n*m equations are generated for the n*m water table elevations (hydraulic head) at each element.

The finite difference model was validated against an analytical solution of the governing equation for a hypothetical two-dimensional, horizontal, unconfined aquifer (See Figure 2). Figure 3 presents the average percent error in predicted head as a function of the discretization scale for the hypothetical aquifer.

Small discretization scales lead to more elements, more variables, and more equations; hence a larger model that often yields more accurate results (if the data are available to support the finer discretization). Large element scales result in fewer elements, variables, and equations, but model accuracy is sacrificed. The optimum grid size should be the largest discretization producing acceptable accuracy which is also commensurate with available field data for model calibration. In this case the two kilometer discretization appeared to be the optimal choice in regard to minimizing model error.

A mass balance check was performed to investigate the presence of model anomalies which introduce or remove water within the bounded hypothetical island. The percent error in the mass balance was small at all scales of discretization. A total gain of 0.008 percent was calculated for the model simulations at 2 kilometer element dimensions. This increase was not regarded as significant. The validation process showed that the numerical ground water flow model could match analytical results within an error standard deviation of
Figure 2. Plan view (a) and Profile view (b) of the hypothetical, two-dimensional, horizontal, and unconfined island aquifer.
Figure 3. Average percent error in predicted head (above mean sea level) as a function of the discretization scale (for the numerical ground water flow model of the hypothetical island aquifer).
+ 5 percent while producing small mass balance errors. It was concluded therefore that the numerical model was successful at describing ground water flow in the test island. For a field study similar to the hypothetical problem presented in this chapter, one would expect, at best, similar numerical accuracies.

A finite difference approximation of the governing contaminant equation was used to construct numerical equations for a discretized hypothetical nonpoint source ground water pollution transport model. The resultant set of algebraic equations were then embedded as continuity constraints in a simple linear program. The solution to the linear program is the numerical estimate of the concentration a dissolved constituent in each element of the discretized aquifer.

For the simplest contaminant transport model the only variables appearing in the finite difference equation are those representing the dissolved contaminant concentrations; the \( C_{i,j}'s \). Each discrete equation is constructed from elemental data on hydraulic head, aquifer transmissivity, node dimensions, node area, contaminant source flows, and contaminant source concentrations.

The numerical contaminant transport model is a set of algebraic equations which are comprised of variables \( C_{i,j} \) (for all \( i \) and \( j \)) representing contaminant concentrations at each node. The set of algebraic equations can be solved with the same algorithm employed to solve linear programs (Gorelick, 1979). The process is one of treating the \( C_{i,j}'s \) as decision variables, while using the set of algebraic equations from the numerical model as continuity constraints in a simple linear program. The objective function in the linear program is,

\[
\text{Maximize } \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j}
\]

The constraint set contains continuity and nonnegativity constraints:

\[
\text{s.t. } [G]{C} = + [C_w]{W} + [C_s]{S} + [C_z]{Z} + [C_u]{U} + [C_q]{Q}
\]

\[
C_{i,j} \geq 0 \quad \forall \ i \text{ and } j
\]

The total number of variables in this problem is same as the total number of constraints, which equals the number of discrete elements in the ground water study area. In the continuity constraints, the terms on the right-hand side of the equal sign are known constants.

A solution to the linear program is a set of values for the \( C_{i,j}'s \) which maximize the value of the objective function and
satisfies the continuity and nonnegativity constraints. The linear program is solved using one among several available algorithms which identify the optimum feasible solution through an iterative procedure.

The accuracy and precision of the linear programming solution to the numerical solute transport model were evaluated against analytical results for the same hypothetical aquifer with an assumed nitrate source.

Results for the average relative error (%) are plotted in Figure 4. Model error increases as numerical discretization increases beyond one kilometer. The numerical nitrate predictions tended to be greater than the analytical estimates; consequently the numerical model provides conservative simulations of the true extent and impact of a ground water pollution scenario. When a two kilometer element dimension was used, the average prediction error was less than .1 mg/L as NO$_3^-$ as NO$_3^-$ or 2.2 percent. The model nitrate concentrations represented average regional predictions over aquifer elements which were each four square kilometers in area. A two percent average error in simulated nitrate concentrations was considered more than acceptable for regional contaminant predictions.

The validation of the numerical contaminant transport model clarifies the level of expected model accuracy at three discretization scales for a ground water pollution scenario comparable to the hypothetical problem. For a numerical discretization of two kilometers it was shown that model errors were 2.2 $\pm$ 4.5 percent. This level of accuracy was considered acceptable given that model results correspond to regional nitrate predictions and not point estimates.
Figure 4. Average percent error in predicted nitrate concentrations as a function of the discretization scale (from the numerical contaminant transport model for the hypothetical island aquifer).
CHAPTER 5

MODELING GROUND WATER FLOW IN WESTERN CAPE COD

This chapter is devoted to the modeling of ground water flows in the aquifer underlying Western Cape Cod. The intense pressures for residential development portend future nonpoint source ground water pollution problems for this area of Massachusetts; consequently, this area was chosen to serve as an application region for the management models.

The chapter begins with a brief description of the primary ground water pollution problems on Cape Cod, followed by a discussion of the hydrology and geology of the study area. The next section of the chapter presents the preliminary assumptions that underlie efforts to model the Cape Cod Aquifer with the validated subsurface flow model. Calibration of the numerical flow model is accomplished with the inverse model. Final simulation results are presented as water table maps and numerical error scatter plots.

5.1. Nonpoint Source Ground Water Pollution on Cape Cod

The aquifer under Barnstable County, Massachusetts (Figure 5) has over a hundred public wells and has been designated a Sole Source Aquifer by the U.S. Environmental Protection Agency. The aquifer is threatened by nonpoint source pollution from road runoff, lawn fertilizers, septage lagoons and pits, and underground fuel and chemical storage tanks. On-site domestic waste disposal systems (septic systems and cesspools), however, are among the largest and most ubiquitous source of distributed ground water contamination on Cape Cod; 88 percent of the year-round residents use septic tanks or cesspools (Quadri, 1984). Another major source of ground water nitrate contamination is the use of lawn fertilizers.

Research has linked the expanded use of on-site domestic waste disposal systems and the application of lawn fertilizers to increases in ground water nitrate levels on Cape Cod and elsewhere (Yates, 1985, Flipse et al., 1984, Quadri, 1984, Porter, 1980, Katz, Linder, and Kagone, 1980). Nitrate pollution of ground waters is recognized as a significant problem because nitrate is a persistent contaminant in the subsurface environment which poses potential health hazards (Virgil and Hayner, 1965, Fraser, et al., 1980) and contributes to the eutrophication of coastal waters. Elevated nitrate concentrations in ground waters may also indicate the presence of
Figure 5. Map of Barnstable County, Massachusetts with associated towns.
other contaminants (e.g., bacteria and viruses), which like nitrate could be public health hazards and expensive to remove from water supplies (Perkins, 1984).

Sewer systems are often installed to mitigate water quality problems derived from septic system pollution. The 208 Water Quality Plan for Cape Cod (Cape Cod Planning and Economic Development Commission, 1987) favored the continued use of septic systems because they are an inexpensive method of treating domestic wastes and because the artificial recharge helps to maintain water levels within the aquifer. To accommodate anticipated population growth, residential development is expected to expand the use of septic systems and lawn cultivation into sensitive regions where major public wells are now situated.

The nitrate pollution on Cape Cod has developed into a regional ground water quality problem. To protect ground waters a cooperative effort will be needed by all communities of Barnstable County to determine where and to what extent residential development should be permitted.

5.2. The Hydrology and Geology of Western Cape Cod

The western section of Barnstable County, Massachusetts, was chosen as the application region for the nonpoint source ground water pollution management models (see Figure 6). Cape Cod is primarily composed of unconsolidated deposits of clay, silt, sand, gravel, and boulders laid down by glacial and fluvial action during the Pleistocene age. The unconsolidated material extends to depths of 25-152 meters below mean sea level with general increases in depth occurring from Bourne eastward (Burns, Frimpter, and Willey, 1975). Underlying the unconsolidated deposits is a granitic basement rock.

The major geological features of the area are the Buzzards Bay Moraine, the Sandwich Moraine, and the Mashpee Outwash Plain. Figure 7 illustrates the location of each formation. The moraines are elevated formations which are composed of unconsolidated deposits that range in grade from clay to larger boulders. The water transmitting properties of the moraines are generally considered poor, because the lithology (and probably the permeability) of the moraine aquifer tends to vary greatly over short distances (Guswa and LeBlanc, 1985). The outwash plain is highly permeable and the depth to the water table is more shallow here than in the moraines. The deposits in the outwash plain are generally sand and gravel with lenses of clays and silt. The grades of the deposits decrease in a direction south and east of the moraines.

The Sole Source Aquifer for the residents of Western Cape Cod is an unconfined lense of fresh water floating on top of salt water (see Figure 8). Surface waters in this region are primarily water table
Figure 6. Map of Western Cape Cod which serves as the hydrologic study site for the application of the numerical ground water flow model.
Figure 7. Geologic map of Western Cape Cod. Adapted from Guswa and LeBlanc (1985).
ponds and a few minor streams which receive flow principally through ground water seepage (with small contributions from surface runoff). Annual variation of pond surface elevation is greatest within the interior regions. Marshes and streams near the coast have relatively constant surface elevations and operate as significant points of ground water discharge (Guswa and LeBlanc, 1985).

Annual recharge (from precipitation only) on the western Cape aquifer is 30–56 centimeters (Guswa and LeBlanc, 1985, LeBlanc, 1984, Burns, Frimpter, and Willey, 1975, and Strahler, 1972). Figure 8 shows the cross-sectional flow paths for water entering the aquifer from surface precipitation. Near the morainal deposits (central rib) water travels in horizontal and vertical directions. As some of the water flows vertically, it is deflected in a horizontal direction due to decreasing vertical permeability of outwash deposits with depth. Figure 8 does not show the presence of fine sand, silt or clay layers; however, LeBlanc (1984) reported the existence of such layers in Falmouth and Strahler (1972) identified a layer of silt/clay 150 feet thick from data collected from a well drilled in Harwich. Both surmised that these formations have much lower permeabilities than overlying outwash deposits and both located these layers approximately 30 meters below Mean Sea Level (MSL). Strahler (1972) suggested that the principal ground water movement could be limited to the highly permeable unconsolidated deposits overlying this layer. LeBlanc (1984) investigated a contaminant plume in the outwash plain of Falmouth, Massachusetts and observed that a layer of fine sands and tills underlying the coarse sand and gravel deposits precluded vertical movement of the plume. Thus, the highly permeable layer of sand/gravel outwash which overlies the fine sand and till deposits appears to be the major conduit of ground waters and ground water contaminants.

Figure 9 shows a gently sloping water table in the outwash plain which suggests that ground water moves easily and horizontally from the central rib through the sand and gravel deposits of the glacial outwash plain toward the coast (LeBlanc, 1984 and Bear, 1979). Near the coast the flows are deflected upward by the denser sea water at the freshwater/saltwater interface (see Figure 8). Because ground water recharge is seasonal, annual fluctuations in the water table have been observed; the greatest range of fluctuations occur inland (2–3 feet) while only a few inches of fluctuation occur near the coast (Strahler, 1972).

5.3. Assumptions

The following assumptions were made regarding the characteristics of the aquifer;

1) Previous work from Guswa and LeBlanc (1985), Burns, Frimpter, and Willey (1975), and Sterling (1963) had shown
Figure 8. Profile view of the unconfined aquifer of Western Cape Cod showing the idealized ground water flow field (a) and the aquifer drawn to scale (b). Adapted from Strahler (1972).
Figure 9. Geologic Map of Western Cape Cod with the steady-state water table elevation contours overlaid. Adapted from Guswa and LeBlanc (1985).
that the aquifer on Western Cape Cod is composed of layered unconsolidated nonhomogeneous deposits of boulders, gravel, sand, silt, and clay which suggests that the aquifer is nonhomogeneous. The horizontal deposition of sediments have created an aquifer which is anisotropic with respect to fluid flow in the vertical direction, but relatively isotropic with respect to fluid flow in the horizontal direction.

2) To use the numerical ground water flow model, the aquifer is discretized into elements where the physical properties of the aquifer are assumed piecewise constant within each element. Large discretizations are acceptable, if the phenomenon of interest is greater than the scale of aquifer inhomogeneities (Bear 1979).

3) The difference in density between freshwater and saltwater prevents significant mixing along the freshwater/saltwater interface; therefore, the elevation of that interface wherever it intersected the highly permeable outwash deposits was treated as an impermeable bottom boundary.

4) Both LeBlanc (1984) and Strahler (1972) identified the basement rock as an aquifuge; however, LeBlanc also observed that the less permeable layers of fine sands, silts, and clays behaved as impermeable boundaries to significant fluid flow and contaminant transport when compared to the overlying highly permeable sand and gravel outwash deposits. It was assumed therefore, that the top elevation of layers of fine sand, silt, or clay would be used as the elevation of the impermeable boundary except where the highly permeable glacial outwash deposits extend all the way to bedrock.

Several assumptions were made regarding the nature of subsurface flow on Cape Cod.

1) Piezometric readings by LeBlanc (1984), Guswa and LeBlanc (1985), and Burns, Frimpter, and Willey (1975) support assumptions that, on a regional scale, ground water flows are principally horizontal. It was assumed that regional ground water flow in Western Cape Cod aquifer is unconfined and horizontal; vertical flows are minor over most of the aquifer except near the coast, but these flows are not considered.

2) Surface water elevations for ponds in the interior of the study area were assumed to reflect local water table elevations.
5.4 Model Formulation

A uniform numerical discretization was applied over the aquifer (see Figure 10). The two kilometer discretization was used because excellent results were obtained at this scale during the validation of both the flow and the contaminant transport models. The area of

3) Seasonal changes in the regional piezometric surface are a reflection of seasonal variations in the ground water recharge buffered by a huge reservoir of fresh water stored in the Western Cape Cod aquifer. Consumptive losses are estimated to be less than one percent of the total annual natural recharge. It was assumed that averaged water table elevations reflect the approximate steady-state conditions.

4) Since the density of septic tank effluent is close to that of distilled water, it is assumed that ground water flow is density independent.

5) Because the ground water model considers only horizontal flows, the dimensions of the discrete elements must be sufficiently large that horizontal flows dominate. Bear (1979) noted that vertical flows resulting from local hydraulic perturbations (e.g., pumping) become minor over horizontal distances of 1.5-2.0 times the thickness of the saturated zone (the distance between the water table boundary and impermeable bottom). On Western Cape Cod the horizontal dimensions of each element were 2000 meters which is over seven times the maximum thickness of the aquifer.

Regarding hydrologic stresses in the study area, there were a few other assumptions;

1) Based upon the work of Guswa and LeBlanc (1985), Burns, Frimpter, and Willey (1975), and Strahler (1972) it was assumed that the principal sources of ground water recharge were precipitation and return water from septic systems.

2) Throughout any discrete element, it was assumed that the aquifer receives evenly delivered stresses from constant recharge originating from precipitation, septic tank effluents, or other identified sources and constant discharge from pumping or other forms of natural and artificial ground water withdrawal.

3) ground water discharge along the shore (i.e., through coastal marshes) and through the seabed is controlled by the elevation of the coastal waters. It is assumed that the hydraulic head along the coast is a constant taken to equal mean sea level above some stated datum.
HYDROLOGIC STUDY SITE

Figure 10. Map of two-dimensional, and horizontal discretized Western Cape Cod showing regions included in the ground water flow model. Elements are addressed by the i,j coordinate system.
each element was assumed to equal four square kilometers unless it was determined that the effective recharge area was less (e.g., for elements along the coastline). Where elements had less than four square kilometers of area the recharge was reduced proportionately.

Two primary boundary conditions were incorporated into the mathematical modeling of subsurface flow on Western Cape Cod. The first condition specified that the aquifer floor was impermeable and that no flow was lost to leakage. This assumption is reasonable since a granitic basement rock and/or clay underlie the permeable sandy aquifer (Strahler, 1972).

The second condition concerns the proper approximation of the boundary where ground water discharge occurs along the shore (i.e., through coastal marshes) and through the sea bed. It was assumed that the elevation of the coastal water elevation determined water table elevations and ground water discharge along the coast. Ground water elevations along the coast are somewhat constant with elevations fluctuating from 0 to 0.6 meters (Frimpter and Fisher 1983); consequently, subsurface flow at the coast may be approximated with a specified head boundary condition. For all elements along the coast, the hydraulic heads were specified at mean sea level; see Figure 11.

The numerical flow model requires data on spatial changes in ground water recharge rates, pumpage rates, and aquifer transmissivity. Data was collected from the U.S. Geological Survey on hydrologic stresses that would yield elemental recharge rates and pumpage rates. It was not possible to calculate discrete transmissivities from aquifer thickness and elemental permeability information because existing data was inadequate to define regional permeabilities at each node. Therefore the discrete transmissivities were estimated with the inverse model. The inverse model required hydraulic head estimates at each node and the estimated transmissivity at some point along each flow line in addition to the hydraulic stresses data. The specifics of the data and the methods of acquisition are presented below.

Aquifer recharge on Cape Cod is derived from natural precipitation and artificial recharge from septic system effluents and wastewater treatment plants. Guswa and LeBlanc (1985) calculated natural recharge as the difference between average annual precipitation and average annual evapotranspiration. Precipitation was calculated from rainfall data for 1947-1976. The annual evapotranspiration rate was calculated by the Thornthwaite method (Strahler, 1972). Data from 1947-1976 on mean monthly temperature was used in conjunction with the geographic latitude of the area to obtain estimates of the annual rate of evapotranspiration. The estimated natural recharge rate increased from 48 cm/year in Yarmouth (in the east) to 56 cm/year in Falmouth (in the west).
Figure 11. Map of discretized Western Cape Cod showing hydrologic boundary elements of specified hydraulic head and elements where the hydraulic head is predicted.
Over all of Cape Cod, 88 percent of the population uses septic systems as a means of disposing domestic wastes. Of this population approximately 90 percent use municipal water (U.S. Census, 1980); consequently, the water exported from municipal wells appears as artificial recharge from septic systems. The total artificial recharge from each water district was calculated from the average daily volume of water pumped by the water district. Table 1 shows the average daily pumpage from each district during 1975-76. Uniform artificial recharge rates from each water district were determined from the ratio of a district's average daily pumpage and the area served. Artificial recharge rates for each element were adjusted proportionally to the fraction of element area served by each municipal water supply. Nodes which received artificial recharge are identified in Figure 12.

Aquifer recharge from the two sewage treatment plants (the Otis and Barnstable plants) were equated to the flows reported by Guswa and LeBlanc (1985) in their three-dimensional model for Western Cape Cod.

Municipal water demands comprise the greater part of all ground water withdrawal on Cape Cod. Pumpage occurs throughout the Western Cape Cod Area. Figure 13 shows which elements had active wells during 1975-76. Table 2 lists each active well and the average daily pumpage over 1975-76.

5.5 Water Table Elevation Data

The inverse model was applied over Western Cape Cod as a means of estimating elemental aquifer transmissivities. The data requirements of the inverse model include hydraulic head estimates at each element in addition to the recharge and pumpage data presented above. The hydraulic head at each element was estimated from a combination of observation wells, pond levels, and interpolations from water table maps. Observed average water levels from 1950-82 were obtained from Guswa and LeBlanc (1985) and Letty (1984). Additional pond levels were obtained from U.S. Geological Survey topographic maps for Cotuit, MA (1974), Dennis, MA (1974), Falmouth, MA (1979), Hyannis, MA (1979), Onset, MA (1967), Pocasset, MA (1979), Sagmore, MA (1979), Sandwich, MA (1972), and Woods Hole, MA (1967). In a few areas of Cape Cod, well water and surface water levels were not available; for these areas the hydraulic head was equated to the average graphic interpolation from three water table maps (Guswa and LeBlanc, 1985, Redfield, in Strahler, 1972, and Cape Cod Planning and Economic Development Commission, 1982). Figure 14 distinguishes nodes where hydraulic heads were estimated from observation data or from interpolation.
Table 1. Artificial Recharge Sources Over Western Cape Cod in 1975/76.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Barnstable Fire District</td>
<td>1079</td>
<td>20</td>
<td>100</td>
</tr>
<tr>
<td>Barnstable Sewage Treatment Plant</td>
<td>2650</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>Barnstable Water Company</td>
<td>5856</td>
<td>24</td>
<td>100</td>
</tr>
<tr>
<td>Bourne Water District</td>
<td>2306</td>
<td>24</td>
<td>100</td>
</tr>
<tr>
<td>Centerville-Osterville and Cotuit Fire Districts</td>
<td>6149</td>
<td>32</td>
<td>100</td>
</tr>
<tr>
<td>Falmouth Water District</td>
<td>10138</td>
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<td>100</td>
</tr>
<tr>
<td>Highwood Water Company</td>
<td>484</td>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>Otis Air Force Base</td>
<td>2298</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>Sandwich/South Sagmore Water District</td>
<td>1490</td>
<td>21</td>
<td>100</td>
</tr>
<tr>
<td>Yarmouth Water District</td>
<td>9270</td>
<td>28</td>
<td>100</td>
</tr>
</tbody>
</table>
Figure 12. Map of discretized Western Cape Cod showing which elements received artificial recharge as of 1976, residential/commercial development patterns.
Figure 13. Map of discretized Western Cape Cod showing which elements contained pumpage as of 1976 for residential/commercial needs.
Table 2. Summary of Active Wells Over Western Cape Cod in 1975/76

<table>
<thead>
<tr>
<th>U. S. Geological Survey Well</th>
<th>Location</th>
<th>Discharge [m$^3$/d]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1W59</td>
<td>8,11</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>9,11</td>
<td>9</td>
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<tr>
<td>A1W158, A1W160</td>
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<td></td>
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<tr>
<td>A1W224</td>
<td>10,11</td>
<td>113</td>
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<td>A1W228</td>
<td>6,16</td>
<td>421</td>
</tr>
<tr>
<td>A1W226</td>
<td>8,15</td>
<td>291</td>
</tr>
<tr>
<td>A2W227</td>
<td>8,16</td>
<td>291</td>
</tr>
<tr>
<td>A1W368</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A1W229, A1W384</td>
<td>8,16</td>
<td>2877</td>
</tr>
<tr>
<td>A1W251</td>
<td>9,11</td>
<td>274</td>
</tr>
<tr>
<td>A1W259, A1W373</td>
<td>8,13</td>
<td>614</td>
</tr>
<tr>
<td>A1W369</td>
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<td>276</td>
</tr>
<tr>
<td>A1W370</td>
<td>6,17</td>
<td>658</td>
</tr>
<tr>
<td>A1W371, A1W372</td>
<td>7,13</td>
<td>798</td>
</tr>
<tr>
<td>A1W376</td>
<td>8,16</td>
<td>1108</td>
</tr>
<tr>
<td></td>
<td>7,18</td>
<td>779</td>
</tr>
<tr>
<td></td>
<td>8,17</td>
<td>779</td>
</tr>
<tr>
<td></td>
<td>8,18</td>
<td>779</td>
</tr>
<tr>
<td>A1W402</td>
<td>6,17</td>
<td>88</td>
</tr>
<tr>
<td></td>
<td>7,17</td>
<td>88</td>
</tr>
<tr>
<td>A1W403, A1W383, A1W387</td>
<td>7,17</td>
<td>1329</td>
</tr>
<tr>
<td>BHW 22,232</td>
<td>2,7</td>
<td>303</td>
</tr>
<tr>
<td>BHW 23</td>
<td>8,5</td>
<td>1241</td>
</tr>
</tbody>
</table>
Table 2. Summary of Active Wells Over Western Cape Cod in 1975/76, Continued.

<table>
<thead>
<tr>
<th>U. S. Geological Survey Well</th>
<th>Location Node (i,j)</th>
<th>Discharge [m$^3$/d]</th>
</tr>
</thead>
<tbody>
<tr>
<td>BHW233, BHW1-3, BHW136</td>
<td>5,4</td>
<td>1600</td>
</tr>
<tr>
<td>BHW199</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long Pond</td>
<td>12,3</td>
<td>3385</td>
</tr>
<tr>
<td></td>
<td>12,4</td>
<td>3385</td>
</tr>
<tr>
<td></td>
<td>13,4</td>
<td>3385</td>
</tr>
<tr>
<td>MIW32, MIW32</td>
<td>12,9</td>
<td>484</td>
</tr>
<tr>
<td>SDW 27,37</td>
<td>3,9</td>
<td>1037</td>
</tr>
<tr>
<td>SDW 155</td>
<td>7,7</td>
<td>1059</td>
</tr>
<tr>
<td>SDW 249,250</td>
<td>2,8</td>
<td>152</td>
</tr>
<tr>
<td>YAW42, YAW43</td>
<td>6,18</td>
<td>629</td>
</tr>
<tr>
<td></td>
<td>6,19</td>
<td>629</td>
</tr>
<tr>
<td>YAW53, YAW144, YAW146</td>
<td>6,21</td>
<td>1754</td>
</tr>
<tr>
<td>YAW54</td>
<td>6,21</td>
<td>1116</td>
</tr>
<tr>
<td>YAW58</td>
<td>7,19</td>
<td>759</td>
</tr>
<tr>
<td>YAW61, YAW63</td>
<td>6,20</td>
<td>1192</td>
</tr>
<tr>
<td>YAW64, YAW65</td>
<td>7,20</td>
<td>1600</td>
</tr>
<tr>
<td>YAW103</td>
<td>5,20</td>
<td>1091</td>
</tr>
<tr>
<td>YAW126, YAW127</td>
<td>5,21</td>
<td>89</td>
</tr>
<tr>
<td></td>
<td>6,21</td>
<td>89</td>
</tr>
<tr>
<td>YAW128</td>
<td>7,19</td>
<td>306</td>
</tr>
</tbody>
</table>
Figure 14. Map of discretized Western Cape Cod showing the location of observed, assumed, and interpolated water table elevations.
5.6 Aquifer Transmissivities

Inadequacies in available field data precluded direct estimation of transmissivities for the Cape Cod aquifer. Available information on aquifer properties represents data compiled from well logs and pump tests which was collected over aquifer intervals that average less than ten percent of the total aquifer thickness. Few wells actually log the aquifer down to bedrock. Most pumping wells penetrate no more than 15 meters into the saturated zone (Horsley, 1981). For virtually all of Western Cape Cod little is known about the lower 75 to 90 percent of the aquifer.

To obtain transmissivity estimates for each element of the discretized aquifer, therefore, the inverse model was employed. (See Hatfield, 1987.)

The final estimates for the discrete transmissivities are presented in the Appendix. The hydraulic conductivity at each node was calculated by dividing the elemental transmissivity by the estimated discrete aquifer thickness. Hydraulic conductivities for interior nodes (the hydraulic conductivity of coastal nodes was not calculated because the saturated thickness of the aquifer was not known) ranged from 5 to 116 m/d. This range of hydraulic conductivities was much greater than the range (3.4 to 36 m/d) reported by Burns, Frimpter, and Willey (1975), but much smaller than the range of 0.3-183 m/day used by Guswa and LeBlanc (1985) to model three-dimensional flows on Western Cape Cod. Hydraulic conductivity values obtained from the inverse model represent regional estimates which may not agree with any point value; this is because aquifer lithology is known to vary over length scales shorter than the discretization scale.

5.7 Ground Water Flow Simulation Results

The discrete transmissivity estimates were used with the pumpage data, recharge data, and boundary conditions to perform the ground water flow simulations of the Western Cape Cod aquifer. Total recharge was 771,572 m$^3$/day. Total daily pumpage was 41,596 m$^3$/day which is five percent of the total recharge. Mass balance errors were very small, averaging in length 0.002 percent, indicating a very slight gain in subsurface flows.

Predicted heads were plotted as contours in Figure 15. The contour map depicts slight hydraulic gradients to the south and east sections of the aquifer and steeper gradients along the north shore. The steeper water table reflects the energy required to force flow through the less permeable moraine deposits.
Figure 15. Contour plots of predicted steady-state water table elevations (meter above mean sea level) from the groundwater flow model for Western Cape Cod.
The errors in predicted heads are summarized in Table 3. The average percent error in calculated head was 0.08 percent with a standard deviation of five percent. Actual error was always less than one meter and on the average was 0.05 meters. Model prediction errors (Figure 16) were generally positive (over estimates) regardless of where the node was located (small values of observed water table elevations indicate regions near the coast). Because nodes near the coast have water table elevations near mean sea level (0) the model was less accurate for these nodes in terms of percent error (see Figure 17).

Calibration of the ground water flow model was considered successful from the perspective of satisfying three goals. First, a set of transmissivities were identified for the region which yield hydraulic conductivities that were not only within the range of previous observations, but were also distributed spatially in a pattern commensurate with known geologic formations. Secondly, the standard deviations of the errors of the hydraulic predictions were within five percent. Finally, the low mass balance errors corroborated the excellent numerical predictions of the piezometric surface.
Table 3. Summary of Flow Model Predictions Errors Following Calibration to Western Cape Cod.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Head Error; [m]</td>
<td>0.051</td>
</tr>
<tr>
<td>Standard Deviation of Average Head Error; [m]</td>
<td>0.224</td>
</tr>
<tr>
<td>Average Percent Error in Predicted Head</td>
<td>0.080</td>
</tr>
<tr>
<td>Standard Deviation of Average Percent Error in Predicted Head</td>
<td>5.040</td>
</tr>
<tr>
<td>Percent Mass Balance Error</td>
<td>0.002</td>
</tr>
</tbody>
</table>
Figure 16. Scatter plots of error in predicted hydraulic head from the numerical ground water flow model of Western Cape Cod as a function of observed water table elevations (meters above mean sea level).
Figure 17. Scatter plot of percent errors in predicted hydraulic head (above mean sea level) from the numerical ground water flow model of Western Cape Cod as a function of observed water table elevations (meters above mean sea level).
CHAPTER 6

MANAGEMENT MODELS TO ELUCIDATE CRITICAL RECHARGE AREAS

Two non-point source ground water pollution management models are formulated and applied in this chapter. The first model identifies areas within the regional ground water flow system which are most critical to the preservation of area-wide ground water quality. The second model can be implemented to delimit critical recharge zones surrounding municipal water supplies.

Both models are applied to a section of Western Cape Cod. Long term regional ground water nitrate distributions from the 1980 development pattern in Falmouth and Bourne are projected. The projections are based on nitrate contamination from septic systems, lawn fertilizers, leaky sewers, background loads, and subsurface recharge from secondary municipal sewage.

Both management models identify the water quality tradeoffs associated with alternative subsurface disposal patterns, regardless of whether water quality impacts are regional in nature or simply at a water supply well. Modeling results include contour plots of the steady-state regional nitrate distributions from the 1980 land use patterns for Bourne and Falmouth, Massachusetts. In addition maps showing regional and local water quality impact isopleths are presented.

6.1. Formulation of Management Model I

The first management model is formulated to identify the maximum values for contaminant decision variables subject to continuity and nonnegativity constraints. The objective function is formulated as a direct summation of all the elemental contaminant variables in the management area divided by the number of discrete nodes under management. Hence, the objective function is written

\[
\text{Maximize } \frac{1}{nm} \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j}
\] (21)

The product of \( n \) and \( m \) is the total number of elements within the boundaries of the management area. The optimum value of the objective function represents the maximum average elemental
concentration subject to the specified constraints on subsurface disposal activities.

Two inequality continuity constraints are constructed for each node in lieu of a single equality constraint. The continuity constraint set is expressed as:

\[
\begin{align*}
[G][C] & \geq [C_w][W] + [C_s][S] + [C_z][Z] \\
& \quad + [C_u][U] + C_q \cdot [I][Q] \\
[G][C] & \leq [C_w][W] + [C_s][S] + [C_z][Z] \\
& \quad + [C_u][U] + C_q \cdot [I][Q]
\end{align*}
\]

where
- \( [C] \) = \((n \times m) \times 1\) vector of variables for contaminant concentrations at every element;
- \([G]\) = \((n \times m) \times (n \times m)\) vector of coefficients generated from the algebraic manipulation of the finite difference approximation of the governing flow equation;
- \([I]\) = \((n \times m) \times (n \times m)\) identity matrix;
- \([C_w], [C_z], [C_s], [C_u]\) = \((n \times m) \times (n \times m)\) diagonal matrices of elemental contamination concentration in every source flow of each element;
- \([U], [W], [S], [Z]\) = \((n \times m) \times 1\) vector of known recharge fluxes from four sources in every element;
- \(C_q\) = background contaminant concentrations in natural recharge flows;
- \([Q]\) = \((n \times m) \times 1\) vector of known natural recharge fluxes in every element.

The terms on the right-hand side of the equation are known subsurface disposal loadings. Presenting the equality constraint as a combination of two inequality constraints ensures that pollutant concentrations reflect specified contaminant loadings and creates dual variables for use in sensitivity analysis.

The complete formulation of model I follows.

Objective function:

Maximize \( \frac{1}{n \times m} \sum_{i=1}^{n} \sum_{j=1}^{m} C_{ij} \)

s.t.

Continuity Constraints:

\[
\begin{align*}
[G][C] & \geq [C_w][W] + [C_s][S] + [C_z][Z] \\
& \quad + [C_u][U] + C_q \cdot [I][Q] \\
[G][C] & \leq [C_w][W] + [C_s][S] + [C_z][Z] \\
& \quad + [C_u][U] + C_q \cdot [I][Q]
\end{align*}
\]
\[ [C][C] \leq [C_w][W] + [C_g][S] + [C_z][Z] \\
\quad + [C_u][U] + C_q \cdot [I][Q] \]

Nonnegativity constraints:
\[ C_{i,j} \geq 0 \quad \forall \, i \text{ and } j \]

6.2. Formulation of Model II

The formulation of Model II is a simplified version of model I where the scope of the objective function has been reduced to include a smaller number of elements. The objective function of Model II contains only the contaminant concentration decision variables \( C_{i,j} \)'s for elements containing water supply wells. (The same type of formulation could be devised to look at other significant surface or ground water resources. The formulation of the objective function is

\[
\text{Maximize} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j} Y_{i,j} \tag{24}
\]

where \( Y_{i,j} \) is equal to one if the contaminant concentration in element \( i,j \) is the target of interest (i.e., an element containing municipal well), and equal to zero otherwise.

Model II has the same continuity constraints as Model I.

As in Model I this model produces the steady-state contaminant distribution resulting from the specified subsurface disposal rates expressed as constant terms on the right-hand-sides of the continuity constraints. Iso-water-quality-impact contours can be drawn by plotting the optimum values of the dual variables and interpolating between the discrete values. These contours yield estimates of the unit changes in contaminant concentration at the target nodes induced by changes in subsurface disposal activity in all elements on that contour within the managed area.
6.3. Application of Models I and II to Bourne and Falmouth, Massachusetts

Models one and two were applied to a small section (268 square kilometers) of Western Cape Cod (see Figure 18). The region lies west of the ground water divide extending north to south along the eastern borders of Bourne and Falmouth, Massachusetts. Figure 19 depicts the locations of significant municipal water wells, sewage treatment plants, sewers, town borders, the military reservation, and the ground water divide.

The study area was discretized into elements as before (see Figure 20). The validated numerical contaminant transport model served as the continuity constraints for each management model. The solution of the two linear programs revealed the long term regional ground water quality impacts from land use activities producing dispersed nitrate pollution from septic systems, lawn fertilizers, secondary sewage recharge, leaky sewers, and background loading. The simulations were performed with estimates of pertinent nitrate source concentrations and flux rates from the year 1980. That is, these models were used to evaluate the steady state nitrate concentrations that would be attained if the study area never developed beyond the 1980 level of development.

6.3.1. Model Assumptions

The specific assumptions made regarding the boundaries of the study area were:

1) Boundary conditions along the ground water divide: contaminant transport across the ground water divide was assumed negligible. This boundary was treated by specifying zero darcian velocities across the divide; see Figure 37.

2) Boundary conditions at the coast: most contaminants in the aquifer discharge to the ocean along the coast. It was assumed that contaminants entering boundary nodes from upgradient flows and boundary recharge must equal the product of ground water discharge and contaminant levels at the boundary. Figure 21 illustrates how the coastal nodes were treated. The mass of contaminant entering the boundary from interior nodes \( V_{1i} \) plus the mass of contaminant entering the aquifer from surface pollution occurring on the boundary \( D_{Wb} \) is equated to the product of total flow leaving the boundary \( V_{i} + W_{D} \) and the concentration of contaminant at the boundary \( C_{b} \).
Figure 18. Map depicting the study site for the application of the nonpoint source ground water pollution management models.
Figure 19. Map illustrating details of the study-site for the application of the nonpoint source ground water pollution management models.
Figure 20. Map of discretized study site for the application of the nonpoint source ground water pollution management models.
Figure 21. Map illustrating the boundary conditions used to construct the continuity constraints in the groundwater quality management models.
3) Boundary conditions at the ground water mound: the numerical problem is not properly posed unless the contaminant concentration is specified at least somewhere along each characteristic. Because element i,j = 6,7 corresponds to the peak of the ground water mound all characteristics originate from this node; hence, for convenience the contaminant concentrations in the element was specified.

4) No loss of nitrate occurred across the impermeable aquifer floor or the water table.

The assumptions made regarding aquifer characteristics and flow were:

1) Western Cape Cod aquifer is composed of unconsolidated nonhomogeneous deposits of boulders, gravel, sand, silt, and clay. It was assumed that the aquifer was nonhomogeneous, but isotropic with respect to horizontal dissolved contaminant transport;

2) Regional ground water flow in Western Cape Cod aquifer has been unconfined and predominantly horizontal; therefore, regional advective contaminant transport was assumed unconfined and horizontal;

3) Ground water artificial recharge and withdrawal along the ground water divide was negligible for all elements within the management area; consequently, the position of the ground water divide was assumed stationary;

4) The numerical mass transport model was incorporated into a larger ground water quality management model to view long term ground water management schemes; therefore, simulated mass transport was predicated on information derived from a steady-state ground water flow regime defined by future regional water demands, existing and future well locations, existing and potential municipal water distribution systems, and existing and potential sewer systems. This approach would be acceptable as long as deviations from the projected volumes of water exported and consumed did not induce significant changes in the original piezometric surface used in the construction of the ground water quality management model.

Several specific assumptions were made regarding the physical and chemical mechanisms governing the fate and transport of nitrate in the Cape Cod aquifer:

1) Because nonpoint source ground water pollution was being evaluated over long time and large length scales, vertical variations in ground water quality become less important
than horizontal variations (Bear, 1979); consequently, only
depth averaged concentrations were considered;

2) Since vertically averaged regional contaminant
concentrations were being modeled, the scale of horizontal
numerical discretization must be sufficiently large that
sufficient vertical mixing through the entire depth of the
aquifer could be assumed. Bear (1979) noted that
contaminants entering the top of the saturated zone would
occupy most of the saturated layer after travelling
horizontal distances equal to 10-15 times the thickness of
the saturated zone. The average thickness of the aquifer on
Western Cape Cod is 72 meters. The discretization scale
used with the management models (2000 meters) would be 28
times larger than the average thickness of the saturated
zone; hence, complete vertical mixing was assumed between
elements;

3) Because little mixing of waters occurs at the
freshwater/saltwater interface and in the less permeable
fine sand, silt, clay, and bedrock layers, it is assumed
that no loss of nitrate occurs at these boundaries;

4) Because only regional scale ground water quality modeling
was pursued and because the sources of ground water
pollution (i.e., septic systems) to be modeled were assumed
areally distributed, the influence of dispersive contaminant
transport was considered negligible and was ignored;

5) Loss of nitrates in the saturated zone is considered minor
(Freeze and Cherry, 1979); therefore, nitrate was treated as
a conservative contaminant.

Finally there were specific modeling assumptions made regarding
nitrate sources and sinks;

1) Many sources of distributed ground water pollution are
semipermanent and will have a lasting impact on the
subsurface environment. For purposes of viewing the long
term ground water quality implications of semipermanent
nonpoint source pollution, only steady-state transport
conditions were simulated;

2) Local scale descriptions of horizontal variations in nitrate
levels were not considered. And for any element of
discretization, all contaminant loadings were assumed evenly
distributed throughout the element;

3) Source fluxes were based upon estimated per capita loading
rates and assumed nitrate concentrations which actually
reach the water table; transport through the unsaturated
zone was not considered. Values for flux and concentration of nitrates were obtained from the literature;

4) Because ponds on Western Cape Cod are generally phosphorous limited, these bodies of water were not treated as sources or sinks.

6.3.2. Adaptation of the General Management Models to Specific Ground Water Nitrate Sources on Cape Cod

Five sources of nitrate contamination were considered in the modeling effort for Cape Cod. The first three sources were related land use activities. The fourth and fifth sources were respectively the land application of wastewaters at the two sewage treatment facilities and the background nitrate loads delivered to the subsurface flow system through natural recharge.

The three land use activities were composite residential/commercial sources of nitrate pollution which differed from each other in regard to their source of water (municipal or on-site) and their method of disposing residential/commercial wastewaters (through sewers or septic systems). The first land use type used of municipal well water and septic systems. This land use contributes recharge to the aquifer from leaky water mains, and septic system effluents. Nitrate pollution from this composite land use is due to the application of residential lawn fertilizers, from nitrates in municipal waters lost to the aquifer through water distribution system leaks and from residential/commercial septic system effluents.

The second land use uses municipal waters, but disposes wastewaters to sewers. For this source, recharge is produced by leakage from water distribution systems and sewer exfiltration. The sewage treatment and discharge is assumed to occur at the coast. (This can be changed however by including it at an inland node as is done with the existing sewage treatment facilities.) Applied residential lawn fertilizers, sewer exfiltration, and water distributions system losses added nitrates to the subsurface environment.

The third land use has on-site wells and on-site septic systems. Nitrate pollution is due to the combined use of septic systems and fertilizers.

Recall the general vector formulation of the continuity inequalities:

\[
[G] [C] \geq [C_w] [W] + [C_g] [S] + [C_z] [Z] + [C_u] [U] + C_q \cdot [I] [Q] \tag{25}
\]
\[ [G][C] \leq [C_w][W] + [C_s][S] + [C_z][Z] \\
+ [C_u][U] + C_q \cdot [I][Q] \]  

(26)

For the Cape Cod problem nitrate source vectors were defined as follows:

\{W\} = (n\times m) \times 1 \text{ vector of elemental recharge flows from the combined domestic and commercial use of on-site wells, septic systems, and lawn fertilizers;}
\{Z\} = (n\times m) \times 1 \text{ vector of elemental recharges from the combined domestic and commercial use of municipal well water, septic systems, and lawn fertilizers;}
\{S\} = (n\times m) \times 1 \text{ vector of elemental recharge flows from the combined domestic and commercial use of municipal well water, sewers, and lawn fertilizers;}
\{U\} = (n\times m) \times 1 \text{ vector of elemental recharge flows from land application of secondary sewage;}
\{Q\} = (n\times m) \times 1 \text{ vector of elemental natural recharge flows;}

\[ [C_w] \] = (n\times m) \times (n\times m) \text{ diagonal matrix of nitrate concentrations in elemental recharge flows from the combined domestic and commercial use of on-site wells, septic systems, and lawn fertilizers;}
\[ [C_z] \] = (n\times m) \times (n\times m) \text{ diagonal matrix of nitrate concentrations in elemental recharges from the combined domestic and commercial use of municipal well water, septic systems, and lawn fertilizers;}
\[ [C_s] \] = (n\times m) \times (n\times m) \text{ diagonal matrix of nitrate concentrations in elemental recharge flows from the combined domestic and commercial use of municipal well waters, sewers, and lawn fertilizers;}
\[ [C_u] \] = (n\times m) \times (n\times m) \text{ diagonal matrix of effective nitrate concentrations in the elemental recharge from land application of secondary sewage;}
\[ C_q \] = \text{ observed background nitrate concentration in ground waters (for convenience precipitation was treated as the source), (mg/L).}

6.3.3. Estimation of the 1980 Hydrologic Stresses and Nitrate Loads in Bourne and Falmouth

Quantifying the recharge vectors (\{W\}, \{S\}, \{Z\} and \{Q\}) and the source nitrate concentrations was predicated on estimates of discrete artificial recharge (from septic systems, leaky water distribution systems, exfiltration flows, and secondary sewage), ground water pumpage, and estimated nitrate concentrations in all recharge flows. Calculations of nitrate concentrations in all recharge flows were
accomplished after assumptions were made regarding domestic nitrogen loads, commercial nitrogen loads, lawn fertilizer application rates and percent nitrogen losses in the vadose zone (estimated at 50 percent). Nitrate concentration estimates for other sources (i.e., secondary sewage recharge and sewer exfiltration) were made from literature values.

Estimates of discrete hydrologic and contaminant stresses were made after land use and population data for 1980 were compiled on Bourne and Falmouth.

6.3.3.1. Estimation of the 1980 Population Distribution in the Management Area

Nonpoint source pollution on Cape Cod is affected by land use activities and population distribution. The average population in 1980 was calculated for each element of the discretized management area. The population could be categorized into two groups; one group represented the elemental population dependent on municipal well water, while the other group relied on water from on-site wells. The 1980 elemental population pattern of each group is presented in Figures 22 and 23.

The population of each group was calculated by multiplying the average number of people per housing unit times the number of housing units in each element connected to on-site wells or municipal water supplies. The number housing units of each type were obtained by superimposing the finite difference grid over 1980 population enumeration district maps (U.S. Census Bureau). The average number of people per housing unit was interpreted as the ratio of total year-round population for each town to the year-round occupied housing units. Table 4 summarizes the population data by town.

6.3.3.2. Estimation of 1980 Pumpage Pattern

Records on the 1980 municipal and private well pumpage were obtained from the pertinent water districts and the Cape Cod Planning and Economic Development Commission (CCPEDC). The CCPEDC identified the safe yields for each well. Total water pumpage by Falmouth was apportioned among the three wells in accordance with their relative safe yields. Long term pumpage at the Otis Air Force Base was calculated with 1975 records on population and ground water pumpage and the long term population projections given in the draft 208 plans developed by CCPEDC (1978). The pumpage at the Sandwich well was conservatively allowed to function at the maximum capacity. Table 5 summarizes the locations and long term pumpage rates for each municipal or private well within the boundaries of the management study area.
Figure 22. Map illustrating the 1980 elemental population using on-site wells within the ground water quality management study area.
Figure 23. Map illustrating the 1980 elemental population using municipal water within the ground water quality management study area.
Table 4. 1980 Population Data for the Portion of Each Town Within the Management Area.

<table>
<thead>
<tr>
<th></th>
<th>Bourne</th>
<th>Falmouth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total households</td>
<td>4385</td>
<td>11658</td>
</tr>
<tr>
<td>connected to public supplies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total households</td>
<td>188</td>
<td>1756</td>
</tr>
<tr>
<td>connected to on-site</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Persons/household</td>
<td>2.35</td>
<td>2.20</td>
</tr>
<tr>
<td>Average daily</td>
<td>10304</td>
<td>27886</td>
</tr>
<tr>
<td>population using</td>
<td></td>
<td></td>
</tr>
<tr>
<td>public supplies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average daily</td>
<td>442</td>
<td>3863</td>
</tr>
<tr>
<td>population using on-site</td>
<td></td>
<td></td>
</tr>
<tr>
<td>well water</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 5. The 1980 and Long Term Pumpage Pattern in Falmouth and Bourne.

<table>
<thead>
<tr>
<th>Water District and Well Name</th>
<th>Location (i,j)</th>
<th>Safe Yield [m$^3$/d]</th>
<th>Fraction of water capacity</th>
<th>1980 Assumed Pumpage Rate [m$^3$/d]</th>
<th>Long term pumpage rate [m$^3$/d]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bourne</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Well #1</td>
<td>5,4</td>
<td>2241</td>
<td>0.332</td>
<td>1061</td>
<td>946</td>
</tr>
<tr>
<td>Well #2</td>
<td>7,4-8,4</td>
<td>1254</td>
<td>0.167</td>
<td>850</td>
<td>476</td>
</tr>
<tr>
<td>Well #3</td>
<td>5,4</td>
<td>1254</td>
<td>0.167</td>
<td>472</td>
<td>476</td>
</tr>
<tr>
<td>Well #4</td>
<td>5,4</td>
<td>1254</td>
<td>0.167</td>
<td>466</td>
<td>476</td>
</tr>
<tr>
<td>Well #5</td>
<td>7,4-8,4</td>
<td>1254</td>
<td>0.167</td>
<td>466</td>
<td>476</td>
</tr>
<tr>
<td>Falmouth</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long Pond</td>
<td>12,3-12,4</td>
<td>53429</td>
<td>0.989</td>
<td>10908</td>
<td>10911</td>
</tr>
<tr>
<td>Fresh Pond</td>
<td>11,7</td>
<td>3789</td>
<td>0.011</td>
<td>124</td>
<td>121</td>
</tr>
<tr>
<td>Otis AFB (South of weeks)</td>
<td>7,7</td>
<td>&gt;3309</td>
<td>1.000</td>
<td>3414</td>
<td>3309</td>
</tr>
<tr>
<td>Sandwich</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Well #5</td>
<td>7,7</td>
<td>2554</td>
<td>-</td>
<td>-</td>
<td>2544</td>
</tr>
</tbody>
</table>

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6.3.3.3. Estimation of Water Usage Patterns for Bourne, Falmouth and the Otis Air Force Base

Data on municipal water use were obtained from available annual water district reports and recent engineering studies. Records on the Falmouth water distribution system were incomplete at the town, county, and state levels. No records were available on the percent unaccounted water. The town estimated commercial usage to be ten percent.

Estimates of the breakdown of water use in Bourne were obtained from a recent water system study performed by Whitman and Howard, Inc. (1984). The breakdown is shown in Table 6. Virtually all homes in Bourne are connected to septic systems or cesspools. Based on the Bourne study, the daily per capita domestic usage was 216 liters (57 gal) which compares well with the U.S. Environmental Protection Agency (USEPA) (1980) estimate of 170/d (45 gal/d) for average septic system flows. The USEPA estimate was the result of averaging several studies where observed septic system flows ranged from 30-3 to 385 L/day.

In 1980 the fraction of households occupied year-round in Bourne (81 percent) was only slightly higher than Falmouth (74 percent); consequently, the per capita domestic usage rate from Bourne was considered a reasonable estimate of the per capita domestic use in Falmouth. Only 54.5 percent of all water pumped in Falmouth could be accounted to domestic use (using the 216 L/day-person and a 1980 service population of 27886) (see Table 6). The amount of annual pumpage devoted to commercial needs in Falmouth was therefore assumed to be 15 percent. The assumed commercial fraction was similar to that found in the contiguous town of Mashpee (14 percent in 1984) where the economic profile of the population was the same as Falmouth's (U.S. Census Bureau, 1980). After commercial and domestic usage, 30.5 percent of Falmouth's municipal water flows remained unaccounted. Water losses through the Falmouth distribution system appeared quite high even against observed losses in Yarmouth (20.4 percent, in 1984) and Cotuit (23 percent in 1980 and 1984). Not all of the water lost could be attributed to leakage; however, whether leaked or used by unmetered users all was assumed to recharge the aquifer.

Water withdrawn from the aquifer underlying the military reservation fell into two water use categories: domestic and unaccounted. The percent breakdown for each category was estimated from pumpage records and wastewater flows for the year 1979 (LeBlanc, 1984). Wastewater flows at the Otis sewage treatment facility amounted to 78 percent of the annual pumpage. Twenty-two percent of all pumpage or 278 m³/d of water was lost enroute to military housing units through the Otis water distribution system and from homes to the Otis wastewater treatment plant through sewer exfiltration.
### Table 6. Municipal and Military Water Use Patterns.

<table>
<thead>
<tr>
<th></th>
<th>Bourne</th>
<th>Falmouth</th>
<th>Otis AFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total pumpage for 1980 (m³/d)</td>
<td>2850.5</td>
<td>11032</td>
<td>3309</td>
</tr>
<tr>
<td>% Domestic usage</td>
<td>78.2</td>
<td>54.5</td>
<td>86.7</td>
</tr>
<tr>
<td>% Commercial</td>
<td>4.4</td>
<td>15.0 (assumed)</td>
<td>0.0</td>
</tr>
<tr>
<td>% Unaccounted</td>
<td>17.4</td>
<td>30.5 (assumed)</td>
<td>13.3</td>
</tr>
<tr>
<td>Total Population Served</td>
<td>10304</td>
<td>27886</td>
<td>2000</td>
</tr>
<tr>
<td>Per capita domestic usage</td>
<td>.216</td>
<td>0.216 (assumed)</td>
<td>1.655</td>
</tr>
</tbody>
</table>
Using the ten percent exfiltration rate reported by (Porter, 1980), the total loss of flow through sewer leaks estimated 8.7 percent. The remaining 13.3 percent water loss totalled as leakage from the water distribution system. The flow rate listed in Table 5 as the 1980 pumpage for Otis AFB was calculated from per capita usage in 1975 and the long term population projection for the military reservation (CCPEDC, 1978). The quoted pumpage is only three percent lower than the observed 1979 flow (see Table 5); hence, Otis had attained the maximum projected development level as of 1979.

6.3.3.4 Estimation of the Total Artificial Recharge from the Combined Domestic and Commercial Use of Municipal Well Water and Septic Systems

Within the management study area, there are several sources of artificial recharge. Among those sources are flows from leaky water distribution systems and septic system effluents generated from domestic and commercial activities.

Estimates of the combined recharge from these sources were predicated on the following assumptions:

1) the ratio of flows associated with domestic and commercial use remained constant within a town regardless of source of water (i.e., municipal or on-site);

2) the ratio of flow associated with domestic and commercial use is uniform in space.

With the above assumptions, the combined recharge from domestic and commercial septic systems, and leakage from water distribution pipes was calculated as

\[ Z = (1 + K_c + K_u) \cdot q_c \cdot P_z \]  

(27)

where

- \( Z \) = combined recharge from septic system effluent derived from domestic and commercial activities plus recharge from water distribution system leakage, 
  \( (m^3/d) \);
- \( q_c \) = per capita domestic usage rate, which is .216 m³/d;
- \( K_c \) = ratio of commercial flows to domestic flows;
- \( K_u \) = ratio of unaccounted water loss for the water distribution system to domestic flows;
- \( P_z \) = average daily population served by municipal water.
Table 10 listed, for each town and Otis AFB, the values of constants used in the above equation.

6.3.3.5 Estimation of Artificial Recharge from Sewered Residential Areas

Housing units overlying sewered areas have both public water and wastewater services. Artificial recharge from these areas originates from sewer exfiltration and leakage from water distribution system.

Wastewater flows from housing units on Otis AFB are collected in sewers and transported to the Base treatment plant. In Falmouth approximately 375 properties will be connected to a small sewer system. The collected sewage will be conveyed north to a treatment facility located between Route 28 North and the Falmouth Sanitary Landfill between Blacksmith Shop Road and Thomas Landers Road.

Recharge from households connected to sewers was calculated under the assumed conditions that: 1) unaccounted losses of water service flows result in direct ground water recharge, and 2) the approximate sewer exfiltration rates are 10 percent of the flows discharged to the sewers. Recharge fluxes were calculated using the following equation;

\[ S = \left(0.10 \cdot (1 + K_c) + K_u \right) \cdot q_c \cdot P_s \]  \hspace{1cm} (28)

where

\[ S \] = combined recharge from sewer exfiltration and water distribution leakage, (m$^3$/d);

\[ q_c \] = per capita domestic usage rate (m$^3$/d);

\[ P_s \] = average daily service population for a sewered area;

\[ K_c \] and \[ K_u \] were defined in equation (27).

Values for constants used the above equation are listed in Table 7. Actual recharge estimates appear in Table 8 along with the location of these flows in the management area. The flows for Falmouth were calculated by allowing the estimated service population to equal the product of the number of properties connected to the sewer and the number of people per household. Recharge flows for Otis AFB were estimated from the projected long term service population of 2000 people (CCPEDC, 1978).

6.3.3.6. Calculation of Recharge Flows from Sewage Treatment Facilities

There are two sewage treatment facilities in the management area. The Otis plant is located in the south east corner of Bourne,
Table 7. Constants for the Distributed Artificial Recharge Equation.

<table>
<thead>
<tr>
<th></th>
<th>Bourne</th>
<th>Falmouth</th>
<th>Otis AFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_c$</td>
<td>0.216</td>
<td>0.216</td>
<td>1.655</td>
</tr>
<tr>
<td>[m$^3$/d]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_c$</td>
<td>0.056</td>
<td>0.275</td>
<td>0.000</td>
</tr>
<tr>
<td>$K_u$</td>
<td>0.223</td>
<td>0.560</td>
<td>0.154</td>
</tr>
</tbody>
</table>
Table 8. Recharge from Sewered Neighborhoods and Waste Water Treatment Facilities.

<table>
<thead>
<tr>
<th>Source</th>
<th>Element</th>
<th>Flow $m^3/d$</th>
<th>Service Population within the Management Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Otis Sewers</td>
<td>8.5</td>
<td>364</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>8.6</td>
<td>364</td>
<td>1000</td>
</tr>
<tr>
<td>Falmouth Sewers</td>
<td>14.3</td>
<td>42</td>
<td>280</td>
</tr>
<tr>
<td></td>
<td>14.4</td>
<td>81</td>
<td>546</td>
</tr>
<tr>
<td>Otis Treatment Plant</td>
<td>9.6</td>
<td>2581</td>
<td>2000</td>
</tr>
<tr>
<td>Falmouth Plant</td>
<td>11.3</td>
<td>1136+218</td>
<td>1106</td>
</tr>
</tbody>
</table>
while the Falmouth plant is located south of the Crocker Pond Watershed. The Otis plant discharges secondary effluent to 24 sand beds (each are .2 ha). The treated sewage percolates to the water table which lies six meters below the surface of the sand beds. Further details on the design of the treatment system can be obtained from LeBlanc (1984). Recharge from the plant was estimated as 78 percent of the projected long term pumpage rate: 3309 m³/d.

The Falmouth sewage treatment plant is a combination spray irrigation and rapid infiltration system. Upon completion of a two phase sewer construction project, the design flow capacity of the plant will be 4700 m³/day (1.25 mgd). Phase I is just nearing completion; phase II may never begin. With the completion of phase I, the maximum capacity of the plant is projected at 2800 m³/day (.75 mgd).

Recharge from the Falmouth plant was expected equal to the sum of 1137 m³/d (300000 gpd) originating from Woods Hole (USEPA, Maguire, Inc., and Camp, Dresser and McKee, Inc., 1981) and 218 m³/d collected from the newly constructed sewers in Woods Hole, Falmouth Beach, Falmouth Center, Davis Straits, and Main Street East. The newly sewer section totals 400 property connections. Recharge was calculated from projected combined domestic and commercial flows that were reduced by ten percent to account for exfiltration.

6.3.3.7. Estimation of Nitrate Concentrations in Septic System Effluents

On-site domestic and commercial waste disposal systems are designed to capture the solids and partially degrade influent wastewaters anaerobically before a clarified effluent is distributed over a seepage field and allowed to percolate through the soils where further aerobic and anaerobic treatment of the wastewaters may occur. As the effluent percolates through the soils, organic nitrogen and ammonia are oxidized to nitrate. Approximately 40 percent of the nitrate is then reduced through denitrification to nitrogen gas and nitrous oxide (Porter, 1980). Of the total mass of nitrogen delivered to these waste disposal systems approximately 50 percent reaches the saturated zone as nitrate. Little if any degradation or adsorption of nitrate occurs in the saturated zone, unless conditions are anaerobic with sufficient dissolved organic substrate to support denitrifying organisms; thus, nitrate is often assumed to behave as a conservative soluble constituent in the subsurface environment. Ground waters of Cape Cod aquifer are generally oxygenated and low in dissolved organics; therefore, significant reduction of nitrate is not expected.
For domestic waste flows, Porter (1980) estimated the per capita nitrogen production at 4.08 kilograms/year (9 pounds/y). Of the total nitrogen produced, the tank and the vadose should remove 50 percent. Thus the effective daily per capita nitrate load would total 5592 mg. Using the daily per capita flow rate from Bourne (216 L/d) the projected nitrate concentration at the top of the saturated zone was 25.9 mg/L as N. This concentration and flux appeared consistent with estimates presented by the USEPA (1980) and observed nitrate concentrations in effluent discharges (Dudley and Stephenson, 1973 in Porter, 1980).

Nitrate concentrations from nonresidential activities would vary greatly between commercial activities (i.e., hotels, swimming pools, restaurants); however, USEPA (1980) determined that many nonresidential wastewater generating sources produce effluents having similar water quality as residential sources. Because additional data to characterize the nitrate concentration from commercial septic system flows was unavailable, the domestic wastewater nitrogen levels were used.

6.3.3.8. Estimation of Nitrate Concentrations in Otis Wastewater Flows

Estimates were needed for the total nitrogen in wastewater influent and effluent streams of the Otis sewage treatment facility. Total nitrogen concentrations were used as estimates of the potential nitrate concentration from streams of completely nitrified exfiltration flows and secondary effluent recharge flows. The total nitrogen load would enter the aquifer as nitrate, ammonia and organic nitrogen. If aerobic conditions exist the ammonia and organic nitrogen would be ultimately oxidized to nitrate as a ground water plume moves down gradient from the recharge beds or the leaky sewer.

The inorganic fraction of secondary domestic wastewater effluent typically represents 79 percent of the total nitrogen and 56 percent of the total nitrogen in raw sewage (Metcalf and Eddy, Inc., 1979). From the total inorganic nitrogen concentrations reported by Vaccaro et al., (1979) in their study of Wastewater Renovation and Retrieval on Cape Cod, the total nitrogen in the Otis raw sewage was estimated 31.96 ± 5.80 mg/L-N. The calculated nitrogen in the effluent discharged to the sand beds was 22.67 ± 4.12 mg/L-N. LeBlanc (1984) found an average total nitrogen level of 19 mg/L as N for Otis secondary effluent.

In a well situated close to the recharge beds LeBlanc (1984) reported total ground water nitrogen at 24 mg/L as N. If this nitrogen concentration reflects the level of nitrogen removal as sewage percolates through the vadose zone, then nitrate concentrations in recharge beneath leaky sewers may have similar water quality. Nitrogen concentrations in the exfiltration flows and
the secondary effluent recharge were assumed similar; hence, the value of 25 mg/L as N was used for both flows as an average of the extreme estimate derived from Vaccaro et al., (1979) and the observed level from LeBlanc (1984).

6.3.3.9. Estimation of Nitrate Concentrations in Falmouth Wastewater Flows

Wastewater in Falmouth is primarily of domestic and commercial origin with significant amounts of septage added. Research by Vaccaro et al., (1979) illustrated that proper operation of the facility could achieve ground water nitrate concentrations of 1-3 mg/L as N. In this work a nitrate concentration of 2 mg/L as N was used for the sewage recharge.

Because exfiltration flows percolate through the vadose zone, the fate of nitrogen in those flows was presumed similar to that of nitrogen in septic system effluents. A fifty percent reduction of nitrogen occurs with the use of septic systems; therefore, under the equal nitrogen reduction assumption, the total nitrogen level in exfiltration flows reaching the saturation zone was calculated as 26 mg/L. After the installation of a planned spray irrigation system, the Falmouth Wastewater Treatment Plant expects to reduce total nitrogen discharges to 15 mg/L (as N).

6.3.3.10. Calculation of Lawn Fertilizer Loads

The application of fertilizers for lawn cultivation could be a major source of ground water nitrates on Cape Cod. The Cape Cod Planning and Economic Development Commission (1979) estimated that the average home owner applies 1.46 kg of nitrogen (as N) per 100 m² of lawn area per year.

The Suffolk County Department of Environmental Control (in Porter, 1980) investigated nitrogen loading from lawn fertilizers in a sewered housing development in central Long Island, New York. A nitrogen loading rate of 1.07 kg/100 m² was determined from a survey of homeowners.

The capacity for turf to assimilate nitrogen had been estimated at 0.5 kg/100 m² y for lawns ten years or older (Porter, 1980). For Cape Cod, the consequence would be that as much as 60 percent of the nitrogen applied leaches to the ground water (Flipse, 1984 and CCPEDC, 1979). In an effort to calculate nitrate loading from lawn fertilizers, the CCPEDC estimated that each household applied an average of 6.8 kg of nitrogen per year. Of the total mass of nitrogen applied, 60 percent (or 4.08 kg) was expected to leach below
the root zone to the water table. The effective daily per capita nitrate loading rates were estimated for Bourne and Falmouth by dividing the number of residents per household into the daily effective household rate (0.60 x gross application rate).

6.3.3.11. Estimation of Background Nitrate Loading

The background nitrate concentration for Cape Cod ground waters was reported by Frimpter and Gay (1979) to be 0.5 mg/L as N. The source of this nitrogen could be from precipitation, animal wastes, or possibly leaching of nitrates produced from the natural oxidation of animal and vegetable matter. Likens et al. (1977) reported a weighted annual mean of 1.47 mg/L nitrate-N in bulk precipitation from 1965-1974 in the Hubbard Brook Experimental Forest, New Hampshire. Other records of nitrates in precipitation revealed concentrations on the order of 1 mg/L or less (Frizzola in Flipse, 1984). The actual source(s) of background nitrate were not ascertained for Cape Cod. In this modeling effort, background nitrate levels were generated through a fixed nitrate concentration of 0.5 mg/L as N in all natural recharge flows.

6.3.4. Calculation of the Components in the Recharge and Source Concentration Vectors

The components of the recharge and source concentration vectors, that appeared in the continuity constraints (see section 6.3.2), were calculated or directly obtained from the Cape Cod data presented above on the 1980 hydrologic stresses and nitrate pollution.

The equations used to calculate the various discrete recharge and discrete source concentration vector components are presented below. Constants used in the equations that follow are given in Table 9. The results of calculations are summarized in Table 10.

6.3.4.1. Calculation of the Nitrate Flux Associated with Recharge from the Use of Municipal Well Water, Septic Systems and Lawn Fertilizers

Several parameters were needed to calculate the effective nitrate concentration in artificial recharge derived from a composite land use activity using municipal well water, septic systems and lawn fertilizers. Those parameters included: 1) the volume of flow from septic system; 2) the recharge flows attributed to loses from water distribution systems; 3) the nitrate concentration at the water table for septic system effluents, 4) the nitrate concentration in leakage from the water distribution system; and 5) the effective per capita nitrogen loading rate from fertilizers.
Table 9. Constants Used in the Source Recharge and Nitrate Flux Equations.

<table>
<thead>
<tr>
<th></th>
<th>Bourne</th>
<th>Falmouth</th>
<th>Otis AFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q_{e_{i,j}}$ [m$^3$/d]</td>
<td>0.216</td>
<td>0.216</td>
<td>1.655</td>
</tr>
<tr>
<td>$K_{c_{i,j}}$</td>
<td>0.056</td>
<td>0.275</td>
<td>0.000</td>
</tr>
<tr>
<td>$K_{u_{i,j}}$</td>
<td>0.223</td>
<td>0.560</td>
<td>0.154</td>
</tr>
<tr>
<td>$C_{ss}(mg/l \text{ NO}_3-N)$</td>
<td>26.000</td>
<td>26.000</td>
<td>-</td>
</tr>
<tr>
<td>$F_{i,j}$ (mg/d·person)</td>
<td>4760</td>
<td>5070</td>
<td>-</td>
</tr>
<tr>
<td>$C_{E_{i,j}}$</td>
<td>-</td>
<td>26.000</td>
<td>25.000</td>
</tr>
</tbody>
</table>
Table 10. Summary of Nitrate Source Concentrations (mg/L as N).

<table>
<thead>
<tr>
<th>Source (by Model Symbol)</th>
<th>Bourne</th>
<th>Falmouth</th>
<th>Otis AFB</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{i,j}$</td>
<td>38.71</td>
<td>30.86</td>
<td>-</td>
</tr>
<tr>
<td>$W_{i,j}$</td>
<td>46.96</td>
<td>44.42</td>
<td></td>
</tr>
<tr>
<td>$S_{i,j}$</td>
<td>-</td>
<td>38.90</td>
<td>9.87</td>
</tr>
<tr>
<td>$U_{i,j}$</td>
<td>-</td>
<td>2.00</td>
<td>25.00</td>
</tr>
<tr>
<td>$Q_{i,j}$</td>
<td>0.50</td>
<td>0.50</td>
<td>0.50</td>
</tr>
</tbody>
</table>
The equation used to calculate discrete recharge from the use of municipal well water, septic systems, and fertilizer was developed from equation (27):

\[
Z_{i,j} = \frac{[1 + K_{c_{i,j}} + K_{u_{i,j}}] \cdot q_{c_{i,j}} \cdot P_{z_{i,j}}}{\Delta x \cdot \Delta y} \quad \forall \ i \text{ and } j
\]

where

- \( Z_{i,j} \) = recharge in element \( i,j \) from septic system effluent derived from domestic and commercial use of municipal well water plus recharge from water distribution system leakage, (m/d);
- \( P_{z_{i,j}} \) = average population in element \( i,j \) using municipal water and septic systems;
- \( q_{c_{i,j}} \) = elemental per capita domestic usage, (m³/day);
- \( K_{c_{i,j}} \) = ratio of commercial flows to domestic flows in element \( i,j \);
- \( K_{u_{i,j}} \) = ratio of unaccounted water loss for the water distribution system to domestic usage in element \( i,j \);
- \( \Delta x \) and \( \Delta y \) = the x and y dimensions of the numerical element, (m).

Notice that no direct flows were attributed to lawn watering. If lawn watering were a significant component of domestic usage, than the recharge would be over estimated because calculated domestic recharge flows ignored losses due to evapotranspiration. The consequence would be underestimated nitrate concentrations in the combined recharge flows and in the predicted ground water quality impacts, because the total nitrate load would remain the same regardless of consumptive water losses. The regional water quality impacts of the consumptive losses were considered minor, were consequently ignored. This conclusion was predicated on ten percent consumptive losses (Quadri, 1984) in total pumpage, which amounted to less than five percent of total natural recharge; hence, a minor 0.5 percent loss of steady ground water flow.

The equation used to calculate the nitrate concentration in the discrete recharge flow \( Z_{i,j} \) for all \( i \) and \( j \) was

\[
C_{z_{i,j}} = \frac{C_{ss} \cdot (1 + K_{c_{i,j}}) \cdot q_{c_{i,j}} + F_{i,j} \cdot \sum K_{u_{i,j}} \cdot q_{c_{i,j}}}{(1 + K_{c_{i,j}} + K_{u_{i,j}}) \cdot q_{c_{i,j}}} \quad \forall \ i \text{ and } j
\]

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where \( C_{z_{i,j}} \) = effective nitrogen concentration in all recharge flows in element \( i,j \) derived from the combined septic systems, and lawn fertilizers, (mg/L as N);
\( C_{ss} \) = effective concentration of nitrate in recharge from domestic and commercial use of septic systems, (mg/L);
\( C_{m} \) = effective concentration of nitrate in recharge from leaking municipal water distribution systems, (mg/L);
\( F_{i,j} \) = effective per capita nitrate load from lawn fertilizer, (mg/d as N).

Notice that nitrate loads associated with leakage in the water distribution system were considered. The nitrate concentration in municipal water supplies was low which meant that the expected ground water nitrate contributions from leaky water distribution systems would be minor.

6.3.4.2. Calculation of the Nitrate Flux Associated with Recharge from the Use of On-site Wells, Septic Systems, and Lawn Fertilizers

If on-site wells were used in conjunction with septic systems and cesspools, little or no impact on the hydrologic balance of flows would be expected. The effective nitrate concentration from the use of on-site wells, septic systems, and lawn fertilizers was calculated from: 1) the volume of flow from the septic system, 2) the septic system effluent nitrate concentration after percolation to the water table, and 3) the effective per capita nitrogen loading rate from fertilizers.

The discrete recharge flows from the use on-site wells was made with the following equation:

\[
W_{i,j} = \frac{[1 + K_{c_{i,j}}]}{\Delta x \cdot \Delta y} \cdot q_{c_{i,j}} \cdot P_{w_{i,j}}. \quad \forall i \text{ and } j
\]

where \( W_{i,j} \) = recharge in element \( i,j \) from septic system effluent derived from domestic and commercial use of on-site well water, (m/d);
\( P_{w_{i,j}} \) = average daily population in element \( i,j \) using on-site wells.

Notice that consumptive losses were not considered.
The formula developed to calculate the nitrate concentrations at the water table from the combined influence of domestic and commercial activities plus lawn cultivation was

$$ C_{w_{i,j}} = \left[ \frac{C_{33} \cdot (1 + K_{c_{i,j}}) \cdot q_{c_{i,j}} + F_{i,j}}{1 + K_{c_{i,j}}} \cdot q_{c_{i,j}} \right] \forall i \text{ and } j \tag{32} $$

where $C_{w_{i,j}}$ = effective nitrate concentration in all recharge flows in element $i,j$ derived from the combined domestic and commercial use of on-site wells, septic systems, and lawn fertilizers, (mg/L as N).

6.3.4.3. Calculation of the Nitrate Flux Associated with Recharge from the Use of Municipal Well Water, Sewers, and Lawn Fertilizers

The effective nitrate concentration in recharge from the use of municipal water, sewers, and lawn fertilizers was evaluated with information on: 1) volume of flow associated with exfiltration, 2) recharge flow attributable to unaccounted water distribution system losses, 3) nitrate concentration at the water table for exfiltration flows, 4) the nitrate concentration in leakage from the water distribution system, and 5) the effective per capita nitrogen loading rate from fertilizer.

To calculate the discrete recharge flows from the use of municipal well water, sewers, and fertilizer, the following equation was used:

$$ S_{i,j} = \frac{(0.10 \cdot (1 + K_{c_{i,j}}) + K_{u_{i,j}}) \cdot q_{c_{i,j}} \cdot p_{s_{i,j}}}{\Delta x \cdot \Delta y} \forall i \text{ and } j \tag{33} $$

Notice that sewer exfiltration was limited ten percent of the domestic and commercial flow collected from each household and business as discussed in section 6.3.3.5.

The effective nitrate concentration in the recharge flow ($C_{s_{i,j}}$) was calculated with the following equation;
\[ C_{s_{i,j}} = \frac{C_{E_{i,j}} \cdot 0.1 \cdot (1 + K_{c_{i,j}}) \cdot q_{c_{i,j}} + C_m \cdot K_{u_{i,j}} \cdot q_{u_{i,j}} \cdot F_{i,j}}{[0.1 \cdot (1 + K_{c_{i,j}}) + K_{u_{i,j}}] \cdot q_{c_{i,j}}} \quad \forall \ i \text{ and } j \]  

where \( C_{s_{i,j}} \) = effective nitrate concentration in all recharge flows in element \( i,j \) derived from the combined domestic and commercial use of municipal water, sewers, and lawn fertilizers, (mg/L as N);

\( C_{E_{i,j}} \) = effective nitrate concentration in element \( i,j \) for sewer exfiltration recharge, (mg/L as N).

6.3.4.4. Calculation of the Nitrate Flux Associated with Recharge from Land Application of Secondary Sewage and Background Loads

Estimates of recharge from the land application of secondary sewage at Falmouth and Otis amounted to 90 percent of the sum of domestic and commercial flows from the sewered areas. Thus,

\[ U_{i,j} = 0.9 \sum_{i=1}^{n} \sum_{j=1}^{m} (1 + K_{c_{i,j}}) \cdot q_{c_{i,j}} \cdot p_{s_{i,j}} \cdot l_{i,j} \quad \forall \ i \text{ and } j \]  

where \( U_{i,j} \) = the recharge in element \( i,j \) from land application of secondary sewage collected from elements where underlying sewers convey flows to site \( i,j \).

\( o_{i,j} \) = integer variable which is equal to one if sewers underlying an element convey flows to land application site \( i,j \).

Sections 6.3.3.8 and 6.3.3.9 talked about the effective nitrate concentration in recharge flows from the land application of secondary sewage at both sewage treatment facilities.

The final nitrate flux to be incorporated in the model was the nitrogen load responsible for observed background nitrates of 500 mg/L as N. The source or sources of background nitrates were not elucidated but their groundwater quality impacts were approximated by specifying a nitrate concentration in natural recharge equal to 500 mg/L nitrate as N.
6.3.5. Constructing and Solving Management Models I and II

Fortran programs were developed that create computer files containing the objective functions and continuity constraints for the two nonpoint source ground water pollution management models. Each Fortran program embodied the ground water flow model to define necessary fluid velocity coefficients which appeared in the continuity constraints. The data required to create the computer files included 1) the area and dimensions of the elements, 2) the elemental recharge rates of all sources, 3) the elemental pumpage rates, 4) the piecewise aquifer transmissivities, 5) the hydraulic model and contaminant model boundary conditions, and 6) the nitrate concentrations in all recharge flows.

Model II contained the same decision variables and constraints as Model I. This model was used to elucidate regional nitrate impact isopleths around Long Pond: Falmouth's major municipal water supply. Long Pond intersects three elements (I,J = 12,3 - 12,4 - 13,4). Equal pumpage at each node was used to approximate the aquifer stress produced when water was withdrawn from this pond. The objective function, formulated as a summation of contaminant decision variables from the pond elements, appeared as;

Maximize \( C_{12,3} + C_{12,4} + C_{12,5} \)

Model I and II were solved using the regular simplex algorithm made available through the Multi Purpose Optimization System package (Northwestern University, 1978). Files representing the linear programming formulations of Models I and II were submitted as input data to the optimization system package.

6.3.6. Results of Model I and II

The results from solving Model I included the steady-state ground water nitrate predictions for each element of the discretized management area for 1980 levels of development and the optimum values of dual variables. Regional ground water quality changes effected through alternative nitrate disposal stresses were interpreted through the dual variables.

Figure 24 presents the predicted steady-state nitrate concentration contours for ground waters underlying Bourne and Falmouth. For the entire management area, the average simulated water quality was 1.5 mg/L nitrate nitrogen. The extent of ground water nitrate pollution reflected the location, intensity and type of land use activity in 1980. In general, the highest levels of simulated ground water nitrates occurred in areas of the highest density of residential/commercial land use (i.e., South Falmouth). Elemental nitrate concentrations were predicted under 5 mg/L as N.
Figure 24. Map depicting the steady-state nitrate nitrogen concentration (mg/L) contours predicted by Model I from 1980 development patterns.
every where except in the element \((i=9, j=6)\) where the Otis wastewater treatment facility is located.

The concentration of nitrate nitrogen in the center of the Otis plume was 5.9 mg/L which was much lower than the observed 20 mg/L total nitrogen reported by LeBlanc (1984). Part of this discrepancy is explained through the exaggerated predicted width of plume near the sewage treatment facility: LeBlanc estimated the plume width at 1000 meters whereas the Model I predicted the plume width at 2000 meters. Nitrogen contour lines drawn by LeBlanc (1984) were integrated to obtain the nitrogen mass per unit longitudinal length of the plume; this figure, when divided by the 2000 meter width, produced an average plume concentration of 5.5 mg/L as N which was similar to the above estimate from the model. The length of the simulated Otis plume was 3000 meters (using the 3 mg/L contour as a boundary). Elevated ground water nitrates found in observation wells (LeBlanc, 1984) indicated that the plume was at least that long.

For the elements which contain water supply wells and ponds, the water quality ranged from 0.8 to 1.4 mg/L as Nitrate nitrogen. The highest steady state nitrate levels were predicted at Long Pond and Fresh Pond which were both in Falmouth (see Table 11).

Postoptimality analysis retrieved the optimum values of dual variables associated with the less-than-or-equal-to continuity constraints. The values of the duals were used to interpret regional water quality impacts effected by unit changes in nitrate nitrogen loading at each element. The values of the dual variables give the increases in the value of the objective function (which is the average nitrate concentration for the whole management area) obtained from unit relaxations of the less-than-or-equal-to continuity constraints (i.e., from a unit increase in the nitrate loading rate). The discrete dual variables were plotted and contour lines were drawn. These represent regional water quality impact isopleths.

The iso-impact contour plot is portrayed in Figure 25. The numbers represent the increased nitrogen concentration (ug/L as N) (i.e., \(C_{i,j}\)'s summed over the whole region—it does not indicate where in the region the increases will occur.) resulting from an increased nitrate nitrogen loading rate of one kilogram per day per square kilometer along the contour line. This change in the loading rate is equivalent to increasing the year-round resident population by 100 per square kilometer.

The iso-regional-water quality impact map indicated that increased nitrate loadings over the interior region would lead to greater increases in the regional nitrate concentrations than would equivalent increases in nitrate loading along the coast. The iso-impact contours followed a pattern similar to the water table contours (see Figure 15).
Table 11. Model I and II Predicted Regional Steady-state Nitrate Concentrations at Wells from 1980 Land Use Activities.

<table>
<thead>
<tr>
<th>Water District and Well Name</th>
<th>Location (i,j)</th>
<th>Predicted Nitrate Concentration (mg/L as N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bourne</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Well #1</td>
<td>5,4</td>
<td>1.3</td>
</tr>
<tr>
<td>Well #2</td>
<td>7,4-8,4</td>
<td>0.8</td>
</tr>
<tr>
<td>Well #3</td>
<td>5,4</td>
<td>1.3</td>
</tr>
<tr>
<td>Well #4</td>
<td>5,4</td>
<td>1.3</td>
</tr>
<tr>
<td>Well #5</td>
<td>7,4-8,4</td>
<td>0.8</td>
</tr>
<tr>
<td>Falmouth</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long Pond</td>
<td>12,3-12,4-13,4</td>
<td>1.4</td>
</tr>
<tr>
<td>Fresh Pond</td>
<td>11,7</td>
<td>1.3</td>
</tr>
<tr>
<td>Otis AFB (south of Weeks)</td>
<td>7,7</td>
<td>1.0</td>
</tr>
<tr>
<td>Sandwich</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Well #5</td>
<td>7,7</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Figure 25. Map depicting the global nitrate nitrogen loading impact isopleths constructed from the optimum values of dual variables associated with continuity constraints of Model I. Numbers represent the ug/L increase in average ground water nitrate nitrogen over the region for an increase nitrate nitrogen load of 1 kg/day-sqkm.
The differences between coastal and interior regions are explained when consideration is given first to the cumulative impacts of pollution occurring in the interior regions and secondly to the small marginal changes in local contaminant levels effected by nitrate loading near the coast. Sources located at interior nodes have a cumulative effect on regional contaminant levels because they affect nitrate concentrations in all down gradient nodes. Sources located nearer to discharge zones have a small cumulative contaminant impact because dissolved solutes do not travel through the ground water flow system to the same extent. The cumulative pollution effect of locating sources over recharge areas has lead some to recommend that location of potential sources of ground water pollution be located near discharge zones. Furthermore, sources located near recharge areas have a greater marginal impact on local ground water quality than sources located near discharge zones; this is because the available ground water flows to dilute the pollution are much less than the cumulative flows found near discharge zones.

A primary implication of the iso-regional-water-quality-impact contour map for Bourne and Falmouth is that certain regions of the area (especially areas occupied by the Military Reservation) are more important with regard to the preservation of regional water quality.

The results from solving Model II included the same steady-state ground water nitrate predictions over the towns of Bourne and Falmouth. Values of the dual variables associated with the less-than-or-equal-to continuity constraints were plotted. As with the last model, the values of the dual variable are used to identify increases in the value of the objective function (which is the average nitrate concentration for the three elements intersected by Long Pond: \(i,j = 12,3\) and \(12,4\) and \(13,4\)) obtained from unit relaxations of the less-than-or-equal-to continuity constraints. Iso-water-quality-impact contours were drawn between discrete values of the dual variables (see Figure 26).

The numbers associated with each contour represent the approximate ug/l increase in ground water nitrate nitrogen observed over ground waters equally withdrawn from elements \(i,j = 12,3\) and \(12,4\) and \(13,4\) from an increased nitrate load of one kilogram nitrogen per day per square kilometer in elements along that contour. The configuration of contours in Figure 15 illustrate the areal extent to which water quality around Long Pond is determined by upgradient sources. The contours extend for several kilometers into the regional water table contours (See Figure 15). The shape of the iso-impact contours is as one would expect for nodes which lie down gradient of a ground water mound or near discharge zones. After nitrate is introduced to the aquifer, the fraction reaching the target nodes is determined by the change in the magnitude and direction of the darcian velocity field. As the assumed source moves towards the target elements the iso-impact contours increase because a larger fraction of the contaminant loaded to the aquifer...
Figure 26. Map depicting the nitrate nitrogen loading impact isopleths around Long Pond constructed from the optimum values of dual variables associated with continuity constraints of Model II. Numbers represent the ug/L. increase in ground water nitrate nitrogen around Long Pond for an increase nitrate nitrogen load of 1 kg/day·sqkm.
reaches these elements. The contours delineate zones which differ in their significance to efforts to protect long term availability of ground water resources around Long Pond. Zones close to Long Pond are most important, especially if they are upgradient. The contours map out the steady-state response of the ground water system at elements containing Long Pond due to increased nitrate loading anywhere within the bounded study area. The long term water quality impact of modified subsurface disposal activities can be evaluated with Figure 26 if the source location and nitrate loading rate are known and the effects on ground water hydraulics are minor.

6.4. Conclusions

It must be remembered that the objective of these management models is to relate regional nonpoint source pollutant loadings to regional water quality. Thus the model's strength is in explaining the relative impacts of alternative development patterns (i.e., alternative placement and strength of sources) and at evaluating areally averaged water quality. The concentrations predicted by the model may not correspond to concentrations found at a given well in the assigned element. The concentrations at a specific well may be greater or less than the predicted concentrations depending on the location of the well relative to the local sources of pollution. The regional contaminant concentrations indicate that if a given well in an element has pollutant concentrations greater than that predicted, then there must also be another well (i.e. alternate well location and screen depth) within that element which will produce water with contaminant concentrations less than that predicted (this is because the regional contaminant predictions represent average elemental contaminant levels).

Model I and Model II are both nonpoint source ground water pollution management models. Both models were applied to a specific area, Western Cape Cod, to examine regional nitrate pollution from land development. The objective function for Model I was a linear summation of the contaminant decision variables (i.e., nitrate concentrations) from the entire management area. The primary constraints of the model were continuity constraints constructed from a defined ground water flow field. Model II is a simplified version Model I that used an objective function which was a summation of contaminant decision variables of elements that were targeted for investigation.

Approximations of hydraulic stresses and nitrate loads were predicated on the 1980 population estimates and information gathered from the literature. Solutions were obtained using available computer algorithms.

Models I and II were used to identify regions in the study area which are critical to the long term protection of regional
ground water quality availability over the study area and in a specific subarea of the region (i.e., the area of the Falmouth municipal well field). Results from Model I and II included steady-state nitrate distributions for Bourne and Falmouth for 1980 development conditions. The nitrate nitrogen in elements containing municipal wells was predicted to be below 1.5 mg/L.

Other results included the optimal values of dual variables associated with relaxing the continuity constraints (i.e., investigating the impacts of marginal increases in nitrate loadings from each element). Plotting contours around the values of the dual variable created iso-water quality impact isopleths.

For Model I, the iso-water quality impact isopleths related changes in nonpoint source nitrate loads to regional changes in ground water quality. A primary implication of the iso-regional-water-quality-impact contour map for Bourne and Falmouth was that certain regions of the area (especially areas occupied by the Military Reservation) should be ranked higher with regard to their importance to the preservation of regional ground water quality.

With Model II the isopleths could predict changes in water quality in a few elements as a result of increased nitrate loadings anywhere in Bourne and Falmouth. The iso-water quality impact plots developed from Model II can be used to identify critical recharge zones: areas most important to the long term preservation of ground water quality at target elements. In this case the elements containing Falmouth's municipal water supply (Long Pond) were chosen as the target elements. As expected, the implication of the Model II results is that activities situated close to the pond have the greatest impact on water quality at the pond. The isopleths give an indication of the extent of areas having significant water quality impact (note the increased areal extent of the contours in the upgradient direction). In addition, the isopleths define the relative significance of separate zones within the recharge area around Long Pond which are critical to the preservation of water quality at the pond. The water quality impacts of placing sources within the region containing a municipal water supply can be evaluated in terms of the approximate water quality impacts on target elements. The actual long term water quality at municipal wells could be more or less than the elemental nitrate concentration shown depending on the actual positions of the well.
CHAPTER 7
NONPOINT SOURCE GROUND WATER POLLUTION MANAGEMENT MODEL TO ELUCIDATE MAXIMUM DEVELOPMENT OF MULTIPLE POLLUTING LAND USE ACTIVITIES

This chapter describes the development and application of a third nonpoint source ground water pollution management model to determine the maximum feasible development of multiple land use activities given restrictions on available resources (i.e., land and water), imposed water quality standards, and specified land use density regulations. The pattern and combination of surface activities at optimality incorporate the present pattern of land use and development.

Model III is applied to the section of Western Cape Cod that constitutes the town Falmouth, Massachusetts. The general formulation was adapted to ascertain the regional patterns of the three forms of residential/commercial land use. The feasible development patterns satisfy constraints on desirable nitrate nitrogen levels at municipal water supplies and for the rest of the region, constraints on available water supplies, and constraints representing imposed regulations on specific land use activities.

Model results include: 1) contour plots of the steady-state regional ground water nitrate distribution from the maximum potential residential/commercial land use; 2) maximum feasible population predictions for each element, and for each land use type in each element; 3) maps illustrating optimal locations of development; 4) maps showing which numerical elements have land use densities approaching zoning restrictions and which elements have predicted nitrate concentrations on the verge of violating standards; and 5) figures depicting the optimal values of dual variables associated with constraints on source densities and water quality.

7.1. Formulation of Management Model III

Model III incorporates all five components of the general management model (recall Chapter 3). The formulation discussed below is presented to evaluate the management of a combination of three composite land use activities known to contribute areal contamination of ground waters. The formulation is general, however, and the variety of surface activities considered could be expanded. Details on model components are presented below.
7.1.1. Decision Variables

Two groups of decision variables are used in this model. The first is the contaminant concentration variables \( C_{i,j} \) for each discrete element \( i,j \). The second group embodies the elemental subsurface recharge flows \( Z_{i,j}, W_{i,j}, \) and \( V_{i,j} \) contributed by the specific land use activities being evaluated with the model.

7.1.2. Objective Function

The objective is a summation of all elemental subsurface recharge flows generated from the various land use activities and sources presently under scrutiny:

\[
\text{Maximize } \sum_{i=1}^{n} \sum_{j=1}^{m} D_{W_{i,j}} \cdot W_{i,j} + D_{Z_{i,j}} \cdot Z_{i,j} + D_{V_{i,j}} \cdot V_{i,j} \quad (36)
\]

where

\[
D_{W_{i,j}} = \text{the reciprocal of the per capita generation of recharge flow } W_{i,j}, \quad \left( \frac{\text{t-person}}{L^3} \right);
\]

\[
D_{Z_{i,j}} = \text{the reciprocal of the per capita generation of recharge flow } Z_{i,j}, \quad \left( \frac{\text{t-person}}{L^3} \right);
\]

\[
D_{V_{i,j}} = \text{the reciprocal of the per capita generation of recharge flow } V_{i,j}, \quad \left( \frac{\text{t-person}}{L^3} \right);
\]

The optimum value of the objective function is the maximum total population in the management area.

7.1.3. Continuity Constraints

The continuity constraints tie together elemental contaminant decision variables with the variables for the contaminant-laden recharge flows flows \( Z_{i,j}, V_{i,j}, \) and \( W_{i,j} \) for all \( i \) and \( j \) in the management area. The form of the continuity constraints is:

\[
[C_{i,j}] = [C_{W}] + [C_{Z}] + [C_{V}] = [C_{i,j}]
\]

[110]
\[ [C_s]\{S\} + [C_u]\{U\} + C_q\{I\}\{Q\} \quad \forall i \text{ and } j \] \hspace{1cm} (37)

where all vectors are defined as above.

Model variables appear in vectors on the left side of the equation sign while the terms on the right reduce to matrices of constants. The terms on the right hand side of the equation sign represent sources of ground water contamination not subject to control or other constant sources.

7.1.4. Management Constraints

Several types of management constraints are employed which bound the feasible values of decision variables. One form of management constraint establishes minimum values on recharge decision variables. The minimum levels reflect the existing intensity of polluting land use activities.

Another form of management constraint imposes upper bounds on allowable values of contaminant and recharge decision variable; these constraints respectively secure the attainment of water quality standards and ensure the satisfaction of various land use regulations. Finally there is a type of management constraint which is applied to restrict the upper limit on the summed values of recharge decision variables to correspond to available resources (i.e., land, water, etc.).

7.1.4.1. Constraints to Incorporate Present Levels of Land Use Activities in all Feasible Development Alternatives

The purpose of these management constraints is to ensure that minimum feasible values of decision variables are set to reflect the existing intensity of permanent land use activities in each element. Generally, two constraints are used in each element to reflect the combinations and intensities of existing land use activities contributing to the nonpoint source pollution of ground waters. The present formulation allows for two land use activities which produce recharge flows \( W_{i,j} \) and \( Z_{i,j} \). A third recharge flow variable \( V_{i,j} \) represents flows generated from switching surface activities from those contributing flows \( W_{i,j} \) to surface activities generating recharge flows \( V_{i,j} \). The three constraints for each element are written as follows:

\[ D_{Z_{i,j}} \cdot Z_{i,j} \geq \text{(total existing population engaged in the} \]
land use activity generating recharge $Z_{i,j}$ in element $i,j$; \( \forall i \) and \( j \)

\[
D_{W_{i,j}} \cdot W_{i,j} + D_{V_{i,j}} \cdot V_{i,j} \geq (\text{total existing population in the land use activity generating recharge } W_{i,j} \text{ in element } i,j) \quad \forall i \text{ and } j.
\]

Constraint (38) sets minimum values for the decision variables $Z_{i,j}$ to reflect permanent discrete populations responsible for these elemental recharge flows. Opportunities to switch from a land use activity producing recharge flow $W_{i,j}$ to one generating flow $V_{i,j}$ are feasible through constraint (39) as long as the total long term combination of polluting activities reflect a population greater than or equal to the original number of people contributing to the elemental flows $W_{i,j}$.

7.1.4.2. Constraints Limiting Maximum Levels of Land Use Activities

Constraints reflecting maximum limits on land use activities take two forms which either prohibit specific surface activities, or place limits on maximum allowable levels on land use activities per unit area of polluting activities. The first constraint type prohibiting activities is expressed simply as:

\[
W_{i,j} \leq 0 \quad \forall i \text{ and } j \text{ where appropriate} \quad \text{(40)}
\]

\[
Z_{i,j} \leq 0 \quad \forall i \text{ and } j \text{ where appropriate} \quad \text{(41)}
\]

\[
V_{i,j} \leq 0 \quad \forall i \text{ and } j \text{ were appropriate} \quad \text{(42)}
\]

The use of constraint (40) and constraint (39) in the model will force the solution to change any existing surface activities producing flows $W_{i,j}$ to those generating flows $V_{i,j}$. Constraint (42) is applied only where the option does not exist to switch land use activities from those generating flow $W_{i,j}$ to those producing flows $V_{i,j}$. Use of constraint (41) is possible only in elements where the land use activity one producing flows $Z_{i,j}$ does not already exist and will not in any future time.

The second form of management constraint to regulate land use activities is implemented if the combination of surface polluting activities compete for resources which cannot be transported between elements. The formulation of this constraint is
\[ L_Z Z_{i,j} + L_W W_{i,j} + L_V V_{i,j} \leq L_{s_{i,j}} \]  \hspace{1cm} (43)

where

- \( L_Z \) = the resource requirement per unit flow of \( Z_{i,j} \);
- \( L_W \) = the resource requirement per unit flow of \( W_{i,j} \);
- \( L_V \) = the resource requirement per unit flow of \( V_{i,j} \);
- \( L_{s_{i,j}} \) = supply of resource in element \( i,j \).

7.1.4.3. Constraints on Available Transferable Resources

If a combination of land use activities compete for common transferable resources, then these constraints ensure that the optimal combination of surface activities does not require more resources than are presently available. Constraints of this type are formulated as summation of all elemental demands for a resource which is set less-than-or-equal-to the available supply of that resource. An example formulation:

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} R_Z Z_{i,j} + R_W W_{i,j} + R_V V_{i,j} \leq R_s \]  \hspace{1cm} (44)

where

- \( R_Z \) = the unit resource requirement per unit recharge flow of \( Z_{i,j} \);
- \( R_W \) = the unit resource requirement per unit recharge flow of \( W_{i,j} \);
- \( R_V \) = the unit resource requirement per unit recharge flow of \( V_{i,j} \);
- \( R_s \) = resource supply.

7.1.4.4. Water Quality Constraints

The last management constraints incorporated in Model III are the water quality constraints. These constraints ensure the optimum pattern of land use activity (which also accommodates the maximum year-round population) which will satisfy desired steady-state ground water quality standards. The formulation is simply:

\[ C_{i,j} \leq Std_{i,j} \]  \hspace{1cm} (45)

where

- \( Std_{i,j} \) = the discrete steady-state water quality standard for element \( i,j \).
7.1.5. The General Formulation of Model III

To summarize, the complete formulation of Model III is:

\[
\text{Maximize } \sum_{i=1}^{n} \sum_{j=1}^{m} D_{wi, j} \cdot w_{i,j} + D_{zi, j} \cdot z_{i,j} + D_{vi, j} \cdot v_{i,j}
\]

s.t.
Continuity Constraints:
\[
\begin{align*}
\text{sgn} & \text{[G][C]} - [C_w][W] - [C_z][Z] - [C_v][V] \\
& = [C_s][S] + [C_u][U] + C_q \text{[I][Q]}
\end{align*}
\]

Management Constraints:
\[
\begin{align*}
D_z \cdot z_{i,j} & \geq \text{(total existing population engaged in the} \\
& \text{land use activity generating recharge } z_{i,j} \text{ in} \\
& \text{element } i,j) \ \forall \ i \text{ and } j;
\end{align*}
\]
\[
\begin{align*}
D_w \cdot w_{i,j} + D_v \cdot v_{i,j} & \geq \text{(total existing population} \\
& \text{engaged in the land use activity} \\
& \text{generating recharge } w_{i,j} \text{ in element} \\
& \text{element } i,j) \ \forall \ i \text{ and } j;
\end{align*}
\]
\[
\begin{align*}
w_{i,j} & \leq 0 \ \forall \ i \text{ and } j \text{ where appropriate};
\end{align*}
\]
\[
\begin{align*}
z_{i,j} & \leq 0 \ \forall \ i \text{ and } j \text{ where appropriate};
\end{align*}
\]
\[
\begin{align*}
v_{i,j} & \leq 0 \ \forall \ i \text{ and } j \text{ where appropriate};
\end{align*}
\]
\[
\begin{align*}
z_{i,j} + w_{i,j} + v_{i,j} & \leq L_{zi,j};
\end{align*}
\]
\[
\begin{align*}
\sum_{i=1}^{n} \sum_{j=2}^{m} R_z \cdot z_{i,j} + R_w \cdot w_{i,j} + R_v \cdot v_{i,j} & \leq R_s
\end{align*}
\]

Nonnegativity Constraints:
\[
\begin{align*}
C_{i,j}, w_{i,j}, z_{i,j} \text{ and } v_{i,j} & \geq 0
\end{align*}
\]

7.2. Application of Model III to Falmouth, Massachusetts

It was initially planned that Model III would be used to investigate the management of nonpoint source nitrate contamination of ground water over the towns of Falmouth and Bourne, Massachusetts (depicted in Figure 19). The resultant size of the linear program would have exceeded the capacity of the optimization software available. To reduce the size of the problem, the optimal pattern of
The optimum placement and intensity of the three composite land use activities were evaluated over the study area. The first land use activity generated nitrate contaminated recharge flows from the use of municipal well water, septic systems, and lawn fertilizers. Nitrate loads from this type of residential/commercial land use were introduced to the aquifer through elemental flows represented by decision variables $Z_{i,j}$ for all $i$ and $j$ elements in Falmouth.

The second source was also from a residential/commercial land use; however here recharge flows were produced from the use of on-site well water (as opposed to municipal water), septic systems, and lawn fertilizers. Nitrate loads were delivered to the subsurface environment by recharge flows represented by decision variables $W_{i,j}$ for all $i$ and $j$ elements in Falmouth.

The third and final ground water polluting surface activity is equivalent to the first land use type, but represents sources transformed from the second type to the first type; commercial and domestic activities that have abandoned on-site wells for municipal water. Recharge from these converted sources was represented in values of decision variables $V_{i,j}$ for all $i$ and $j$ elements in Falmouth. Note that the recharge from the first and third land use activities represents artificial recharge of imported water originating from the municipal water supply source; whereas, the second land use activity receives its water from on-site wells and recharges it to ground water via on-site septic systems.

Other sources of ground water nitrate contamination in Falmouth (i.e., sewer exfiltration, natural recharge, and subsurface disposal of secondary sewage) were treated as constant fluxes in the continuity constraints (terms on the right-hand-side). Because land use activity in Bourne was not evaluated with the model, all sources of nonpoint source ground water nitrate contamination in Bourne were treated as constant nitrate fluxes in the continuity constraints.
7.2.1. Data Requirements for Model III

The underlying hydrologic and contaminant transport conditions applied in Model III were the same as those used in Models I and II; consequently, Model III required the same basic input data as Models I and II. Beyond the basic data, Model III required additional information to define coefficients appearing in the model components.

7.2.1.1. Data Requirements for Construction of the Objective Function

The objective function is a summation of decision variables representing the elemental recharge flows from the three land use activities under evaluation. Associated with each decision variable is a coefficient that converts recharge flows into the population equivalent; hence the value of the objective function is the total population contributing to the sum of the recharge from the three types of land use. The coefficients \( D_{w,i,j} \), \( D_{z,i,j} \), and \( D_{v,i,j} \) from the objective function (36) were calculated using the equations below:

\[
D_{w,i,j} = \frac{\Delta x \cdot \Delta y}{(1 + K_{c,i,j}) \cdot q_{c,i,j}} \quad \forall \ i \text{ and } j
\]

\[
D_{z,i,j} = \frac{\Delta x \cdot \Delta y}{(1 + K_{c,i,j} + K_{u,i,j}) \cdot q_{c,i,j}} \quad \forall \ i \text{ and } j
\]

\[
D_{v,i,j} = \frac{\Delta x \cdot \Delta y}{(1 + K_{c,i,j} + K_{u,i,j}) \cdot q_{c,i,j}} \quad \forall \ i \text{ and } j
\]

where all coefficients and constants appearing on the right side of the equal sign are defined in Chapter 6, Equation (29), and the values of the constants and coefficients are specified for the field problem in Table 9.

7.2.1.2. Data Requirements for the Construction of the Continuity Constraints

The continuity constraints of Model III embody the same components as those constructed for Model I and II with the following additions: first the recharge vectors \([Z]\) and \([W]\) are brought over to the left side of the equal sign because they are now treated as variable vectors; secondly, the variable recharge vector \([V]\) and source concentration vector \([C_{u}]\) are added to the constraint set. The
components of the source concentration vector are the same as those appearing in the source concentration vector \([C]\). The value of the decision variable \(V_{i,j}\) represents the elemental flows created from switching Domestic and Commercial usage from onsite well water to municipal well water; consequently the nitrate concentration in recharge flows are expected to be the same as flows from the combined domestic and commercial use of municipal water, septic systems, and lawn fertilizers.

7.2.1.3. Data Requirements for Construction of Constraints to Incorporate Present Land Use Activities

Management model constraints (38) and (39) also require the coefficients calculated above for the objective function. These constraints restrict the lowest feasible values of the recharge decision variables to reflect the existing elemental population engaged in the two land use activities.

The right-hand-sides of the constraints (38) and (39) are respectively \(P_{z_{i,j}}\) and \(P_{w_{i,j}}\) for all \(i\) and \(j\). The respective populations were calculated from 1980 U.S. census data as described in section 6.3.3.1.

7.2.1.4. Data Requirements for Construction of Constraints Limiting Maximum Levels of Land Use Activities

Constraints which prohibit specific land use activities were used only in two elements: the use of on-site well water was prohibited in the area downgradient from the Falmouth sewage treatment facility to protect the health of local residents; hence,

\[ W_{i,j} < 0 \text{ for elements } i,j = 11,2 \text{ and } 11,3. \]

All the land use activities under evaluation compete for available land. Constraints similar to (43) were written to ensure that optimal values of recharge decision variables reflect feasible intensities of combined land use activities given real restrictions on available land. The land use density (or source density) constraints were written for each element in Falmouth in the form of constraint (43). The constraint coefficients represent the number of housing units required per unit flow of recharge (\(m^3/d\)), and the constant on the right-hand-side is set equal to the permissible number of housing units in each element. The construction of the land use density constraints requires data on the number of households presently situated in each element plus specification of the residential zoning regulations and the area of each element.
7.2.1.5. Data Requirements for Construction of Constraints on Available Resources

The CCPEDC (1986) provided the data required to construct a single constraint to limit total elemental municipal water use \( (V_{i,j} \text{ plus } Z_{i,j}) \) for all \( i \) and \( j \) in Falmouth to the present capacity of the town's water supply (55132 m\(^3\)/d). In the general constraint \( (84) \), \( R_z \) and \( R_v \) equal one, and \( R_w \) equals zero.

7.2.1.6. Data Requirements for Construction of the Water Quality Constraints

Water quality constraints were constructed for each element of Falmouth. Data from the steady-state nitrate nitrogen predictions produced in Model I (or Model II) were used with information on municipal well locations and a specified global ground water nitrate standard, i.e., a nitrate standard applicable in all elements unless specifically superceded by a more stringent constraint.

The elemental nitrate standards varied between elements. For all elements having predicted steady-state nitrate nitrogen concentrations (from 1980 development) in excess of the arbitrary global regional standard, and for all elements containing municipal water supplies, the imposed elemental nitrate standard was the predicted long term concentration produced from Model I for 1980 development patterns. These constraints operated as nondegradation constraints. They precluded the placement of land use activities in those elements or in other elements if such additional land use activities would cause further degradation of ground waters at municipal supplies or in areas already unable to meet the global nitrate nitrogen standard due to existing development. For all other elements, ground water quality was allowed to be degraded to a prespecified level of degradation. This was accomplished by using the global nitrate nitrogen standard in their respective discrete water quality constraints.

7.2.2. Constructing and Solving Management Model III

A Fortran program was written to create a computer file containing the objective function and constraints of Model III. The resulting linear programming problem embodied the ground water flow model to define fluid velocity coefficients necessary for the construction of the continuity constraints. Data required by the Fortran program included 1) the regional nitrate standard, 2) the residential housing density regulation for each element (applied uniformly over the region in these runs), 3) the steady-state

\[ v \]
predicted nitrate concentrations from 1980 development (obtained from solving Model I or II), 4) the area and dimensions of the elements, 5) the elemental recharge rates of all sources, 6) the elemental pumpage rates, 7) the piecewise aquifer transmissivities, 8) the hydraulic and contaminant model boundary conditions, and 9) the nitrate concentrations in all recharge flows.

Model III was solved using the regular Simplex algorithm available through the Multi Purpose Optimization System Package (Northwestern University, 1978). Several executions of the general model were performed using different global nitrate standards and residential/commercial density regulations. Source density regulations and water quality standards are two recognized methods of controlling the water quality impacts of nonpoint source ground water pollution. The multiple model runs were intended to reveal insights into the effectiveness of source density restrictions and water quality standards in protecting ground water resources, plus characteristics of the relationship between source density restrictions and water quality standards when used together to achieve optimal land use development.

7.2.3. Model III Results: Effects of Water Quality Standards on Ground Water Protection and Development

Multiple runs of Model III were performed to investigate the effects of water quality constraints on the predicted maximum feasible development of the three residential/commercial land use activities in Falmouth. The development of each composite land use activity was expressed in terms of the population engaged.

During each model run a constant housing density regulation was imposed through constraint type (43). The zoning regulation limited total residential/commercial land use to no greater than 500 household per square kilometer. For elements containing municipal water supplies (and also elements having a predicted nitrate concentration from Model I which exceed the global standard) the elemental long term nitrate nitrogen predictions from 'present' (1980) land use activity were used as the standards; these are referred to as nondegradation constraints. In all remaining water quality constraints the imposed right-hand-side was the specified global nitrate nitrogen standard.

Several Model III runs were performed where only the global nitrate standard was changed. The solutions to the multiple linear programs yielded data revealing the influence of regional and elemental ground water nitrate standards on residential/commercial development and on protection of ground water resources in Falmouth.
7.2.3.1. Effects of Nitrate Standards on the Magnitude and Pattern of Maximum Development

The magnitudes and patterns of the three residential/commercial composite land use activities evaluated in this field problem were initially expressed in terms of the equivalent discrete populations which would generate recharge flows represented in the optimum values of decision variables. Working with large sets of numbers proved to be cumbersome and to a large extent obscured observed patterns of development. Patterns were identified through the use of optimum land use development maps.

Figure 27 shows the maximum feasible population of Falmouth (that is within the boundaries of the modeled area) under different regional nitrate standards. The distribution of this population will be discussed later. When the regional nitrate standard was varied between 5 and 8 mg/L as N, the capacity of the study area to accommodate more people increased. As the regional standard was relaxed, land use activity expanded to take advantage of an apparent increase in the assimilative capacity of the aquifer. Under regional ground water nitrate standards from 5-8 mg/L as N, maximum feasible development (maximum combined residential/commercial land use activity) was constrained by nondegradation for elements containing municipal wells, global water quality, and land use density constraints. The land use density constraints precluded additional growth in areas where existing (1980) development exceeded the zoning restriction (500 household/km²). In addition these source density constraints assured that new development when combined with existing development was always less-than-or-equal-to the specified zoning regulation.

When a regional nitrate nitrogen standard greater than 8 mg/L was used, the capacity of Falmouth to include more people remained essentially constant (see Figure 27). Maximum feasible development of Falmouth was no longer dependent on the global nitrate standard because additional town growth was restricted by binding water quality constraints (of the nondegradation type) and binding land use density constraints.

From Figure 28, it is evident that as the global nitrate standard is relaxed, the number of water quality constraints which are binding decreases, from a total of 16 down to the five nondegradation water quality constraints embodied in the problem (one at each element containing a municipal water source). Simultaneously, the number of binding land use density constraints doubled. Figure 44 shows that when higher global nitrate standards are used nitrate levels in fewer regions (elements) approach the global standard before development is curtailed by the land use density regulations.
Figure 27. Predicted maximum population of Falmouth as a function of global nitrate nitrogen standards (for development scenarios under the constant land use density limit of 500 houses/sqkm).
Figure 28. Number of binding land use density, nondegradation, and global water quality constraints as a function of the global nitrate nitrogen standard (in the optimal solutions of Model III using a constant land use density limit of 500 houses/sqkm).
The optimal patterns of residential/commercial development for Falmouth under a range of global nitrate nitrogen standards are depicted in Figures 29-32. Elements which are lightly shaded are designated areas where additional growth (above 1980 levels) is desirable. The presence of a letter 'D' indicates development proceeded to the maximum feasible level allowed by the land use density constraints. Elements with a letter 'N' are predicted to have steady-state nitrate levels which just satisfy water quality constraints for their elements if the optimal development plan is implemented. Municipal water supplies are located in elements containing a letter 'W'.

Figure 29 shows the optimal pattern of development under a land use density restriction of 500 households per square kilometer, a regional nitrate nitrogen standard of 5 mg/L, and six nondegradation constraints associated with elements containing municipal water supplies and the Otis wastewater treatment plant (i,j = 8,4; 11,7; 12,3; 12,4; 13,4). The maximum population associated with this residential/commercial land use pattern is 60,108; this population represents a 131 percent increase over the 1980 population for the modeled area of Falmouth.

From Figure 29 it is obvious that additional growth above the 1980 levels is feasible in most elements. Pre-existing land use densities precluded new development in only two elements (i,j = 13,7 and 14,3). For most coastal nodes, development occurred up to the maximum feasible level, and nitrate nitrogen concentrations in the coastal areas increased to the maximum allowable under the global standard. Land use activity for interior nodes (nodes upgradient of the coastal elements) did not reach maximum allowable levels (in terms of density regulations), otherwise nitrogen concentrations in the coastal elements would have exceeded the global nitrate standards. In most elements containing municipal water supplies no new development appeared because the nondegradation constraints were binding. Elements located up gradient from Long Pond (i,j = 12,3; 12,4; 13,4) were identified by the model to remain at the 1980 development levels to satisfy nondegradation constraints on elements around Long Pond: this region corresponds to the critical recharge zone of Long Pond as delineated by Model II (see Figure 26).

With model runs using a global nitrate nitrogen standard of 7 mg/L, changes in the regional development pattern began to appear (see Figure 30). Under the existing land use density regulations few coastal nodes attained nitrate concentrations of 7 mg/L as N even though with several interior nodes developed to the maximum level allowed by the zoning regulation. Nondegradation constraints continued to be binding at all elements containing municipal supplies.

The land use development impacts of relaxing the global nitrate standard to 8 mg/L were dramatic (See Figure 31). Only two global
Figure 29. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 30. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 7 mg/L).
nitrate constraints remained in the linear programming solution (i.e., were binding). Land use activities in most elements increased to the limits allowed by the zoning. It would appear that under the present land use density restriction development could proceed in most regions to the maximum extent without causing ground water nitrate concentrations to exceed the 8 mg/L regional standard.

When the global nitrate standard was raised to 9 and 10 mg/L as N, model results were the same. Figure 32 shows that the global water quality constraints were not binding anywhere. Development proceeded to the maximum feasible level in most areas (82,838 people total). The only binding water quality constraints were those protecting municipal water supplies from further degradation.

Through the sequence of model runs (Figures 29-32), 71-76 percent of Falmouth model elements received added residential/commercial land use above 1980 levels (see Figure 33). Of these elements, the percentage in danger of violating water quality standards decreased as the global nitrate standard was relaxed (see Figure 34). But, as shown in Figure 35, increasing the allowable ground water nitrate level encouraged expansion of residential/commercial land use activities to maximum limits.

The discussion to this point has focused on the total combined population due to the three residential/commercial land use types. The model was designed to identify the optimal combinations of the three sources (land use types) that would emerge under various regional nitrate standards and residential/commercial land use density restrictions.

The most obvious trend describing a relationship between preferred land use types and global water quality standards is presented in Figure 36. This figure shows that with the relaxation of the global ground water nitrate standard there was an increase in the percentage of town area where municipal water was selected as the sole form of water used. The nonpoint source ground water pollution management model selected residential/commercial land use activities which require on-site well water over equivalent land use types requiring municipal water wherever water quality constraints were binding; that is, when both water quality and housing density constraints were defining the boundary, the model selected the source (on-site well water users) which delivered the lowest per capita nitrate loadings.

Use of municipal water generates higher nitrate loads to the aquifer than usage of on-site well water because of the nitrates in the imported municipal water. The management model perceived only that the mass loadings were higher with municipal water and not that the nitrate concentration in the recharge was lower. Consequently, the model was unable to distinguish a diluted source from a concentrated source because the linear continuity equations could not
Figure 31. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 8 mg/L).
Figure 32. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 10 mg/L).
Figure 33. The percent area of Falmouth to receive additional growth as a function of global nitrate nitrogen standards (for maximum development of Falmouth under the constant land use density limit of 500 houses/sqkm).
Figure 34. The percent new-growth area of Falmouth polluted to allowable nitrate levels as a function of global nitrate nitrogen standards (for maximum development of Falmouth under the constant land use density limit of 500 houses/sqkm).
Figure 35. The percent new-growth area of Falmouth to reach the allowable land use density limit as a function of global nitrate nitrogen standards (for maximum development of Falmouth under the constant land use density limit of 500 houses/sqkm).
Figure 36. Percent area of Falmouth entirely dependent on municipal water as a function of global nitrate nitrogen standards (for maximum development of Falmouth under the constant land use density limit of 500 houses/sqkm).
incorporate the nonlinear hydraulic effects induced from disposing large volumes of dilute wastewater to the aquifer. The result of this modeling limitation was a consistent selection of the most conservative land use development patterns wherever binding water quality constraints were precluding additional growth.

The general patterns of the three residential/commercial land use activities evaluated are illustrated in a series of figures (37-40) which span the multiple solutions of Model III using various regional nitrate standards ranging from 5-10 mg/L as N. The figures show in which elements municipal water was the principal source of water. On-site well development was prohibited in elements i,j = 11, 2 and 11,3 because these nodes were situated downgradient from the Falmouth wastewater treatment facility. In a few elements, development activities requiring on-site well water were introduced where they never existed before (i.e., in elements i,j = 13,2 and 14,3); this occurred when the optimum development pattern depended on satisfying global nitrate standards. Element i,j = 12,2 never allowed land use activities needing on-site well water. Land use changes occurred in a few elements where existing residential/commercial activities dependent on on-site well water were converted to the municipal water supply (i.e., elements i,j = 8,3; 9,3; 10,3; 12,5; 12,6; 12,7 and 14,4). Most conversions from land use activities generating recharge flows $W_{i,j}$ (from on-site well water usage) to activities producing flows $V_{i,j}$ (from municipal water usage), occurred after the global nitrate nitrogen standard was increased above 7 mg/L.

The nonpoint source pollution management model never increased residential/commercial use of municipal water in an element without first connecting all on-site well water users to the town water distribution system; two observation were drawn from this. First, any areas depicted in Figures 37-40 as having only municipal water users were also the only elements where residential/commercial use of municipal water increased. The second observation was that all other elements known to acquire new development saw an increase in only on-site well water use. Therefore, wherever elemental development occurred either on-site well water or municipal water usage increased but not both.

The constraint (see constraint 44) specifying an upper limit on available town water was never binding in any of the model runs. At most, maximum municipal water use never required more than 36 percent of the available supply.
Figure 37. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 38. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 7 mg/L).
Figure 39. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 8 mg/L).
Figure 40. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 10 mg/L).
7.2.3.2. Effects of Nitrate Standards on Ground Water Protection

Ground water protection can be effected through the enforcement of ground water quality standards and land use zoning regulations. This section discusses the effects of nitrate nitrogen standards on efforts to protect the long term availability of ground waters and the development tradeoffs associated with relaxing these standards.

The protection of ground water was evaluated in terms of ground water nitrate changes effected through additional development above 1980 levels. The regional preservation of ground water was achieved through limited land use development over Falmouth in a pattern indirectly determined by binding land use density constraints and directly through binding nondegradation water quality constraints. Figure 41 shows the average (i.e., average overall Falmouth elements) steady-state nitrate nitrogen concentration in Falmouth under maximum feasible development conditions for a specified global nitrate standard and a residential land use density restriction of 500 households per square kilometer. Average nitrate nitrogen concentrations slightly exceeded the 5 mg/L goal set by the CCDEPC (1978) after the global nitrate standard was elevated to 8 mg/L as N. As expected, the percent area of Falmouth polluted to allowable nitrate levels decreases as the global nitrate nitrogen standard is elevated (see Figure 42). However, no less than 15 percent of Falmouth can be at allowable nitrate levels because 15 percent of the area corresponds to the region protected with nondegradation constraints (which is already at the permissible limit of nitrate contamination and is consequently, not affected by relaxing the global nitrate nitrogen standard).

Nondegradation constraints for elements containing municipal water supplies preserved water quality at steady-state levels predicted from present (1980) land use activities. Additional development was all but precluded near municipal water supplies which forced most development to occur down gradient and between Falmouth's water supplies. The nondegradation constraints were a dominant factor affecting the pattern of optimal development in the study area.

Global water quality constraints protected ground water quality near the coasts by limiting development upgradient where expanded residential/commercial activities were possible without affecting nitrate changes in municipal water supplies.

Figures 43-46 illustrate the steady-state nitrate nitrogen distributions in Falmouth from maximum development under various global nitrate nitrogen standards. Most evident from leafing through the figures are the preservation of water quality around municipal supplies (centered at nodes 8,4; 12,3; 12,4; 13,4 and 11,7) and extensive nitrate pollution between supplies. Notice that in every
Figure 4.1. Average nitrate nitrogen concentration in Falmouth as a function of global nitrate nitrogen standards (for maximum development scenarios under the constant land use density limit of 500 houses/sqkm).
Figure 42. Percent area of Falmouth polluted to allowable nitrate levels as a function of global nitrate nitrogen standards (for maximum development scenarios under the constant land use density limit of 500 houses/sqkm).
Figure 43. Predicted steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 44. Predicted steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 7 mg/L).
Figure 45. Predicted steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 8 mg/L).
Figure 46. Predicted steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 10 mg/L).
elemental ground water nitrate levels were less than the
specified global standard.

The development tradeoffs associated with relaxing land use
density and water quality constraints were investigated through the
optimal values of the dual variables. Each model run produced values
for dual variables associated with each model constraint. For the
binding constraints, the values of the dual variables were used to
interpret marginal changes in the maximum population of Falmouth for
unit relaxations of the constraints. Figure 47 presents the values
of the dual variables associated with binding land use density
constraints. These were obtained from the solution of Model III
with a housing density restriction of 500 households per square
kilometer and a regional nitrate nitrogen standard of 5 mg/L. The
numbers appearing inside the elements represent the additional people
which could be located in Falmouth if one more household could be
added in those elements. To relax the housing density constraints of
elements which contain no number, would not permit more people to
live in Falmouth, because the land use activity in these elements has
not yet exhausted available land for development.

The values of the dual variables for the land use density
constraints did not change with the relaxation of the regional
nitrate standard. Dual variable values less than the residential
occupancy rate (2.203 people/household) indicated an added household
in one place would require a reduction in residential/commercial
activity elsewhere to ensure other model constraints remain
satisfied. When the values of dual variables were larger than the
occupancy rate the effect of adding one more housing unit allowed a
shift in upgradient land use activities which in turn permitted
additional residential/commercial development elsewhere; hence, the
net population gain was greater than the average occupancy rate.

The development tradeoffs generated from relaxing water quality
constraints are displayed in Figure 48. The numbers appearing in
specific elements represent the marginal increase in the number of
people which could be located in Falmouth if the water quality
constraints for those elements were relaxed by 1 mg/L as N.

For binding global water quality constraints the values of dual
variables decreased as the global standard increased from 5 to 10
mg/L as N. The fall in values is probably indicative of the extent
to which feasible development would assume a pattern determined less
by global water quality constraints and more as by nondegradation and
land use density constraints.

Values of dual variables generated from binding nondegradation
water quality constraints increase as the global nitrate standard is
increased. That is, as the potential development in surrounding
areas increases, the actual development possible due to a relaxation
Figure 47. Optimum values of dual variables associated with binding elemental land use density constraints (from the solution of Model III under a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L). Numbers represent the additional population growth from allowing one more housing unit in appropriate elements.
Figure 48. Optimum values of dual variables associated with binding elemental water quality constraints (from the solution of Model III under a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L). Numbers represent the additional population growth from allowing one more mg/L of nitrate nitrogen in appropriate elemental ground waters.
of water quality limits in the area with more stringent standards increases.

The opportunity cost (in terms of development foregone) due to stringent water quality constraints in downgradient elements increases as the potential for development in the region increases (i.e., as global nitrate nitrogen standards are increased).

7.2.4. Model III Results: Effects of Density Constraints on Ground Water Protection and Development

An investigation was made with multiple runs of the third nonpoint source ground water pollution management model to elucidate the effects of source density constraints (also known as land use density constraints and residential/commercial zoning constraints) on the maximum feasible residential/commercial development and the protection of ground water resources in Falmouth. During each model run a different land use density regulation was imposed. Nondegradation and global water quality standards remained constant between model executions. The global nitrate nitrogen standard was always 5 mg/L. Nondegradation water quality constraints were constructed for each element containing a municipal water supply; the other nondegradation nitrate standards were generated as before from the long term nitrate predictions from Model I.

7.2.4.1 Effects of Density Constraints on the Magnitude and Pattern of Maximum Development

A range of specified land use density limits was used in Model III to ascertain optimal magnitudes and deployment patterns of the three land uses in Falmouth. Residential/commercial density limits were expressed in units of allowable houses/km$^2$. The maximum population projections are plotted against density restrictions from 200-500 houses/km$^2$ (one house per 48,900 to 16,500 ft$^2$) in Figure 49. The figure exhibits a curve showing increased maximum development potential (expressed in population) for Falmouth under successively more relaxed regulations on allowable housing densities.

The shape of the curve in Figure 49 reflects a change in the type of constraints which become boundary equations when more sources are permitted per unit area. The numbers and types of binding water quality and land use density constraints associated with each solution of Model III are illustrated in Figure 50 as a function of the different land use density regulations. For source density limits under 200 houses/km$^2$, maximum potential residential/commercial development was independent of global nitrate nitrogen standards. Development occurred to the maximum allowable density, which was
Figure 49. Predicted maximum population of Falmouth as a function of land use density limits (for development scenarios under the constant global nitrate nitrogen standard of 5 mg/L).
Figure 50. Number of binding land use density, nondegradation, and global water quality constraints as a function of land use density limits (in the optimal solutions of Model III using a constant global nitrate nitrogen standard of 5 mg/L).
sufficiently low that global water quality constraints were never binding. The nondegradation constraints, however, either were binding or were on the verge of becoming binding constraints. When density regulations greater-than-or-equal-to 375 houses/km² were used, Model III yielded optimal development patterns which were determined by binding global water quality, nondegradation, and land use density constraints. Wherever development was feasible it occurred to maximum density levels when stringent land use density limits were used. Under more relaxed density regulations maximum allowable growth occurred primarily among coastal nodes.

The solutions of Model III for each of the four land use density limits are illustrated in Figures 51-54. Each figure depicts the location of municipal water supplies and the status of development and water quality for each element in the management area.

Figure 51 displays the optimal pattern of development under a land use density restriction of 200 houses/km². The maximum potential population from combining the three land use activities was 40,193 under a global nitrate standard of 5 mg/L as N. Pre-existing residential/commercial development precluded new development in elements i,j = 9,3; 13,5; 13,6; 13,7; 14,3 and 14,4 because source densities were higher than the specified desirable limit. Nondegradation constraints for elements containing municipal water supplies (Long Pond and Fresh Pond), discouraged development around and up-gradient from the sources of town water. In virtually all other elements, new development expanded to 200 houses/km². The only binding water quality constraints were the nondegradation constraints specified at elements i,j = 8,4; 9,6; 11,7; 12,3 and 13,4.

When the restriction on allowable residential/commercial density was raised to 250 houses/km² the pattern of development was similar to that achievable under a source density of 200 houses/km² (see Figure 52). Again, growth reached the maximum level allowed in most elements because elemental nitrate levels were not yet approaching the global standard (5 mg/L as N).

Under a land use density restriction of 375 houses/km², the intensity and pattern of development changed to reflect the influence of binding global water quality constraints (see Figure 53). Maximum residential/commercial activity was situated along the coast, where most global nitrate nitrogen constraints were binding. Upgradient from the coast, development proceeded to levels which would allow down gradient coastal elements to meet global nitrate nitrogen standards. Nondegradation constraints were binding constraints for every element where applicable except in element i,j = 9,6. Here, predicted long term water quality was slightly less than the imposed nondegradation standard. As the density regulation was increased,
Figure 51. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 200 houses/sqkm and a global nitrate nitrogen standard of 5 mg/l).
Figure 52. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 250 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 53. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 375 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 54. Maximum residential/commercial land use development pattern for Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
land use activity began to expand in a few elements \((i,j = 9,3; 13,6; 14,3)\) where more stringent source density limits had precluded growth because pre-existing development was already unacceptably high.

A last model run was performed in which allowable levels of residential/commercial activity were increased to 500 houses/km\(^2\). The solution, shown in Figure 54 indicated continued expansion of land use activities along the coast and in most coastal elements to the maximum allowable level. Development in the interior regions decreased to compensate for the additional coastal nitrate loads, in order that nitrate concentrations along the coast would satisfy global water quality constraints. The nondegradation constraints were binding in all elements containing municipal water supplies. A five percent increase in population \((117\ \text{people})\) was observed in element \(i,j = 13,4\) where a nondegradation constraint was imposed to protect a municipal water supply. The increased population did not impact water quality in the element. It appears that small changes in nitrate load can be tolerated without serious impact on the approximation of the nitrate concentration gradients in the continuity equations. This is probably due to the large discretization scales used in the model.

It was noted that the percentage of Falmouth area to receive additional development increased when more residential/commercial activity was allowed per unit area (see Figure 55). The observed increases were due to new development in areas where pre-existing land use activities had, under more stringent land use density regulations, precluded additional growth. Relaxing the source density limit increased the growth potential for Falmouth. Of the elements accepting new growth, the percentage in danger of violating the global nitrate nitrogen standard increased with every increase in the source density limit (see Figure 56). As the nitrate concentrations in ground water increased with allowable source densities, the number of binding global water quality constraints became more prevalent. This in turn curtailed the number of nodes where potential development could approach the density limit (see Figure 57). The increased growth potential led to greater growth along the coastline, but for reasons of maintaining coastal ground water quality this was at the cost of reduced growth potential for interior regions. New development in elements where nondegradation constraints were imposed was for the most part not observed. Specifically, for the nodes upgradient from Long Pond, no new development was possible without relaxing the nondegradation constraints at elements \(i,j = 12,3; 12,4; 13,4\).

Beyond the total residential/commercial land use picture, the magnitude and patterns of the three component land use types were reviewed for possible relationships between them and the specified source density regulations. Figure 58 suggests that a distinct preference exists for residential/commercial activities dependent on
Figure 55. The percent area of Falmouth to receive additional growth as a function of land use density limits (for maximum development of Falmouth under the constant global nitrate nitrogen standard of 5 mg/L).
Figure 56. The percent new-growth area of Falmouth polluted to allowable nitrate levels as a function of land use density limits (for maximum development of Falmouth under the constant global nitrate nitrogen standard of 5 mg/l).
Figure 57. The percent new-growth area of Falmouth to reach the allowable land use density limit as a function of land used density limits (for maximum development of Falmouth under the constant global nitrate nitrogen standard of 5 mg/L).
Figure 58. Percent area of Falmouth entirely dependent on municipal water as a function of land use density limits (for maximum development of Falmouth under the constant global nitrate nitrogen standard of 5 mg/L).
municipal water when the source density limits are stringent, while activities dependent on on-site water are more desirable with land use density restrictions allowing more development than 375 houses/km$^2$. A regional profile for municipal water use reveals that this model selected at most 38 percent of Falmouth for the use of municipal water over the use of on-site wells. The nonpoint source ground water pollution management model selected for residential/commercial activities dependent on on-site water wherever water quality constraints were restricting development in a region. Under opposite conditions, where development proceeded unhindered by global water quality constraints, the choice of water source is municipal. The model selects for the land use type which contributes the lowest load when source density limits are sufficiently relaxed that ground water quality constraints play a part in effectively defining the upper limit on allowable levels of residential/commercial land use activities.

The patterns of the three residential/commercial land use activities evaluated are illustrated in the series of Figures 59-62 which cover the series of solutions of Model III under global source density restrictions between 200-500 houses/km$^2$. As in previous figures (see Figures 37-40), the shaded elements are those where municipal water was the principal source of water. With the exception of elements i,j = 11,2 and 11,3 options exist to develop residential/commercial activities which depend on municipal water, on-site well water, or to convert existing on-site well users to municipal water users. For element i,j = 14,2, on-site wells did not exist as a source of residential/commercial water, in 1980; however, when additional development was feasible (when the land use density regulation was elevated above 250 houses/km$^2$), on-site well water use was introduced over increased municipal water usage. This occurred because land use activities dependent on municipal water were perceived by the model to deliver higher per capita nitrate loads, and to add municipal water consumption would have led to a suboptimal increase in elemental population under development conditions where water quality changes were going to constrain development. The optimal development in node i,j = 13,2 was always that which required town water until relaxation of the land use density limit portended nitrate levels which encouraged optimal development with on-site well water uses. Development in element i,j = 12,2 was the same as displayed above; land use activities needing on-site well water were never permitted. For all other elements illustrated as having only town water, residential/commercial use never increased in an element without first connecting all elemental on-site well water users to the municipal water distribution system. It was also the case with multiple model runs with the various source density limits that wherever elemental development occurred, either on-site well water or municipal water usage increased but not both. Had the demand for town water ever exceeded the supply it could have been possible for
Figure 59. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 200 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 600: Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 250 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 61. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 375 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 62. Water usage patterns for residential/commercial land use in Falmouth (for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
all three land use types to have appeared simultaneously in any one element.

7.2.4.2 Effects of Density Constraints on Ground Water Protection

Ground water protection through uniform source density controls can be an equitable and effective means of protecting ground waters. Figure 63 shows the average steady-state nitrate nitrogen concentrations in Falmouth under maximum feasible development conditions for different specified regional land use density limits and a global nitrate nitrogen standard of 5 mg/L. Average steady-state nitrate concentrations increased from 2.1 mg/L from 1980 development to a maximum of 3.6 mg/L. Changes in the average concentration above a housing density of 375 houses/km$^2$ were small because additional development occurred along coastal nodes where the impact of subsurface loading on the average nitrate nitrogen concentration (recall Figure 25) is small. Because much of the high density development occurs along the coastline, and because much interior development is precluded by the nondegradation constraints, excellent water quality in nondeveloped areas compensates for poor water quality elsewhere. Of the area of Falmouth falling within the boundaries of the study area, the areal extent of ground water which had degraded to allowable nitrate levels was 15 percent under a density regulation of 200 houses/km$^2$, but increased to 47 percent when 500 houses/km$^2$ were allowed (see Figure 64). The rapid increase in the percentage of the area reaching allowable nitrate limits was entirely in areas where global water quality constraints became binding constraints; hence, the water quality constraints were precluding additional growth where density constraints were not sufficient to prevent unacceptable ground water contamination. From Figure 64 a source density limit of 375 houses/km$^2$ will ensure protection of ground waters where global standards prevail and yet allow most development outside the 'no development zones' (defined by the nondegradation constraints) to proceed to the maximum land use density. The cost of a more stringent land use density limit is lower feasible population growth or lower total feasible land use development. When the residential/commercial density limit was relaxed the affect was to allow water quality constraints to play a larger role in determining the optimal pattern of development. By allowing more water quality constraints (beyond the nondegradation constraints) to establish the limits of development, the optimal pattern of growth becomes less uniform: recall that most development occurred along the coast which precluded equal development upgradient due to coastal water quality problems. Under the relaxed uniform density limits, Model III identified a pattern of development which could accommodate a larger population and satisfy water quality
Figure 63. Average nitrate nitrogen concentration in Falmouth as a function of land use density limits (for maximum development scenarios under the constant global nitrate nitrogen standard of 5 mg/L).
Figure 64. Percent area of Falmouth polluted to allowable nitrate levels as a function of land use density limits (for maximum development scenarios under the constant global nitrate nitrogen standard of 5 mg/L).
constraints, but would necessitate the enforcement of nonuniform development restrictions if the plan were ever enacted.

The steady-state nitrate nitrogen distributions generated from solving model III under each of the four land use density limits are presented in Figures 65 through 68. Where density constraints and nondegradation constraints defined the solution space (Figures 65 and 66) stringent source density limits allowed equal degradation of ground water between municipal water supplies. Higher source loads generated steeper gradients around supply wells and ponds. At density limits of 375 and 500 houses/km² (Figures 67 and 68) the pattern of nitrate contours emphasizes the effect of high development density on the coast with high nitrate contours protruding upgradient from the coastline. The differences between Figures 67 and 68 are small with the most visible difference being the smaller extent of clean water around Long Pond.

The development tradeoffs associated with relaxing elemental land use density and water quality constraints were again investigated through the optimal values of the dual variables. Figure 69 presents the dual variables associated with the land use density constraints for the model solution where the density limit was 250 houses/km². The values of these dual variables decreased with increases in the source density limit (the right-hand-side of these constraints). The reduction in the magnitudes of the dual variables occurs with increases in the number of binding water quality constraints; hence the decreased marginal opportunity cost associated with relaxing elemental source density constraints could reflect a reduction in flexibility to shift development patterns in a manner which could yield a positive population gain.

The optimal values of dual variable associated with binding water quality constraints are displayed in Figure 70 from the solution where the applied density restriction was again 250 houses/km². For the water quality constraints which specified nondegradation of water quality at municipal water supplies, some of the dual variables decreased as more land use activity was allowed per unit area. The values of the dual variables express the marginal increase in the number of people who could be located in Falmouth if the associated nondegradation constraint was relaxed by 1 mg/L as nitrate nitrogen. The reduction in the value of the dual variable could indicate that Falmouth would have less to gain (measured in terms of population increases) by allowing more pollution in their water supplies if development elsewhere has proceeded to the extent of binding global water quality constraints (such as when the allowable source density limit is greater than 250 houses/km²). This is equivalent to saying that the preserved area around a well becomes less important in providing additional room for development if growth
Figure 65. Predicted steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 200 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 66. Predicted steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 250 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 67. Predicted steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 375 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 68. Predicted steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth (at maximum feasible development of Falmouth for a specified land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 69. Optimum values of dual variables associated with binding elemental land use density constraints (from the solution of Model III under a land use density limit of 250 houses/sq km and a global nitrate nitrogen standard of 5 mg/L). Numbers represent the additional population growth from allowing one more housing unit in appropriate elements.
Figure 70. Optimum values of dual variables associated with binding elemental water quality constraints (from the solution of Model III under a land use density limit of 250 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L). Numbers represent the additional population growth from allowing one more mg/L of nitrate nitrogen in appropriate elemental ground waters.
elsewhere has forced changes in water quality elsewhere to the limits of acceptability.

The dual variables associated with the binding global water quality constraints did not appear to change as the nitrate concentrations in more regions approached the global standard. The stability may reflect that any new growth would occur only in the element where the water quality constraint was relaxed. It also reflects that to pollute an additional 1 mg/L nitrate nitrogen a specified number of people are required to add that nitrogen.

7.3. Conclusions

The application of Model III to Falmouth, Massachusetts illustrated the successful identification of optimal residential/commercial development patterns which incorporate existing development, accommodate maximum population growth, preserve water quality within standards, satisfy source density regulations, and operate within available resource limits. Optimal growth patterns varied with regard to type, location and density of the land use activities developed. Under conditions where global water quality constraints were nonbinding development approached maximum feasible uniformity. Alternatively, when global water quality constraints were effectively constraining development, they operated to restrict development in the interior while growth along the coast reached maximum allowable levels.

The nondegradation constraints defined zones where additional development was unacceptable. These constraints were a dominant factor in the design of optimal development patterns. Outside this 'zero growth zone' expanding development is determined by global water quality constraints and source density constraints. The dual variable associated with the nondegradation constraints generally increased as the global nitrate nitrogen standard was increased; that is, the development opportunities associated with allowing further nitrate pollution in 'zero development' zones increased as the global nitrate nitrogen standard increased.

Relaxing the global water quality standard increased the real assimilative capacity of the aquifer and as a result land use activity expanded to fill the increased capacity. For a given set of nondegradation constraints and a given source density limit there is a minimum global standard above which the optimal development pattern is no longer defined by binding global water quality constraints. For Falmouth this level was 8 mg/L nitrate nitrogen under a maximum source density limit of 500 houses/km², and approximately 5 mg/L for a density of 200 houses/km² (see Figure 7).
Figure 71. Regions of uniform and nonuniform maximum development opportunity in Falmouth for combinations of imposed global nitrate nitrogen standards and land use density limits.
The imposition of global water quality constraints can lead to nonuniform development opportunity because preserving water quality in a two-dimensional flow field may require the restriction of development in some areas to allow preferred growth in other areas. Proposed development scenarios which strive for uniform development opportunities must be sufficiently severe that global water quality constraints remain non-binding. Other causes of nonuniform development opportunity are preexisting sources (e.g., the Otis Plume and intense coastal development) and nondegradation constraints.

There exists a land use density for a specified ground water quality standard above which the development changes from as uniform as possible to a nonuniform pattern where development opportunity is determined by global and land use density constraints. Use of stringent density constraints yields lower regional contaminant concentrations, more uniform development opportunities, but lower maximum feasible growth. Higher density limits generate, reduced average water quality, nonuniform development opportunity, but higher feasible population growth.

Model III was not able to select between sources which varied in concentration of nitrate recharge flows. The model selected sources on the basis of their per capita mass loading rates and was not able to distinguish a dilute source (i.e., recharge from municipal water users) from a concentrated source (i.e., recharge from on-site well water users) because the linear continuity constraints could not incorporate the nonlinear dilution effects of disposing large volumes of diluted wastes as opposed to small volumes of concentrated wastewater to an aquifer. The errors associated with this modeling limitation were on the order of less than one percent. This is because recharge contributed from increased municipal water usage was generally small and situated in coastal nodes (or nodes near the coast) where the underlying ground water flow operates as a buffer against significant water quality changes at the coasts.
Chapter 8

Nonpoint Source Ground Water Pollution Management Model to Elucidate Development Scenarios for Minimizing Ground Water Impacts from Multiple Land Use Activities

The fourth and final nonpoint source ground water pollution management model is formulated to ascertain patterns to expand multiple land use activities, such that the resultant ground water quality impacts are minimized. The optimum pattern and combination of surface activities incorporates the present pattern of land use development and is identified from a specified population projection, stated development restrictions around municipal water supplies, given restrictions on available resources (i.e., land and water), imposed water quality standards, and specified housing density regulations. The specified population projection is the anticipated development level at some future time; it represents the minimum amount of growth which must be included in the study region.

Model IV is applied to the same section of Western Cape Cod (the town of Falmouth, Massachusetts) as Model III, and the general formulation was adapted, as with Model III, to identify the regional patterns of the three forms of residential/commercial land use evaluated in Chapter 7. The fourth model determines the combination of surface activities required to accommodate a projected year-round population. The optimal development pattern is identified from a large set of feasible residential/commercial land use patterns which incorporate present (1980) growth in Falmouth. Each development scenario of the feasible set satisfies constraints on desirable elemental nitrate nitrogen levels over the entire region, constraints on available water supplies, and constraints representing imposed regulations on specific land use activities.

Model results include: 1) contour plots of the minimum feasible steady-state regional ground water nitrate distribution from the projected residential/commercial development; 2) the optimal population predictions for each element and for each land use; 3) maps illustrating optimum locations for growth; 4) maps showing the degree of development and the long term status of water quality in each element; and 5) figures depicting the optimum values of dual variables associated with constraints on source densities and water quality.
8.1 Formulation of Management Model IV

Model IV is a direct adaptation of Model III. The decision variables and the continuity constraints used in Model IV are the same as those used to construct Model III. Changes were made in the management constraint set and objective function. The formulation presented will evaluate the management of a combination of three composite land use activities known to contribute areal contamination of ground waters; however, the formulation is general and the variety of surface activities considered, could be expanded to incorporate any number. Details on model components are presented below.

8.1.1. Objective Function

The formulation of the objective is a summation of all elemental contaminant variables in the management study area.

\[ \text{Minimize } \frac{1}{\alpha} \sum_{i=1}^{n} \sum_{j=1}^{m} C_{i,j} \cdot p_{i,j} \]

where \( p_{i,j} \) is the value of one, if the element \( i,j \) is in both the region where contaminant transport is modeled and in the area where sources of ground water pollution are being managed. It is equal to zero otherwise (such as 1 for Falmouth elements, and 0 for Bourne elements).

\[ \alpha = \sum_{i=1}^{n} \sum_{j=1}^{m} p_{i,j} \]

which equals all the elements in the management area.

The optimum value of the objective function is the minimum average contaminant distribution in the management area that is feasible under the projected population scenario.

8.1.2. Management Constraints

Most of the management constraints employed in Model III were also used in Model IV. Modifications affected management constraints designed to incorporate present levels of land use activities into the optimal solution (see constraints 38 and 39). Rewritten below, the modifications restructured inequality constraints 38 and 39 into equality constraints for elements containing municipal supplies. These changes ensure that feasible values of recharge decision variables associated with the three land use types (\( W_{i,j} \), \( Z_{i,j} \), and \( V_{i,j} \)) reflect existing and only existing levels of land use.
activities in these elements. The modified constraints are written as follows:

\[ D_{z_{i,j}} \cdot Z_{i,j} = p_{z_{i,j}} \quad \forall i \text{ and } j \text{ where a municipal water supply is situated; } \]

\[ D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} = p_{w_{i,j}} \quad \forall i \text{ and } j \text{ where a municipal water supply is situated. } \]

Constraints (50) and (51) preclude any increase in residential/commercial activity in elements containing municipal water supplies. All other elements continue to have management constraints like (38) and (39). For appropriate elements, constraint (50) sets values for the decision variables \( Z_{i,j} \) to reflect permanent discrete populations responsible for these elemental recharge flows. Opportunities to switch from one land use activity producing recharge flow \( W_{i,j} \) to another generating flow \( V_{i,j} \) are feasible in elements containing municipal water supplies; however, constraint (51) ensures that the optimum long term combination of recharge flows \( W_{i,j} \) and \( V_{i,j} \) reflects a population equal to the original number of people \( p_{w_{i,j}} \) contributing to the elemental flows \( W_{i,j} \).

Beyond the specific constraint modifications which restrict new development around municipal water supplies, Model IV contains a constraint which specifies a minimum level of residential/commercial development (expressed as a minimum population level) to be accommodated within the boundaries of the management area. The constraint operates to force consideration of development levels which represent projections of future growth. The formulation of this minimum development constraint is a summation of all elemental subsurface recharge flows generated from the three residential/commercial land use activities. Each decision variable is modified by a coefficient which changes flows to population. The right-hand side of the constraint is the future population to be located in the management area. The minimum population constraint is written as:

\[ \sum_{i=1}^{n} \sum_{j=1}^{m} D_{z_{i,j}} \cdot Z_{i,j} + D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} \geq p_e \]

where \( p_e \) is the projected population to be received in Falmouth
8.1.3. The General Formulation of Model IV

The complete formulation of Model IV is:

Minimize \( \frac{1}{\alpha} \sum_{i=1}^{n} \sum_{j=1}^{m} c_{i,j} p_{i,j} \)

s.t.

Continuity Constraints:
\( [G](C) - [C_w](W) - [C_z](Z) - [C_v](V) \)
\( = [C_g](S) + [C_u](U) + C_q[I][Q] \)

Management Constraints:
\( D_{z_{i,j}} \cdot Z_{i,j} \geq P_{z_{i,j}} \forall i, j \) not containing municipal water supplies;
\( D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} \geq P_{w_{i,j}} \forall i, j \) not containing municipal water supplies;
\( D_{z_{i,j}} \cdot Z_{i,j} = P_{z_{i,j}} \forall i, j \) containing municipal water supplies;
\( D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} = P_{w_{i,j}} \forall i, j \) containing municipal water supplies;
\( w_{i,j} \leq 0 \forall i, j \) where appropriate;
\( z_{i,j} \leq 0 \forall i, j \) where appropriate;
\( v_{i,j} \leq 0 \forall i, j \) where appropriate;

\( L_{z_{i,j}} \cdot Z_{i,j} + L_{w_{i,j}} \cdot W_{i,j} + L_{v_{i,j}} \cdot V_{i,j} \leq L_{s_{i,j}} \)
\( \sum_{i=1}^{n} \sum_{j=1}^{m} R_{z_{i,j}} \cdot Z_{i,j} + R_{w_{i,j}} \cdot W_{i,j} + R_{v_{i,j}} \cdot V_{i,j} \leq R_{s} \)
\( \sum_{i=1}^{n} \sum_{j=1}^{m} D_{z_{i,j}} \cdot Z_{i,j} + D_{w_{i,j}} \cdot W_{i,j} + D_{v_{i,j}} \cdot V_{i,j} \geq P_{e} \)

Nonnegativity Constraints:
\( c_{i,j}, w_{i,j}, z_{i,j} \) and \( v_{i,j} \geq 0 \)
Model IV was used to investigate the management of nonpoint source nitrate contamination of ground water in the town of Falmouth, Massachusetts (depicted in Figure 19). Nitrate transport was simulated over the entire area west of the ground water divide with the use of continuity constraints and contaminant decision variables \( C_{i,j} \) in both Bourne and Falmouth. The investigation of the optimum placement and intensity of the three composite land use activities was carried out using additional decision variables \( Z_{i,j} \), \( W_{i,j} \) and \( V_{i,j} \), the optimal values of which represent elemental recharge flows from three residential/commercial land use activities in elements over Falmouth alone.

The three land use activities contributing nonpoint source ground water nitrate pollution were; 1) recharge flows \( Z_{i,j} \) for all elements \( i,j \) in Falmouth) from the combined domestic and commercial use of municipal water, septic systems, and lawn fertilizers, 2) recharge flows \( W_{i,j} \) for all elements \( i,j \) in Falmouth) produced from the combined domestic and commercial use of on-site well water (as opposed to municipal water), septic systems, and lawn fertilizers, and 3) recharge from domestic and commercial activities which have abandoned on-site wells for municipal water and therefore, produced recharge flows \( V_{i,j} \) for all elements \( i,j \) in Falmouth) from the combined domestic and commercial use of municipal water, septic systems, and lawn fertilizers. Other sources of ground water nitrate contamination in Falmouth (i.e, sewer exfiltration, natural recharge, and subsurface disposal of secondary sewage) were treated as constant fluxes in the continuity constraints (terms on the right-hand-side). Land use activity in Bourne was not evaluated with the model; consequently, all sources of nonpoint source ground water nitrate contamination there were treated as constant nitrate fluxes in continuity constraints.

8.2.1. Data Requirements for Model IV

Model IV has the same underlying hydrologic and contaminant transport conditions as those used in Models I and II. The data requirements are the same as those identified for Model III; for details the reader is referred to Chapter 7. The only additional information required to construct Model IV is the projected residential/commercial development level (expressed as projected population for the entire area) to be situated in an optimal pattern over the study area.
8.2.3. Model IV Results: Optimal Development Which Minimizes Impact on Ground Water Quality

Model IV identified an optimum combination and intensity of elemental residential/commercial land use activities which have a minimum impact on average ground water quality in Falmouth for a projected population level; this was achieved while simultaneously satisfying all other model constraints (i.e., water quality, present development pattern, etc.). Multiple runs of Model IV were performed to span a range of projected population levels (35,000 - 50,000) for Falmouth. The various model runs were intended to identify the minimum ground water nitrate distributions and the associated optimal development pattern for each projected population.

8.2.3.1. Ground Water Protection Through Development Scenarios Effecting Minimum Ground Water Quality Impacts

The protection of ground water resources in Falmouth under the various optimal development plans was evaluated in terms of predicted ground water nitrate changes under different projected population increases for the town.

Figure 72 shows the average steady-state nitrate nitrogen levels for Falmouth under a range of projected population levels. Average
Figure 72. Predicted minimum average nitrate nitrogen concentrations for Falmouth as a function of projected population (for development patterns of minimum ground water impact under a specified land use density limit of 500 houses/sq km and a global nitrate nitrogen standard of 5 mg/l).
nitrate nitrogen concentrations increased from 2.1 mg/L (in 1980 for a population of 26,926) to 2.7 mg/L for a projected population of 50,000. Average ground water nitrogen levels increased at one third the growth rate of population.

The ability to protect the average water quality in Falmouth against degradation from future development was founded in a general pattern of development identified by Model IV. Comparisons of average water quality in Falmouth from different development patterns accommodating the same population are shown in Figure 73. Higher average nitrate concentrations were identified with the management Model III where different land use density constraints were used to obtain maximum population and the associated average nitrate concentrations. For equivalent populations, the development pattern identified with Model III was more uniform and allowed more development in the interior regions than the development scenario that minimized ground water impact (as determined by Model IV). Development at interior elements (recharge zones) has greater impacts on average regional ground water quality than development along the coast (discharge zones). Consequently, for equivalent populations, a uniform development pattern (which forces interior growth) leads to higher average nitrate levels than a development pattern where population is situated near regional discharge zones. These results represent members of a set of potential development patterns which could accommodate equivalent populations. The results show that the different development patterns have different regional ground water quality impacts; however, the set of development scenarios for any given population will be determined by the numerical values of the land use density constraints, the global nitrate nitrogen constraints, nondegradation constraints, the nature of the source, and the prevailing hydrologic conditions.

Several nondegradation constraints were not binding, that is, water quality was predicted to be better than the nondegradation standard. The predicted nitrate nitrogen concentrations were, however, close (< .1 mg/L as N) to the nondegradation standard. Global water quality constraints were not important to the protection of Falmouth ground water resources until the projected population reached 50,000. Further degradation of water quality at coastal nodes was precluded with binding global water quality constraints.

Figures 74-77 illustrate the minimum steady-state nitrate nitrogen distributions in Falmouth for development patterns accommodating projected residential/commercial development from 35,000 to 50,000 people. Water quality in elements containing Falmouth's major water supply (i,j = 12,3; 12,4 and 13,4) is preserved below 2 mg/L as N even at population projections of 50,000 people. Increased development along the coast creates nitrate concentration contours which run parallel to the coastline. The peak nitrate level for the region appears in element i,j = 9,6 where Otis sewage treatment plant is located. At the highest population
Figure 73. Predicted average nitrate nitrogen concentrations in Falmouth as a function of population. Model III predictions were obtained from varying the land use density limit from 200 to 500 houses/sqkm and a global nitrate nitrogen described in Figure 72.
Figure 74. Minimum steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth predicted by Model IV (for a projected Falmouth population of 35,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 75. Minimum steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth predicted by Model IV (for a projected Falmouth population of 40,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 76. Minimum steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth predicted by Model IV (for a projected Falmouth population of 45,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 77. Minimum steady-state nitrate nitrogen concentration (mg/L) contours over Bourne and Falmouth predicted by Model IV (for a projected Falmouth population of 50,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 78. Optimum values of dual variables associated with binding elemental land use density constraints (from the solution of Model IV under a projected Falmouth population of 50,000, a land use density limit of 500 houses/sqkm, and a global nitrate nitrogen standard of 5 mg/L). Numbers represent the µg/L decrease in average ground water nitrate nitrogen over Falmouth from allowing one more housing unit in appropriate elements.
Figure 79. Optimum values of dual variables associated with binding elemental water quality constraints (from the solution of Model IV under a projected Falmouth population of 50,000, a land use density limit of 500 houses/sqkm, and a global nitrate nitrogen standard of 5 mg/L). Numbers represent the ug/L decrease in average ground water nitrate nitrogen over Falmouth from allowing one more mg/L of nitrate nitrogen in appropriate elemental ground waters.
waters would increase). Global water quality constraints were binding along the coast. Increasing the standard from 5 to 6 mg/L as N would allow a shift in development toward the coasts where it would effect a lower impact on the regional average nitrate nitrogen levels.

8.2.3.3. Patterns of Development which Effect the Lowest Ground Water Quality Impacts

The magnitudes and patterns of the three residential/commercial land use activities were evaluated with land use development maps. Figures 80-83 illustrate the patterns of development which produce the lowest nitrate nitrogen levels in Falmouth for several population projections. In general, growth increases from zero development in the interior regions to maximum feasible levels along the coast.

As the population projection was increased, the percent of Falmouth to receive added residential/commercial development increased from 29 to 52 percent for projections of 35,000 and 50,000 people, respectively (see Figure 84). Among those elements to show growth, the percent to achieve maximum allowable densities increased with population until the global nitrate nitrogen constraint precluded additional development along the coast. As shown in Figures 84 and 85, growth occurs over a larger area when the projected population reaches 50,000, but a smaller percent of the growth reaches maximum allowable limits after the population exceeds 40,000.

The pattern of development in Falmouth was primarily determined by binding density and nondegradation constraints for the range of projected populations evaluated with Model IV (see Figure 86). It was not until the population exceeded 40,000 that development to the maximum density in some elements was curtailed because of binding global water quality constraints. During all the model runs an increased use of municipal water was confined to only those nodes where use of septic systems was prohibited (i,j = 11,2 and 11,3). Because municipal water uses generate higher nitrate contaminant fluxes to the subsurface than residential/commercial activities connected on-site well water, the model consistently expanded on-site well water usage to keep per capita nitrate loads at a minimum.

8.3 Conclusions

Application of Model IV to Falmouth, Massachusetts demonstrated the identification of feasible development scenarios which can accommodate specified population increases with minimal additional ground water degradation. The feasible development scenarios prevent additional development in elements containing municipal supplies, but allow development elsewhere as long as ground water quality remains
Figure 80. Falmouth residential/commercial land use development pattern for minimum ground water quality impacts. (For a projected population of 35,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 81. Falmouth residential/commercial land use development pattern for minimum groundwater quality impacts (for a projected population of 40,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 82. Falmouth residential/commercial land use development pattern for minimum groundwater quality impacts (for a projected population of 45,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 83. Falmouth residential/commercial land use development pattern for minimum groundwater quality impacts (for a projected population of 50,000, a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 84. The percent area of Falmouth to receive additional growth as a function of projected population (for the development pattern of minimum ground water impact under a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 85. The percent new-growth area of Falmouth to reach the allowable land use density limit as a function of projected population (for the development pattern of minimum ground water impact under a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
Figure 86. Number of binding land use density, nondegradation, and global water quality constraints as a function of projected Falmouth population (for the development pattern of minimum ground water impact under a land use density limit of 500 houses/sqkm and a global nitrate nitrogen standard of 5 mg/L).
within global and nondegradation standards, present development is left intact, and all source densities fall within zoning limits.

The optimal pattern of growth which leads to lower changes in average ground water quality is one that concentrates sources near the discharge areas. The water quality advantages of coastal development (over interior growth) were elucidated in Chapter 6 with results from Model I. In the several model runs under small population projections development was curtailed primarily by density constraints. For higher development projections the minimum ground water impact pattern for residential/commercial growth was determined by global water quality, nondegradation and land use density constraints.

Land use activities that necessitate conversions from on-site wells to town water or the expanded use of municipal water were all but avoided except in regions down gradient from the Falmouth sewage treatment plant; the exception included elements 1, j = 11, 2, and 11, 3 where use of municipal water was mandated for reasons of protecting health.

Comparisons were made of average nitrate nitrogen levels obtained from Models III and IV for equivalent development levels but different prevailing source density regulation. The results of the comparison point to a potential set of feasible development patterns which can accommodate the same population but have differing impacts on ground water quality. Used in combination Models III and IV could identify many feasible development scenarios.
CHAPTER 9

CONCLUSIONS AND RECOMMENDATIONS

This research developed four models which could be used in the evaluation of strategies for managing multiple land use activities so that long term quality of ground water could be protected from the nonpoint source pollution associated with those activities. All of the management models were linear programs which included linear algebraic equations from a numerical steady-state contaminant transport model as part of the constraint set. The nonpoint source ground water pollution management models were applied to a 'Sole Source Aquifer' underlying the towns of Bourne and Falmouth, Massachusetts.

Models I and II were used to delineate areas within the regional ground water flow system which are most critical to the preservation of ground water quality over the region and in specific sub-areas of the region. Model I revealed the relative importance of subareas within Bourne and Falmouth which are critical to the preservation of regional ground water quality from dispersed nitrate pollution. Model II ascertained the comparable significance of protecting zones within a recharge area surrounding a major municipal water supply.

Models III and IV were formulated to identify optimal patterns and intensities of multiple land use activities. Both Models will locate multiple land use activities, set pollutant fluxes, and predict ground water impacts in a simultaneous fashion. Model III determines the maximum feasible development of a combination of land use activities. Model IV ascertains patterns to expand multiple land use activities, such that resultant ground water impacts are minimized. Alternative development scenarios can be investigated by simple changes in model constraints. Models III and IV were used to evaluate the control of nitrate pollution from three similar residential/commercial land use activities through optimal development patterns which satisfy water quality standards and land use density limits. Land use development scenarios were determined from given development objectives (specific to each model) and constraints imposed on the land surface activities and their associated ground water impacts (represented here by nitrate nitrogen concentrations). The combined results of several model runs produced, for various development scenarios, the resultant regional ground water nitrate nitrogen distributions, population predictions, and development patterns for the three targeted residential/commercial land use activities.
In light of results obtained from the four models, several conclusions were drawn and recommendations for additional work were made.

9.1 Conclusions

1) The models developed in this work characterize where and to what extent future nonpoint source ground water nitrate pollution should be controlled in a study area in order to preserve regional ground water quality at specified levels.

2) Models III and IV identified regions where meeting land use density limits and water quality standards would be difficult if the optimum development pattern was pursued. Postoptimal analysis reveals the development and water quality tradeoffs of relaxing land use density limits and water quality standards.

3) The management model perceives differences between land use development alternatives as differences in unit mass loadings. Under water quality limited conditions the model selects for the land surface activities which generate the lowest contaminant loads.

4) Where nondegradation conditions exist within the system of regional ground water flow, feasible development patterns included areas where potential growth was precluded. These zero-growth areas often extended upgradient from the protected waters.

3) Beyond the boundaries of the zero-growth zones, land use development was determined by existing surface activities, land use density limits and global water quality constraints.

4) Beyond the boundaries of the zero-growth zones, the combination of land use density limits and global water quality standards can result in either uniform or nonuniform development opportunities.

5) Use of stringent density constraints can yield lower regional contaminant concentrations, more uniform development opportunities, but lower maximum feasible growth. Higher density limits generate higher average contaminant levels, nonuniform development opportunity, and higher feasible population growth.

6) For a given population, different combinations of water quality standards and land use density restriction can lead
to different development patterns which in turn affect
different regional ground water quality impacts.

7) For a given population, uniform development brought about
from stringent land use density limits can lead to higher
regional contaminant levels than nonuniform development
allowed with relaxed source density limits.

8) Development in discharge zones is preferable over growth in
recharge zones both from the perspective of maximum
achievable growth and from the perspective of preserving
ground water resources.

9.2 Recommendations

1) Coupled with thorough geologic work, the above models could
be applied to better define the relative significance of
ground water protection efforts in separate zones within
recharge areas around water supplies.

2) Clearly there are many feasible development scenarios for a
given population. Elucidating the noninferior set of
development patterns could be achieved with work that
further specifies development and water quality objectives.

3) More land use activities could be included in the model.
This would test the optimality of development scenarios
identified with simple models which may not incorporate all
the complexities of the planning process.

4) Rarely do multiple land use activities produce one
contaminant that affects groundwater quality. Future
research could address ground water protection from
nonpoint source pollution involving multiple contaminants.
References


Gelhar, L. W., and Wilson, J. C., Ground-Water Quality Modeling, Ground Water, 12(6), 399-408, 1974.


Perkins, R. J., Septic Tanks, Lot Size, and Pollution of Water Table Aquifers, Journal of Environmental Health, 46(6), 298-304, 1984.


LIST OF SYMBOLS

$A_{i,2}$ = a linear algebraic function obtained from the finite difference approximation for the ground water flow (23) at element $(i=1,j=2)$

$B$ = saturated thickness which may equal $h$ for an unconfined aquifer with a horizontal impermeable bottom boundary $(L)$

$C$ = the vertically averaged concentration of dissolved chemical species $(M/L^3)$

$C_{E_{i,j}}$ = effective nitrate concentration in element $i,j$ for sewer exfiltration recharge, $(M/L^3)$

$C_i$ = concentration of contaminant at node $i$, $(M/L^3)$

$C_I$ = nitrate concentration in the recharge $I$, $(M/L^3)$

$C_{i,j}$ = a variable for contaminant concentration in element $(i,j), (M/L^3)$

$C_o$ = background nitrate concentration $(M/L^3)$

$C^P$ = concentration of dissolved chemical at the source $(M/L^3)$

$C_{i}^P$ = concentration of waste injected in element $i$, $(M/L^3)$

$C_q$ = observed background nitrate concentration in ground waters (for convenience precipitation was treated as the source, or generically contaminant concentration in natural recharge flow, $(M/L^3)$

$C_{S_{i,j}}$ = effective nitrate concentration in all recharge flows in element $i,j$ derived from the combined domestic and commercial use of municipal water, sewers, and lawn fertilizers, $(M/L^3)$

$C_{ss}$ = effective concentration of nitrate in recharge from domestic and commercial use of septic systems, $(M/L^3)$.

$C_w$ = nitrate concentration in septic system effluent $(M/L^3)$

$C_{w_{i,j}}$ = effective nitrate concentration in all recharge flows in element $i,j$ derived from the combined domestic and
commercial use of on-site wells, septic systems, and lawn fertilizers, \((\text{M/L}^3)\)

- \(C_{zi,j}\) = effective nitrogen concentration in all recharge flows in element \(i,j\) derived from the combined septic systems, and lawn fertilizers, \((\text{M/L}^3)\)

- \(D\) = decision variable corresponding to the maximum deviation occurring to the maximum deviation occurring between approximate simulated contaminant concentrations in ground water and allowable limits in operational areas \((a = 1, 2, \ldots, n)\)

- \(D_a\) = approximate positive deviation resulting from the management scheme as defined through the optimum values of \(Q_k\)

- \(D_{ao}\) = initial positive deviation from the water quality standard in operational area 'a' under the \(Q_0\) conditions

- \(D_{ij}\) = vertically averaged coefficient of hydrodynamic dispersion which is descriptive of the combined effects of Fickian diffusion and dispersion caused by microscopic variations in fluid velocities within individual pores \((\text{L}^2/\text{t})\)

- \(D_{vi,j}\) = the reciprocal of the per capita generation of recharge flow \(V_{i,j} \left(\frac{\text{t} \cdot \text{person}}{\text{L}^3}\right)\)

- \(D_{wi,j}\) = the reciprocal of the per capita generation of recharge flow \(W_{i,j} \left(\frac{\text{t} \cdot \text{person}}{\text{L}^3}\right)\)

- \(D_{zi,j}\) = the reciprocal of the per capita generation of recharge flow \(Z_{i,j} \left(\frac{\text{t} \cdot \text{person}}{\text{L}^3}\right)\)

- \(F_i\) = flow rate of ground water at land use section \(i\) \((\text{L/t})\)

- \(F_{i,j}\) = effective per capita nitrate load from lawn fertilizer, \((\text{M/t} \cdot \text{person})\)

- \(G\) = water quality goal \((\text{M/L}^3)\)

- \(G_{xi,j}\) = the error in simulated hydraulic gradient in the \(x\) direction at element \(i,j\)

- \(G_{yi,j}\) = the error in simulated hydraulic gradient in the \(y\) direction at element \(i,j\)
\[ H_{i,j} \] observed head at element \( i,j \), \((L)\)
\[ H_L \] elevation of water table at the coast, \((L)\)
\[ I \] recharge in the circle defined by \( r_w \), \((L^3/t)\)
\[ K_C \] ratio of commercial flows to domestic flows
\[ K_{o1,j} \] ratio of commercial flows to domestic flows in element \( i,j \)
\[ K_{i,j} \] vertically averaged hydraulic conductivity tensor which is a physical parameter indicating the ease with which water passes through porous material in the direction \( i,j \), \((L/t)\)
\[ K_u \] ratio of unaccounted water loss for the water distribution system to domestic flows
\[ K_{u1,j} \] ratio of unaccounted water loss for the water distribution system to domestic usage in element \( i,j \)
\[ L \] the radial distance from the center of the well to the coast, \((L)\)
\[ L(L) \] length, \((meter)\)
\[ L_{s1,j} \] supply of resource in element \( i,j \)
\[ L_v \] the resource requirement per unit flow of \( V_{i,j} \)
\[ L_w \] the resource requirement per unit flow of \( W_{i,j} \)
\[ L_z \] the resource requirement per unit flow of \( Z_{i,j} \)
\[ (M_{ini})_i \] decision variable corresponding to the injection flux at proposed injection site \( i \), \((L/t)\)
\[ P \] volumetric flux of withdrawal per unit surface area of aquifer \((L/t)\)
\[ P_{i,j} \] constant corresponding to the total ground water withdrawal per unit area in element \((i,j)\), \((L/t)\)
\[ P_s \] average daily service population for a sewered area
\[ P_{w1,j} \] average daily population in element \( i,j \) using on-site wells
\[ P_z \] average daily population served by municipal water and septic systems
\[ P_{z1,j} \] average population in element \( i,j \) using municipal water
\[ Q \] natural recharge, \((L/t)\)
\[ Q_{i,j} \] natural recharge flow in element \( i,j \), \((L/t)\)
\[ Q_{in} \] volumetric flux of recharge per unit surface area of aquifer \((L/t)\)
\( Q_{\text{in}} \) = assumed volumetric flux of recharge into element \( i \), (L/t)
\( Q_{\text{out}} \) = volumetric flux of withdrawal per unit surface area of aquifer (L/t)
\( Q_{\text{out}}^i \) = assumed volumetric flux of withdrawal from element \( i \), (L/t)
\( R \) = nitrate production rate (M/person*t)
\( R^b_i \) = known boundary conditions in element \( i \) (e.g., constant flux conditions), (L/t)
\( R^s \) = resource supply
\( R^v \) = the unit resource requirement per unit flow \( V_{i,j} \)
\( R^w \) = the unit resource requirement per unit flow \( W_{i,j} \)
\( R^z \) = the unit resource requirements per unit recharge flow \( Z_{i,j} \)
\( S \) = combined recharge from sewer exfiltration and water distribution leakage, (L^3/t)
\( s \) = total number of possible contaminant reactions
\( S_y \) = vertically averaged specific yield (dimensionless), which physically corresponds to the percent of saturated porosity which drains under the force of gravity
\( H^i_{i,j} \) = specified hydraulic head, (L)
\( T_{i,j} \) = transfer coefficient defining the ratio of resultant concentration (at year 2000) at surveillance point \( j \) to the peak concentration at land use Sector:
\( U_{i,j} \) = the recharge in element \( i,j \) from land application of secondary sewage collected from elements where underlying sewers convey flows to site \( i,j \)
\( V_{i} \) = the vertically averaged specific discharge or the mass average flux of fluid flow in the \( i \) direction, (L/t).
\( V_k \) = chemical, biological, or physical reaction \( k \), negative for the addition of solute and positive for the removal of solute (M/L^3*t)
\( V_{x}, V_{y} \) = horizontal darcian fluid velocities in the \( x \) and \( y \) directions respectively, (L/t)
\( W_{i,j} \) = recharge in element \( i,j \) from septic system effluent derived from domestic and commercial use of on-site well water, (L/t)
\( W_{L} \) = maximum recharge rate from septic systems in a mature residential development, (L/t)
\( X_i \) = the decision variable for the unsewered population of land use sector \( i; \ i = 1,2...n \)

\( x_i, x_j \) = horizontal coordinate axis \( i,j \) (L)

\( Y_{i,j} \) = one of the contaminant concentration in element \( i,j \) is the target of interest (i.e., an element containing a municipal well)

\( Z \) = combined recharge from septic system effluent derived from domestic and commercial activities plus recharge from water distribution system leakage, \( (L^3/t) \)

\( Z_{i,j} \) = recharge in element \( i,j \) from septic system effluent derived from domestic and commercial use of municipal well water plus recharge from water distribution system leakage, \( (L/t) \)

\( r_{i,j} \) = the combined parameters generated from the algebraic approximation of the governing ground water flow equation at given element \( i \) in terms of neighboring element \( j \), for \( i,j = 1,...n, (1/t) \)

\( D^+_{i,j} \) = the decision variable for the positive deviation from the water quality goal at surveillance point \( j \), \( j=1,2...m \)

\( d^-_{i,j} \) = the decision variable for the negative deviation from the water quality goal at surveillance point \( j \), \( j=1,2...m \)

\( b_i \) = known boundary conditions for element \( i \) (e.g., contributions of contaminant through natural sources), \( (M/L^2t) \)

\( e_{i,j} \) = the combined parameters generated from the algebraic approximation of the governing mass transport model at given element \( i \) in terms of neighboring elements \( j \), for \( i,j = 1,...n, (L/t) \)

\( h \) = elevation of the water table above the bedrock, \( (L) \)

\( h_i \) = variable corresponding to the hydraulic head in element \( i, (L) \)

\( h_{i,j} \) = a variable corresponding to the hydraulic head in element \( (i,j), (L) \)

\( i,j \) = the respective \( y \) and \( x \) coordinates of an element in a grid superimposed over an aquifer being modeled.

\( n_e \) = vertically averaged effective porosity (dimensionless)

\( n \) = the number of elements in the \( x \) direction in a two dimensional field or \( n \) equals the number of land use sectors.
\( m \) - the number of elements in the \( y \) direction in a two-dimensional field or \( m \) equals the number of surveillance points

\( q_c \) - per capita domestic usage rate, \((L^3/t)\)

\( q_{c_{i,j}} \) - elemental per capita domestic usage, \((L^3/t)\) in element \( i,j \)

\( q_k \) - \( p \) decision variables corresponding to either the quantity of water from supply wells or the quantity of wastewater transported, or the quantity of water recharged in the various regions \((k = 1,2,...P)\) of the basin

\( q_k^0 \) - is the initial condition of the \( k \) decision variable, this could be a water supply pumping rate, wastewater transmission flow rate or recharge rate for \( k = 1,2,...P \)

\( r_w \) - radius of the well, \((L)\)

\( s_k \) - a constant, \((1/L)\)

\( \Delta x, \Delta y \) - the \( x \) and \( y \) dimensions of the numerical element, \( (m) \)

\( w_{i,j} \) - integer variable which is equal to one if sewers underlying an element convey flows to land application site \( i,j \)

\( [R] \) - \( nxn \) vector of coefficients generated from the algebraic approximation of the governing flow equation

\( [A] \) - \((n-m)\times(n-m)\) vector of coefficients generated from the algebraic manipulation of the finite difference approximation of the governing flow equation

\( [C] \) - \((n-m)\times1\) vector of variables for contaminant concentrations at every element or in Chapter 2 a \( nx1 \) column vector of decision variables defining solute concentrations throughout the system

\( [C_e] \) - \( nx1 \) column vector of decision variables corresponding to the concentrations of contaminant \( e = 1,2,...z \), for each element

\( [C_s] \) - \((n-m)\times(n-m)\) diagonal matrix of nitrate concentrations in elemental recharge flows from the combined domestic and commercial use of municipal well waters, sewers, and lawn fertilizers

\( [C_u] \) - \((n-m)\times(n-m)\) diagonal matrix of effective nitrate concentrations in the elemental recharge from land application of secondary sewage

\( [C_w] \) - \((n-m)\times(n-m)\) diagonal matrix of effective nitrate concentrations in elemental recharge flows from the combined domestic and commercial use of waters from on-site wells, septic systems and lawn fertilizers
\[ [C] = (n-m) \times (n-m) \text{ diagonal matrix of nitrate concentrations} \]

in elemental recharges from the combined domestic and commercial use of municipal water, septic systems, and lawn fertilizers.

\[ [G] = (n-m) \times (n-m) \text{ vector of coefficients generated from the algebraic manipulation of the finite difference approximation of the governing contaminant transport equation} \]

\[ [I] = (n-m) \times (n-m) \text{ identity matrix or in Chapter 2 nxn identity matrix} \]

\[ \{M\} = \text{nxl vector of decision variables defining the contaminant injection fluxes (each flux equal to } Q_{in} C_{p} \text{, the solute concentration in the injected waste times the flow rate} \]

\[ [P] = \text{a nxn diagonal matrix with values of one for entries that correspond to the injection sites and values equal to zero for all other entries} \]

\[ \{P\} = (n-m) \times 1 \text{ vector of known pumping fluxes in every element} \]

\[ \{Q\} = (n-m) \times 1 \text{ vector of elemental natural recharge flows} \]

\[ \{Q_{in}\} = \text{nxl vector of assumed or known recharge rates} \]

\[ \{Q_{in} C_{p}\} = \text{nxl vector of known waste injection fluxes} \]

\[ \{Q_{out}\} = \text{nxl vector of assumed or known pumping or withdrawal rates} \]

\[ \{R\} = \text{nxl vector of known boundary conditions} \]

\[ \{S\} = (n-m) \times 1 \text{ vector of elemental recharge flows from the combined domestic and commercial use of municipal well water, sewers, and lawn fertilizers} \]

\[ \{U\} = (n-m) \times 1 \text{ vector of elemental recharge flows from land application of secondary sewage} \]

\[ \{W\} = (n-m) \times 1 \text{ vector of elemental recharge flows from the combined domestic and commercial use of waters from on-site wells, septic systems and lawn fertilizers} \]

\[ \{Z\} = (n-m) \times 1 \text{ vector of elemental recharges from the combined domestic and commercial use of municipal well water, septic systems, and lawn fertilizers} \]

\[ \{b\} = \text{nxl right-hand side vector reflecting boundary conditions (i.e., existing disposal fluxes)} \]

\[ \{f\} = \text{nxl column vector defining boundary conditions and input fluxes of contaminant p as a nonlinear function of the integer decision variable } X_{i} \text{, the chosen treatment received before subsurface injection and the decision variable D, dilution water flows} \]

\[ [g] = \text{a nxn matrix of coefficients derived from a known constant velocity field and finite difference approximation of the mass transport equation} \]
\[ \{h\} = (n \cdot m) \times 1 \text{ vector of variables for hydraulic heads at every element or in Chapter 2 } \]

\[ n \times 1 \text{ vector of variables corresponding to the hydraulic heads at every element} \]