SOURCE CONTROL OF WATER TREATMENT WASTE SOLIDS

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Progress Report for Federal Water Pollution Control Administration

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SOURCE CONTROL OF WATER TREATMENT WASTE SOLIDS

First Annual Report

By

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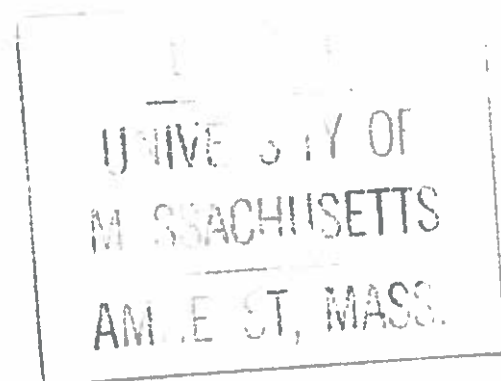
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PREFACE

The following report is the first of a series of progress reports
prepared for the Federal Water Pollution Control Administration research
grant WP-01239, "Source Control of Water Treatment Waste Solids."

Official notification of award of the project was received in mid-
July 1967. Acquisition of research equipment, search of the literature,
liaison with water treatment plant operators, development of theories of
dewatering and drying, testing and characterization of the sludges, and
design of equipment unavailable commercially occupied most of the initial
months of the project. This report represents a condensation of work
accomplished from mid-July 1967 to early March 1968, and as such is intended
to portray a synopsis of the authors' research activities.

The preparation of Chapters 1 and 5 and Appendix B by Dr. Adrian,
Chapters 2, 3, and 4 by Dr. Nebiker, and Chapter 6 and Appendix A by Mr. Lutin
is hereby acknowledged.

Portions of this report are to be presented at the 49th Annual Meeting
of the American Geophysical Union, April, 1968; the 23rd Purdue Industrial
Waste Conference, May, 1968; and the New England Conference on Industrial
Waste Control, July, 1968. In addition, following the stated desire of the
investigators to direct the emphasis of this research to a solution of the
water treatment solids disposal problem, copies of this report will be
brought to the attention of all members of the ASCE-WPCF Subcommittee on
Sludge Dewatering, who including Dr. Adrian and Dr. Nebiker, are preparing
a Manual of Practice for the profession.

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I. INTRODUCTION

NEED FOR STUDY

Water for domestic purposes and general municipal use has traditionally
been obtained from the best available supplies consistent with economy.
With the advent of physical and chemical water treatment late in the Nine-
teenth Century, raw water supplies of lower quality could be upgraded to
acceptable levels. The chemical dosage was roughly inversely proportional
to the raw water quality, with more chemicals, such as alum and lime, re-
quired for poor quality waters, resulting in a concomitant increase in
sludge production.

The central concern of the water utility was the production of ample
supplies of potable treated water under the constraint that this production
be achieved at a minimum cost. Scant attention was paid to the by-products
of water treatment, such as filter wash water and sludge from the sedimen-
tation basin. A convenient and low cost disposal site for filter wash
water was often the supplying stream. Sludge from the sedimentation basin
may also have been discharged to the stream, or, if the stream capacity
was low, discharged to a lagoon.

The occasional usage of a sludge lagoon, rather than direct discharge
of the sludge to the stream, attested to some concern over degradation of
the receiving water quality. However, this mode of operation was subject to
the whim of the utility. Moreover, the stream was always available as a
disposal site should the lagoon capacity become exhausted. In fact, a report
by Louis R. Howson in 1966 indicated that over 90 percent of all water
treatment wastes were being returned to the raw water source.[1] Recently,
the national conscience has become sensitive regarding water pollution and

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it is now recognized that water treatment sludge discharged into a stream
is a pollutant. The concern over water pollution necessitates a re-
examination of water treatment sludge handling and disposal methodologies.
Lagoon disposal may be a satisfactory solution providing additional lagoon
sites are available to replace those which are filled, or if the compacted
solids can be removed and the lagoon reused. Recent studies point out
that lagoons may be unsuitable for water treatment clarifier sludges as com-
paction of the solids in the lagoon is insufficient to produce a low enough
water content to enable the solids to be removed as a solid.[2]

Elsewhere the sludge may be discharged directly to a sanitary sewer
where it becomes the concern of the wastewater treatment plant. However,
lime sludge becomes less acceptable to the wastewater treatment plant as
the level of treatment increases. While lime sludge is acceptable to a
primary treatment plant, it may be unacceptable to a secondary treatment
plant,[3] unless incorporated into a phosphate removal process.[4]

The alternative choice between augmenting treated supplies by utilizing
waters of lower quality, or conserving existing supplies by concentrating
on reduction of water usage, has recently become more clearly delineated.[5]
In many instances immediate temporary relief from an insufficiently treated
water supply is attained by marked reduction in consumption achieved by
eliminating waste or by altering industrial processes. However, the in-
creasing long range demand has focused increased attention on water renovation,
with some processes envisioned as forming a closed system such that complete
reuse will be achieved. Renovation of a municipal wastewater is beyond the
capability of secondary treatment in that it requires one or more of the
advanced waste treatment methods. The high quality product water is obtained

at the expense of a production of voluminous dilute sludges or brines, the disposal of which vastly increases the present formidable solids handling problem. Today, water renovation in the sense of complete recycling of wastewater through successive treatment steps until it can be returned to the domestic water supply, is practiced in laboratory and pilot systems. Rapid advances in science, technology, and engineering promise that wastewater renovation soon will be practiced on a large scale. Indeed, the FWPCA Demonstration Grant program on advanced waste treatment shows that wastewater renovation is evolving to where it will fit into the spectrum of practical treatment methods. As it evolves and is put into practice, the alternative between development of lower quality new raw water sources or renovation of wastewaters for reuse will become clearly defined, so that an economic choice can be made between these alternatives.

Both utilization of lower quality raw waters and wastewater renovation will become more commonplace in the future. Either choice will increase the sludge handling problem. The per capita quantity of sludge to be handled will be greater with water renovation than with conventional water treatment (based on the present day production figures of 15 lb. per capita per year of dry solids from water treatment plants and 130 lb. per capita per year of dry solids from wastewater treatment plants.)[3] The solids from either process will be accompanied by vast quantities of water so that on a wet basis, assuming sludges of 3% solids content, a water treatment plant would produce nearly 500 lb. of sludge per capita per year while a wastewater treatment plant would produce 4,200 lb. of sludge per capita per year.

Ultimate disposal of the sludge solids requires their discharge to the atmosphere, to the ocean, or to the land. Organic compounds, which are oxidizable to CO_2, are most amenable to atmospheric disposal. Water

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treatment sludges are low in organic material so that incineration will leave a substantial fraction of ash. Soluble compounds may be carried by streamflow to the ocean, or, for communities near the ocean, direct discharge of the sludge into the ocean may be practicable. For the over-whelming majority of United States communities the water treatment sludge solids are disposed of on land. Prior to their disposal, volume and weight reduction are necessary to reduce the weight from some 500 lb./cap.-year on a wet basis to approach the 15 lb./cap.-year on a dry basis. This can be achieved by removing water from the sludge.

Reduction of the water content of sludge may be carried out by a variety of mechanical methods or by non-mechanical methods such as compaction and decantation in lagoons, or drainage and evaporation on drying beds. Selection among these alternatives is based on economics. Exhaustive studies by the City of Chicago for wastewater sludges shows the cost of dewatering, drying, and disposal methods varies from approximately $12/ton to $80/ton on a dry solids basis[6] The most economic method was dewatering, drying, and disposing of the solids on land.

Although land disposal is widely used by small treatment plants, there is a paucity of reliable design data with which to design a sand dewatering and drying bed for sludge. Mechanical dewatering methods, although less economic than sand beds, have become increasingly popular with design engineers because of the ready availability of their design data. A typical example is the vacuum filter for sludge dewatering. Sludge solids content, specific resistance of the sludge cake, and its coefficient of compressibility are related to the desired dewatered sludge cake characteristics through the machine operating variables. Variation of the machine operating parameters permits the selection of the optimum vacuum filter. No such procedure is available to the design engineer faced with designing a sand dewatering bed.

He can resort to some empirical rules of thumb, practice at similar installations, or follow his intuition. None of these procedures is as satisfactory as a functioning mathematical model, verified adequately for reliability, which clearly delineates the role of each parameter. There is a great need for such a mathematical model describing the operation of a sludge dewatering and drying bed. Without it, no rigorous economic analysis can be made.

PURPOSE AND OBJECTIVES OF STUDY

The purpose of the study herein reported was to develop methods of approach to be applied to the previously cited problems of land disposal of water treatment sludges. The general objective of the study was to develop rational design formulations with which an engineer could design dewatering and drying beds for water treatment sludges. Specific objectives of the research project and study included:

1. To bring together a research team which would have the expertise to recognize, study, attack, and master the engineering problems of water treatment sludge dewatering and drying.

2. To review the literature on the subject of water treatment sludge and related areas with specific reference to sludge characteristics, theories and practices of dewatering and drying, and the instrumentation utilized in various experimental investigations.

3. To establish liaison with several water treatment plants treating raw water from several sources and employing a variety of treatment processes; and to characterize samples of the sludges by such classification tests as specific resistance, coefficient of compressibility, and solids content.

4. To apply engineering analysis to the problem of sludge dewatering and drying in order to postulate hypotheses which would permit formulation of mathematical models describing these physical processes.

5. To assemble the necessary experimental apparatus to characterize the sludges, to verify published behavior of sludges, to test the applicability of hypotheses which had been formulated, and to gain insight into the behavior of sludges.

6. To promulgate the results of this research to the engineering profession, to other researchers working on similar research endeavors, and to the general scientific and engineering community.

Progress on the aforementioned objectives was to be related to the three-year research program outlined in the original application for this research grant. Therefore, efforts were concentrated on completing item 1, bringing item 2 to a nearly complete state, establishing the liaison discussed in item 3, concentrating on item 4, and starting work on item 5. Initial results appropriate for item 6 will be presented in the form of engineering papers at conferences prior to expiration of the first grant year.

SCOPE OF STUDY

Formulation of the relation between loading rates of water treatment sludges applied to filter beds, and the subsequent dewatering and drying of the sludge, forms the core of the engineering analysis reported in this study. This study presents the essential aspects of filtration theory, dewatering theory, drying theory, dewatering practice, current design

practice, and instrumentation appropriate to the theory and practice of sludge dewatering and drying. In addition, selected water treatment sludges were analyzed for their properties of solids content, specific resistance and coefficient of compressibility.

The dewatering and drying theories that have been formulated by the investigators are subject to few assumptions and restrictions, so that they should have wide applicability. The laboratory tests required for parameter evaluation in these formulations are simple, straight-forward, and easily carried out. The flow resistance of the sludge filter cake is assumed to be far larger than the flow resistance of the filter supporting medium; preliminary tests verify this assumption. Sludge penetration of the supporting sand has been assumed to be negligible. Experimental testing of this assumption brings out the role of sand size as related to sludge penetration of the sand filter. While it is obvious that selection of large sand sizes would permit penetration of the sludge particles into the sand bed, yet it was judged to be good design practice to select a sand size which would minimize sludge particle penetration into the filter. This judgment is based on the desire to prolong the usable life of the sand bed by avoiding clogging of the interstices with subsequent filter blinding. Water treatment sludge particle penetration of a dewatering bed would result in more rapid clogging than would be expected for a similar amount of pene-tration by a wastewater treatment sludge because of lack of biological degradation of the former as contrasted to the latter. Biological degra-dation of the penetrating sludge particles could result in rejuvenation of a wastewater treatment sludge dewatering bed, whereas little rejuvenation of a water treatment sludge dewatering bed would be anticipated.

Water softening sludge characterization was not carried out during

the project period herein reported. Instead, efforts were concentrated on characterizing sludges from the sedimentation basins of water treatment plants which had utilized alum and lime for their chemical treatment. Water softening sludges, some of the advanced waste treatment sludges, and sludges representing a wide spectrum of raw water characteristics are to be investigated during future project periods.

Employment of the gamma radiation attenuation equipment for measuring the solids content necessitated discussion of the applicable statistical theory describing random radiation events. These results were then applied to the electronic equipment in order to interpret the results in terms of the original sludge dewatering and drying system.

ORGANIZATION FOR STUDY

The study herein reported originated as an outgrowth of the research interests of the principal investigator, Dr. D. D. Adrian, and the co-principal investigator, Dr. J. H. Nebiker. In 1966 at Vanderbilt University Dr. Nebiker was awarded a National Science Foundation research grant GK-924 entitled, "Dewatering of Sewage Sludge on Granular Materials," to study dewatering of wastewater sludges on sand beds. Also Dr. Adrian was awarded a University Research Council grant to study "Unsaturated Flow Through Porous Media," while Dr. Nebiker was also awarded a University Research Council grant to study "Hygroscopicity Hysteresis of Sewage Sludge." Later in 1966, at Vanderbilt University, Dr. Nebiker as principal investigator and Dr. Adrian as co-principal investigator were awarded a Federal Water Pollution Control Administration research grant WP-01053, "Dewatering and Drying of Sewage Sludge on Porous Media." Upon relocating from Vanderbilt University to the University of Massachusetts in June 1967, the

National Science Foundation grant expired while Vanderbilt University
retained grant WP-01053 under new leadership.

In June, 1967 grant WP-01239, "Source Control of Water Treatment Plant
Waste Solids" was awarded, although official notification was not received
until July. In August Mr. Philip A. Lutin was retained as a full-time
research engineer to work on the project. Mr. Thomas G. Sanders, master's
degree candidate in Environmental Engineering, while not supported by this
project, was available for consultation from July onward. In January, Mr.
Edward E. Clark, a Ph.D. candidate in Environmental Engineering, began work
as a research assistant. In February, Mr. Kuang-Mei Lo, a Ph.D. candidate
in Environmental Engineering, also began work as a research assistant.
Undergraduate student help on a part-time basis was encouraged, and
the following were appointed: Mr. John Connolly, Mr. Tom Roule, Mr. Walter
Bickford, Mrs. Susan Tomkins Foudy, and Mr. Ronald Michalski. Mr. Connolly,
Mr. Bickford, and Mr. Michalski served as general laboratory technicians,
while Mr. Roule, an Electrical Engineer, specialized in instrumentation.
Mrs. Foudy concentrated on drafting. All work herein reported was conducted
in the Environmental Engineering Laboratories of the Department of Civil
Engineering.

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II. THEORY OF FILTRATION

The elements of filtration theory derive from the concepts of fluid flow through closed conduits. A Newtonian fluid in laminar flow through a circular pipe will be subjected to pressure, gravity, and viscous forces. Force equilibrium under steady flow conditions allows one to write for a cylindrical element of constant radius (see Figure 2-1):

$$\tau = (2\pi r)L - \Delta p \, \pi r^2 = 0$$

2-1

where τ = shear stress along the curved surface of the cylinder $[F][L^{-2}]$

r = radius of cylinder $[L]$

L = length of cylinder $[L]$

$(-\Delta p)$ = pressure drop across the length of the cylinder $[F][L^{-2}]$

Solving for the shear stress

$$\tau = -\frac{r}{2}\frac{\Delta p}{L}$$

2-1a

The basic Newton law of viscosity defines shear stress as

$$\tau = \mu\frac{dv}{dy}$$

2-2

with μ = dynamic viscosity of the fluid $[F][T][L^{-2}]$

$\frac{dv}{dy}$ = velocity gradient $[T^{-1}]$

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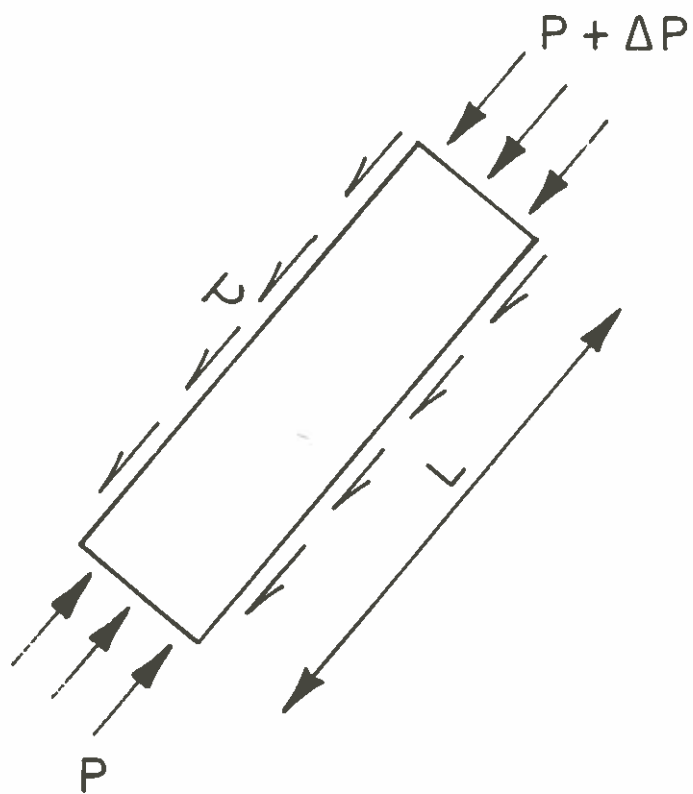
Figure 2-1. Cylindrical fluid element subjected to pressure and
viscous shear forces.

The velocity gradient is measured in the direction transverse to flow, and from the boundary. For the more specific case of flow in a circular conduit, dv/dy is written as -dv/dr, as r increases as y decreases. Thus

$$\tau = -\mu \frac{dv}{dr}$$

2-2a

Equating Equations 2-1a and 2-2a yields

$$\mu \frac{dv}{dr} = \frac{r}{2} \frac{\Delta P}{L}$$

which reorganizes to

$$dv = \frac{\Delta P}{2\mu L} r \, dr$$

HAGEN-POISEUILLE EQUATION

This equation may be integrated to represent velocity distribution in a circular pipe. The velocity at the boundary (r = D/2, with D the diameter of the pipe) is zero.

$$\int_0^v dv = \frac{\Delta P}{2\mu L} \int_{D/2}^r r \, dr$$

$$v = \frac{\Delta P}{4\mu L} \left(r^2 - \frac{D^2}{4} \right)$$

2-3

A further integration is necessary to establish a relationship for the average velocity across the cross-section, $\bar{v}$. This is accomplished by

using the equation of continuity for imcompressible flow

$$\int_{0}^{A} v \, dA = \bar{v} A$$

Rearrangement, noting that

$$A = \pi D^2 / 4$$

and

$$dA = d(\pi r^2) = 2\pi r \, dr$$

allows solving for $\bar{v}$ upon introduction of Equation 2-4:

$$\bar{v} = \frac{4}{\pi D^2} \cdot 2\pi \int_{0}^{r=D/2} v \, r \, dr$$

$$= \frac{8}{D^2} \cdot \frac{\Delta p}{4\mu L} \int_{0}^{r=D/2} \left(r^2 - \frac{D^2}{4} \right) r \, dr$$

$$= \frac{2\Delta p}{\mu L D^2} \left[\frac{r^4}{4} - \frac{D^2 r^2}{4 \cdot 2} \right]_{0}^{r=D/2}$$

$$= \frac{32 \Delta p D^2}{\mu L} \qquad\qquad 2\text{-}4$$

Equation 2-4 is known as the equation of Hagen-Poiseuille. It is generally in the form

$$\Delta p = \frac{32 \mu \bar{v} L}{D^2}$$

$$2\text{-}4a$$

It is here more convenient to express pressure drop in terms of headloss. A dimensionly correct conversion is

$$\Delta p = \rho g\, h_f$$

with

ρ = density of the fluid $[M][L^{-3}]$

g = acceleration of gravity $[L][T^{-2}]$

h_f = equivalent height of fluid $[L]$

Thus

$$\rho g\, h_f = \frac{32\, \mu\, \bar{V}\, L}{D^2}$$

or

$$h_f = \frac{32\, \mu\, \bar{V}\, L}{\rho g\, D^2} \qquad \text{2-4a}$$

FLOW THROUGH POROUS MEDIA

Equation 2-4a may be modified for non-circular cross-sections by use of the concept of hydraulic radius (R_h). The hydraulic radius is defined as the ratio of cross-sectional area A to the wetted perimeter of the conduit P. For circular cross-sections this is

$$R_h = \frac{A}{P} = \frac{\pi\, D^2/4}{\pi D} = \frac{D}{4} \qquad \text{2-5}$$

R_h has the unit of length. It allows expression of the Hagen-Poiseuille equation as

$$h_f = \frac{2\, \mu\, \bar{V}\, L}{\rho g\, R_h^2} \qquad \text{2-6}$$

For the more general case of non-circular cross-sections, Equation 2-6 should be modified to

$$h_f = \frac{K \mu \bar{V} L}{\rho g R_h^2}$$

<div align="right">2-6a</div>

with K a dimensionless variable, dependent on the shape of the cross-section.

Flow through porous media is generally represented as laminar flow through a series of interconnected pores of varying cross-section. An initial assumption of a media composed of identical particles is also made.

If N represents the number of particles in a sample of L length and cross-sectional area A', then the wetted conduit surface P over unit length may be written as

$$P = \frac{N S_p}{L}$$

<div align="right">2-7</div>

with S_p = surface area of a single particle $[L^2]$

The cross-sectional flow area A is some fraction of the total cross-sectional area A'. This fraction is the void ratio ϵ :

$$\epsilon = \frac{V_v}{V_{TOT}}$$

<div align="right">2-8</div>

with $\qquad V_v$ = volume of voids in the sample $[L^3]$

V_{TOT} = total volume of sample, or the volume of voids and solids $[L^3]$

However V_{TOT} constitutes the volume of voids and solids. If the particles are each of volume V_p, then $N \cdot V_p$ represents the volume of solids,

allowing one to write

$$V_{TOT} = V_v + N V_p$$

2-9

Thus

$$\epsilon = \frac{V_v}{V_{TOT}}$$

$$= \frac{V_v}{V_v + N V_p}$$

or rearranging,

$$V_v = \frac{\epsilon N V_p}{1 - \epsilon}$$

2-10

Substitution of Equations 2-7 into Equation 2-5 yields

$$R_h = \frac{A}{P} = \frac{A}{N S_p / L}$$

or

$$R_h = \frac{A L}{N S_p}$$

2-11

Noting that the product AL equals the volume of voids in the sample (V_V), combination of Equations 2-10 and 2-11 yields

$$R_h = \frac{A L}{N S_p} = \frac{V_v}{N S_p} = \frac{\epsilon N V_p}{1 - \epsilon} \cdot \frac{1}{N S_p}$$

or

$$R_h = \frac{\epsilon}{1 - \epsilon} \frac{V_p}{S_p}$$

$$= \frac{\epsilon}{1 - \epsilon} \frac{1}{S_o}$$

2-12

with S_0 = specific surface or ratio of area of particle to volume $[L^{-1}]$

The velocity $\bar{v}$ through the pores of the sample is generally not readily determinable. Instead of $\bar{v}$, a velocity called the superficial velocity, $\bar{v}_S$, is used. It may be thought of as the velocity of flow if the particles are considered non-existant. The equation of continuity defines $\bar{v}_S$:

$$\bar{V_S}\, A' = \bar{V}\, A$$

or

$$\bar{V_S} = \bar{V}\, \frac{A}{A'}$$

Multiplication of the right hand side of this equation by L/L = 1 yields

$$\bar{V_S} = \bar{V}\, \frac{A \cdot L}{A' \cdot L}$$

The product AL is again the volume of voids in the sample, whereas A'·L is the total volume; thus

$$\bar{V_S} = \bar{V}\, \frac{V_v}{V_{TOT}}$$

or

$$\bar{V_S} = \epsilon\, \bar{V} \qquad\qquad \bar{V} = \frac{\bar{V_S}}{\epsilon} \qquad\qquad \text{2-13}$$

as V_V/V_{TOT} is the void ratio (Equation 2-8).

Substitution of Equations 2-12 and 2-13 into Equation of 2-6a follows:

$$h_f = \frac{k\mu\bar{v}L}{\rho g\, R_h^2}$$

$$= \frac{k\mu L}{\rho g} \cdot \frac{\bar{V_S}}{\epsilon} \bigg/ \left[\left(\frac{\epsilon}{1-\epsilon}\right)\frac{1}{S_0}\right]^2 \qquad\qquad \text{2-6a}$$

which simplifies to

$$h_f = \frac{\mu L \bar{v}_s}{\rho g} \frac{K}{\epsilon} \left[\frac{(1-\epsilon)}{\epsilon S_o} \right]^2 \qquad \text{2-14}$$

This equation is frequently written as

$$\bar{v}_s = \frac{\rho g h_f}{L} \frac{\epsilon}{K} \left[\frac{\epsilon}{(1-\epsilon) S_o} \right]^2 \qquad \text{2-14a}$$

The end terms

$$\frac{\epsilon}{K} \left[\frac{\epsilon}{(1-\epsilon) S_o} \right]^2$$

are recognized as intrinsic properties of the porous medium, and are thus given the substitutive symbol K, the intrinsic permeability. The reciprocal of K is R', the filter resistance. It has units of $[L^{-2}]$. Equation 2-14 may therefore be written as

$$h_f = \frac{\mu L \bar{v}_s R'}{\rho g} \qquad \text{2-14b}$$

with

$$R' = \frac{K}{\epsilon} \left[\frac{(1-\epsilon) S_o}{\epsilon} \right]^2 \qquad \text{2-15}$$

At this point it is well to point out that the theoretically derivable Equation 2-14b is limited to laminar flow of an incompressible Newtonian fluid in the absence of inertial effects. In the more general case of

flow in which inertial effects are prevalent, the equation

$$\Delta p = A_o \bar{V} + B_o \bar{V}^2 \qquad \text{2-16}$$

has been found valid.[1] A_0 and B_0 are constants.

Stepochkin[2] proposed the expression of Equation 2-16 as:

$$N_L = \phi \left(2 N_R + 0.1 N_R^2 \right)$$

2-16a

where ϕ = constant dependent on material (dimensionless)

N_L = Lebenson number   (dimensionless)

$$= \frac{\rho R_H^3 \Delta p}{\mu^2 L} \qquad \text{2-17}$$

N_R = Reynolds number   (dimensionless)

$$= \frac{\bar{V} R_H \rho}{\mu} \qquad \text{2-18}$$

with ρ = density of fluid $[M][L^{-3}]$

Substitution of Equations 2-17 and 2-18 into Equation 2-16a yields

$$\frac{\rho R_H^3 \Delta p}{\mu^2 L} = \phi \left(\frac{2 \bar{V} R_H \rho}{\mu} + \frac{0.1 \bar{V}^2 R_H^2 \rho^2}{\mu^2} \right)$$

which simplifies to

$$\Delta p = \frac{2 \phi \mu \bar{V} L}{R_H^2} + \frac{0.1 \phi \rho \bar{V}^2 L}{R_H} \qquad \text{2-18a}$$

Ignoring the second group of terms on the right hand side results in a form of Equation 2-6a. The assumption of laminar flow, or flow subjected to predominantly viscous effects, is thus justified if

$$\frac{2\phi\mu\bar{V}L}{R_H^2} \gg \frac{0.1\phi\rho\bar{V}^2L}{R_H}$$

or

$$\frac{20\mu}{R_H} \gg \rho\bar{V}$$

Substitution of Equation 2-12 and 2-13 into the above leads to

$$\frac{20\mu(1-\epsilon)S_o}{\epsilon} \gg \rho\frac{\bar{V}_s}{\epsilon}$$

or

$$\frac{20\mu(1-\epsilon)S_o}{\rho} \gg \bar{V}_s$$

If one assumes a void ratio of 0.5, water as the fluid, and perfectly spherical particles of one micron; a sample calculation reveals

$$\frac{20\mu(1-\epsilon)S_o}{\rho} = \frac{20\mu(1-\epsilon)\pi d^2}{\rho\pi d^3/6} = \frac{120\mu(1-\epsilon)}{\rho d}$$

$$\frac{120\cdot 0.01(1-0.5)}{1\cdot 10^{-4}} = 0.60 \ cm/sec = \bar{V}_s$$

with the substitutions as follows

μ = 1 centipoise = 0.01 poise = 0.01 gm/cm-sec.

ϵ = 0.5

$$\rho = 1 \text{ gram/cm}^3$$

$$d = 10^{-4} \text{ cm} = \text{diameter of particle}$$

The high velocity calculated above suggests that only laminar flow without inertial effects is a significant possibility in filtration of fine particles.

CARMEN-KOZENY RELATIONSHIP

The variables involved in Equation 2-15

$$R' = \frac{K}{\epsilon} \left[\frac{(1-\epsilon) S_o}{\epsilon} \right]^2 \qquad \text{2-15}$$

bear closer scrutiny. It is clear that if the sample consists of identical geometrically describable particles, the value of S_o may be directly calculable. The values of K and ϵ, however, depend on the packing or placement of the particles. Empirical corrections are applied resulting in

$$R' = \frac{K'}{\epsilon} \left[\frac{(1-\epsilon) S'}{\epsilon} \right]^2 \qquad \text{2-15a}$$

Here

$$S' \propto S_o$$

which is defined as a shape factor. The proportionality is removed by writin

$$K'(S')^2 = K S_o^2$$

Theoretically K' is a function of particle shape and orientation (i.e., the shape of cross-section available for flow). Grace[3] reports that for particles $> 5\,\mu$ in diameter, the value of K' is about 5 ± 10%. However his work shows that for diameters $< 5\,\mu$, K' is a variable, due to compression of the small particles. S', the shape factor, is of course, extremely variable, particularly for flocculent particles.

Rewriting of Equation 2-14b using Equation 2-15a

$$h_f = \frac{\mu L \bar{V}_s R'}{\rho g}$$

2-14b

leads to

$$h_f = \frac{\mu L \bar{V}_s K'}{\rho g \epsilon}\left[\frac{(1-\epsilon)S'}{\epsilon}\right]^2$$

2-19

which is the Kozeny-Carmen relationship.[4] It may be modified for non-uniform materials by reference to a particle size analysis, allowing average values for particle area and volume to be used.

FILTRATION OF COMPRESSIBLE MEDIA

It has been repeatedly proven that Equation 2-19 does not apply to the filtration of very fine suspensions such as wastewater and water treatment sludges.[5,6] This is true for two reasons:

1. The suspended materials are of a gelatinous nature and may flocculate when brought in close proximity to each other.

2. The extremely fine size creates a large headloss through a short distance. The loss is in the form of viscous drag over the particle surfaces resulting in particle deformations and reduction of void sizes.

Hence it is seen that the term R' in Equation 2-14b reflecting the influence of shape, size and orientation of particle, and the void ratio, is a variable for filtration of fine suspensions. Further analysis requires the assumption of a gradual increase in filter resistance with duration of filtration. A later assumption will be the dependence of filter resistance on the pressure applied across the sample.

Assume a filter material of known and constant resistance which will serve as a supporting media to the filter cake formed on its surface during filtration. The resistance of this cake may be called R_f' $[L^{-2}]$ and the depth, L_f. It follows that Equation 2-14b

$$h_f = \frac{\mu \, L \, \bar{v}_s \, R'}{\rho g} \qquad \text{2-14b}$$

may include the effects of the supporting filter by writing

$$h_f = \frac{\mu}{\rho g} \left(\bar{v}_s \, L \, R' + \bar{v}_s' \, L_f \, R_f' \right) \qquad \text{2-20}$$

with $\bar{v}_S$ = superficial velocity in supporting filter.

The superficial velocities are best rewritten in terms of the filtrate volume flow rate dV/dt $[L^3][T^{-1}]$. Thus:

$$\overline{V_s} = \frac{1}{A} \frac{dV}{dt}$$

and

$$\overline{V_s}' = \frac{1}{A} \frac{dV}{dt}$$

Hence Equation 2-20 reads

$$h_f = \frac{\mu}{\rho g A} \frac{dV}{dt} \left(L R' + L_f R_f' \right) \qquad \text{2-20a}$$

which reorganizes to

$$\frac{dV}{dt} = \frac{\rho g A h_f}{\mu \left(L R' + L_f R_f' \right)} \qquad \text{2-20b}$$

Carmen[4] introduced the concept that the length L of the compressible filter cake was proportional to the volume of filtrate:

$$L = v_c \, V/A$$

with v_c = volume of cake deposited per unit volume of filtrate (dimensionless) This leads to

$$\frac{dV}{dt} = \frac{\rho g A h_f}{\mu \left(\frac{v_c V R'}{A} + L_f R_f' \right)} \qquad \text{2-20c}$$

However, the nature of compressible cakes makes it illogical to measure volume. More likely is the measurement of weight. The term c, or weight of

dry cake solids per unit volume of filtrate is thus introduced. Dimensions of c are $[F][L^{-3}]$. This necessitates a new definition of filter resistance, R:

$$V_c\, R' = c\, R \qquad \text{2-21}$$

R, entitled the specific resistance, is in units of $[L][F^{-1}]=[T^2][M^{-1}]$

Equation 2-20c can be reformulated then to

$$\frac{dV}{dt} = \frac{\rho g\, A\, h_f}{\mu \left(\frac{c R V}{A} + L_f R_f' \right)}$$

$$= \frac{\rho g\, A^2\, h_f}{\mu \left(c R V + A L_f R_f' \right)} \qquad \text{2-20d}$$

This equation is the basis for work done by Coackley and Jones[7], utilizing the concept of specific resistance to calculate vacuum filter performance in dewatering sewage sludges.

DETERMINATION OF SPECIFIC RESISTANCE

Vacuum filters maintain a constant vacuum during cake formation. An assumption of constant pressure or headloss, h_f, leads to integration of Equation 2-20d:

$$\frac{dV}{dt} = \frac{\rho g A h_f}{\mu \left(\frac{c R V}{A} + L_f R_f' \right)} \qquad \text{2-20d}$$

$$\int_0^t \rho g A h_f\, dt = \int_0^V \frac{\mu c R V}{A}\, dV + \int_0^V \mu L_f R_f'\, dV \qquad \text{2-21}$$

$$t = \frac{\mu c R V^2}{2 \rho g A^2 h_f} + \mu L_f R_f' V$$

The procedure for determination of R and a in Equation 2-21 is well established. A Buchner funnel or filter leaf is used, and a constant vacuum pressure applied. Compression cylinders have also been adapted for measurement of specific resistance. Running determinations of V versus t are made and plotted as t/V versus V on cartesian coordinates yielding a straight line. This is seen by reorganization of Equation 2-22 to

$$\frac{t}{V} = \frac{\mu c R}{2 \rho g A^2 h_f} V + \mu L_f R_f'$$

2-22a

The slope of the plot is equal to b $[T][L^{-6}]$ with

$$b = \frac{\mu c R}{2 \rho g A^2 h_f}$$

2-23

or

$$R = \frac{2 b \rho g A^2 h_f}{\mu c}$$

2-23a

The values of μ and ρ are readily determinable. For water treatment sludge, values for water may be substituted. Headloss across the cake, held constant, is directly set at the value the researcher desires, generally in the range of 5 to 70 cm of mercury. Some controversy exists over the measurement of filter area. It is thought that a filter leaf contributes more of its entire surface area to filtration than does the Buchner funnel, which allows flow through holes underlying the filter paper. Swanwick reports

a satisfactory procedure using a wire screen underneath the filter paper. Thus the entire plan area can be measured and its value placed in Equation 2-23a.[8]

The value of c is inferred constant (see Equation 2-21). Since normal testing procedure allows opportunity to determine c at only the end of the test, it is important that the test be run until a true cake is formed, at which time the test is stopped and the solids content determined. The value c is therefrom determined as follows:

Let s_0 = solids content at beginning of test [%]

s_f = solids content at end of test [%]

W_{TS} = weight of solids in unfiltered sludge

$\doteq$ weight of solids in filtered cake $[F][L^{-3}]$

W_{TOT_0} = weight of unfiltered sludge $[F][L^{-3}]$

W_{TOT_f} = weight of filtered cake $[F][L^{-3}]$

The volume of filtrate that occurs during testing, assuming no solids loss, is

$$V = \left(W_{TOT_o} - W_{TOT_f} \right) / \rho g \qquad 2\text{-}24$$

with ρg as a weight-volume conversion factor (ρ refers to the filtrate). Referring to the total weights as

$$W_{TOT} = \frac{W_{TS}}{s} \cdot 100$$

allows rewriting of Equation 2-24 as

$$V = \left(\frac{W_{TS} \cdot 100}{S_o} - \frac{W_{TS} \cdot 100}{S_f} \right) \Big/ \rho g \qquad \text{2-24a}$$

which simplifies to

$$V = 100 \frac{W_{TS}}{\rho g} \left(\frac{1}{S_o} - \frac{1}{S_f} \right) \qquad \text{2-24b}$$

The weight of dry cake solids per unit volume of filtrate is c:

$$c = \frac{W_{TS}}{V}$$

which, inserting Equation 2-24b reads as

$$c = W_{TS} \Big/ \frac{100 \cdot W_{TS}}{\rho g} \left(\frac{1}{S_o} - \frac{1}{S_f} \right)$$

$$= \frac{\rho g}{\frac{100}{S_o} - \frac{100}{S_f}} \qquad \text{2-25}$$

Dilute sludges ($s_o \sim 2\%$) dewatered to solids contents of 20% and higher, allow neglect of the s_f term

$$c = \frac{\rho g s_o}{100} \qquad \text{2-26}$$

Neglect of s_f is particularly justified for Buchner funnel testing, as the neglect tends to decrease R. But the measurement of A is on the high side, and thus the deviations in A and R cancel out according to Equation 2-23a. The value of c, calculated by either Equation 2-25 or Equation 2-26 is inserted into Equation 2-23a for the eventual solution of R.

COEFFICIENT OF COMPRESSIBILITY

It must be emphasized that the value of R is dependent not only upon the sludge characteristics but on the pressure at which the test is run. It is known from experimental analysis that a relation between R and $\Delta\rho$ exists.[3] It is in the form

$$R = a_o + a' \left(\Delta p\right)^\sigma \qquad \text{2-27}$$

or using headloss

$$R = a_o + a' \left(\rho g h_f\right)^\sigma \qquad \text{2-27a}$$

with a_o, and a' and σ constants for a given sample, and independent of pressure. σ is commonly known as the cake compressibility factor, and a' the cake constant. Several determinations of R for various pressures may be plotted on bi-logarithmic paper, the slope of the line of best fit revealing the cake compressibility factor. Generally a_o is neglected, simplifying Equation 2-27a to

$$R = a' \left(\rho g h_f\right)^\sigma \qquad \text{2-28}$$

Equation 2-28 proves dimensionally incorrect. It may be modified for dimensional correctness by writing

$$R = a' \left(\frac{\rho g h_f}{\Delta p_a = 1}\right)^\sigma \qquad \text{2-28a}$$

which therefore describes a' as the specific resistance $[T^2][M^{-1}]$ at a

pressure of unity $[F][L^{-2}]$. Equation 2-28a does not, however, differ numerically from Equation 2-28.

GRAVITY DEWATERING

Application of the theory and concept of specific resistance to gravity drainage of water treatment sludges on porous media appears most promising.[9] Assume a sludge is applied on a dewatering bed of sand or other porous material. Referring back to Equation 2-20d

$$\frac{dV}{dt} = \frac{\rho g A^2 h_f}{\mu (cRV + AL_f R_f')}$$

2-20d

the term dV/dt may be rewritten in terms of the head as the rate of drop of the sludge surface by drainage

$$\frac{dV}{dt} = -A \frac{dH}{dt}$$

with H = the head [L]

Also the terms referring to the supporting media may be ignored, as sludge is vastly higher in resistance than permeable soils.[8]

$$-A \frac{dH}{dt} = \frac{\rho g A^2 h_f}{\mu c R V}$$

But h_f is none other than H, assuming of course, that resistance in the filter is minimal. Hence, with cancellation of A

$$\frac{dH}{dt} = \frac{-\rho g A H}{\mu c R V}$$

2-27

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-27

-27a

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2-28

for

2-28a

This equation may be written for a cumulative drop from height H_o at $t=0$ upon substitution of $A(H_o-H)$ for V:

$$\frac{dH}{dt} = \frac{-\rho g A H}{\mu c A (H_o - H) R}$$

2-29

Noting that

$$R = a' (\rho g h_f)^\sigma$$

2-28

or here

$$R = a' (\rho g H)^\sigma$$

Equation 2-29 can be rewritten as

$$\frac{dH}{dt} = \frac{-\rho g H}{\mu c (H_o - H) a' (\rho g H)^\sigma}$$

reorganized to

$$dt = -a' \mu c (H_o - H)(\rho g H)^{\sigma-1} dH$$

2-30

This equation may be integrated from $H = H_o$ at $t = 0$:

$$\int_o^t dt = -\int_{H_o}^H a' \mu c (H_o - H)(\rho g H)^{\sigma-1} dH$$

$$= -a' \mu c (\rho g)^{\sigma-1} \left(\int H_o H^{\sigma-1} dH - \int_{H_o}^H H^\sigma dH \right)$$

which yields

$$t = -a'\mu c (\rho g)^{\sigma-1} \left[\frac{H_0 H^\sigma}{\sigma} - \frac{H^{\sigma+1}}{\sigma+1} \right]_{H_0}^{H}$$

$$= -a'\mu c (\rho g)^{\sigma-1} \left(\frac{H_0 H^\sigma - H_0^{\sigma+1}}{\sigma} + \frac{H_0^{\sigma+1} - H^{\sigma+1}}{\sigma+1} \right)$$

2-31

If the sludge is initially a dilute suspension, then Equation 2-26 is valid:

$$c = \frac{\rho g s_0}{100}$$

2-26

and thus Equation 2-31 may read

$$t = -\frac{a'\mu s_0}{100} (\rho g)^\sigma \left(\frac{H_0 H^\sigma - H_0^{\sigma+1}}{\sigma} + \frac{H_0^{\sigma+1} - H^{\sigma+1}}{\sigma+1} \right)$$

2-31a

Equation 2-31a allows determination of the drainage rate of a sludge on porous material, provided the initial sludge solids content is evenly distributed. Laboratory determinations of s_0, a', and σ are all that are necessary, and these are easily carried out. The equation has been experi-

mentally verified.[9]

Water treatment sludges are known frequently to settle out rapidly when
allowed to dewater. A modification of Equation 2-31a may be developed for the
case of a supernatant draining on a settled sludge layer. Another analysis
of great potential interest is the drainage of sludge on a previously
applied sludge. These analyses will shortly be tested in the laboratory.

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III. THEORY OF DRYING

The drying rate of a sludge varies with its physical state and the evaporation potential of the air. If supernatant is present, the drying rate would be expected to approximate that of a free water surface, modified by the greater heat absorption of the darker sludge natant and suppression characteristics. This suppression is caused primarily by dissolved substances.

The drying of a sludge solids surface would not be expected to differ substantially from the evaporation of supernatant, provided ample water were available at the evaporating surface to fulfill the evaporation potential of the air, which is determined by the moisture gradient in the air-vapor layer above the sludge. Such water would be that contained in the larger menisci of the sludge mass and surface film water. As drying continues, a moisture gradient in the sludge mass develops, and water is drawn from below to the surface predominantly by capillary forces. Work by Vater[1], Swanwick et. al.[2,3] and Lundesgaard[4] on wastewater sludges substantiates this conclusion for initial phases of drying. See Figure 3-1.

FIRST CRITICAL POINT

A drop in drying rate is noticed the instant moisture transport to the surface fails to fulfill the evaporation potential of the air. This is defined as the first critical point. Drying beyond this point occurs with ever-increasing difficulty as the evaporating menisci recede deeper into the sludge mass. A second critical point occurs when the majority of remaining water is of hygroscopic nature held by menisci $\leq 10^{-6}$ cm diameter, and by adsorption. Much of this water, including, of course, hydrate water, cannot be removed under atmospheric conditions, as its

37

38

EVAPORATION RATIO e (%)

	Type of Sludge	s₀(%)	Initial Depth (cm)	Average In- tensity of Evaporation	Research Area	Researcher
o	Digested	7.63	28.0	0.259	Laboratory	Lundesgaard
∇	Digested	7.63	28.0	1.33	Laboratory	Lundesgaard
Δ	Digested	10.52	28.0	1.06	Laboratory	Lundesgaard
□	Digested	10.52	28.0	1.32	Laboratory	Lundesgaard
	Digested	18.2	1.2	0.077	Laboratory	Vater
— — —	Raw	—	30.0	0.549	Outdoors	Swanwick (1960)
....	Digested	—	30.0	0.549	Outdoors	Swanwick (1961)
— · — ·	Raw	—	30.0	0.117	Outdoors	Swanwick (1963)

$$\text{Evaporation Ratio} = e = \frac{\text{Drying Rate of Sludge}}{\text{Evaporation Rate of Water}}$$
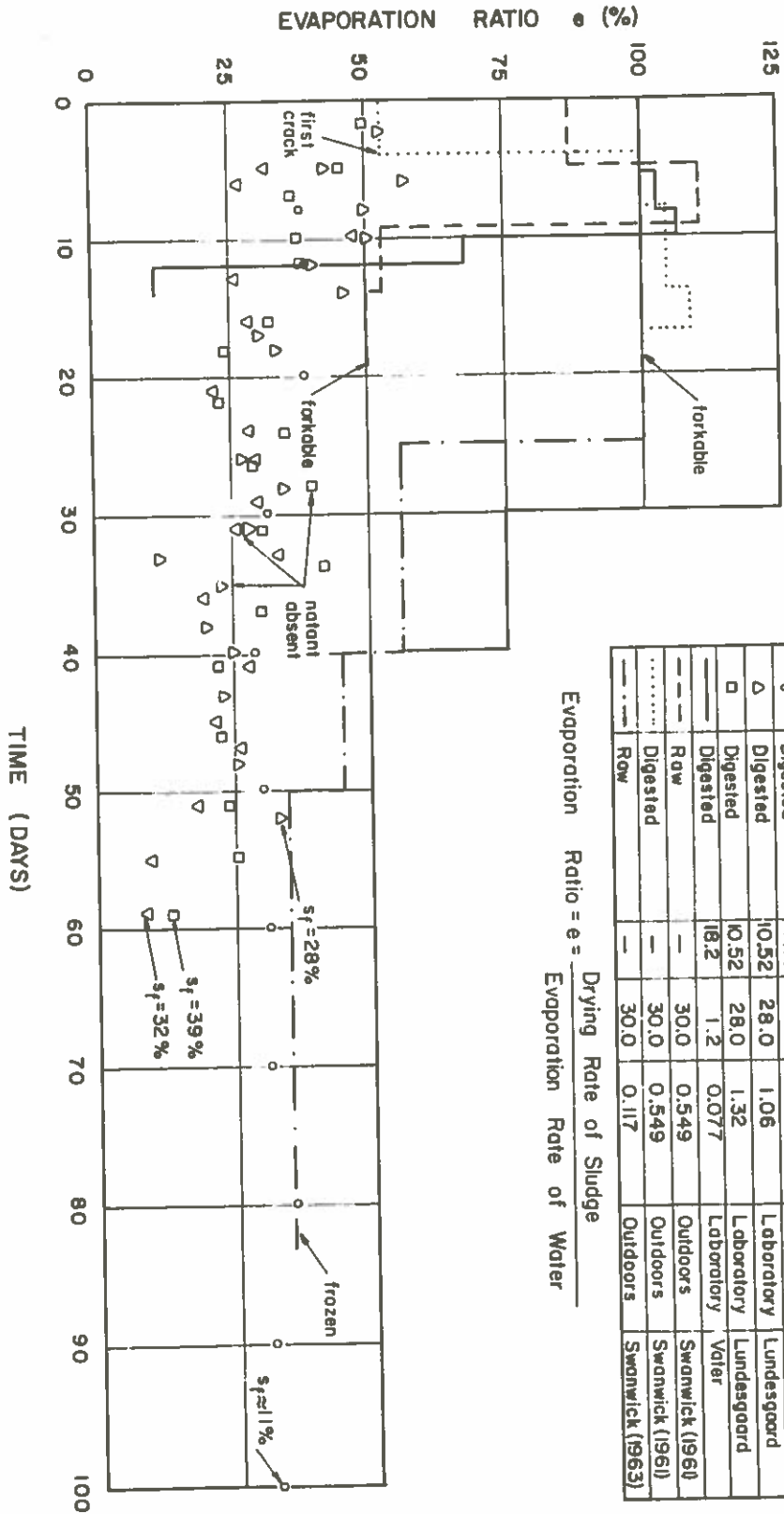
TIME (DAYS)

Figure 3-1. Drying rate of wastewater sludges as a ratio to the evaporation rate of a free water surface, plotted versus time. The beginning drying rates are approximately equal to the evaporation rate. The results of Lundesgaard represent an unusually high surface suppression.

degree of eventual removal is a function of vapor pressure and tempera-
ture.

Coackley and Allos[5], and Quon and Ward[6] have traced the drying rate
for several wastewater sludges (see Figure 3-2) under conditions of constant
evaporation potential. The drying rate or weight loss/time is plotted right
to left against water content (wt. of water/wt. of sludge.) The drying
rate occurring previous to the first critical point is known as the constant-
rate drying rate; that after is called the falling-rate drying rate.

The moisture content* at the first critical point, or the so-called
first critical moisture content, will differ for various samples possessing
different internal characteristics and subjected to different external
drying conditions.

In the case of two identical sludge samples, the first critical mois-
ture content will occur at a higher value for the sludge dried under a
more intense drying rate. Sludges with lower internal moisture transport
rates will also exhibit higher values for the critical point. The effect
of depth of the sludge sample is also important to the first critical mois-
ture content. At the instant of transition to falling rate drying, a pro-
nounced difference in moisture contents is found between the surface of the
sludge and the interior. The moisture content of the actual drying surface

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*Drying technology generally utilizes moisture content (U or u) determined
on a dry basis so as to utilize a constant denominator during a drying opera-
tion. Conversion of water content w (% wet basis) may be made by the formula
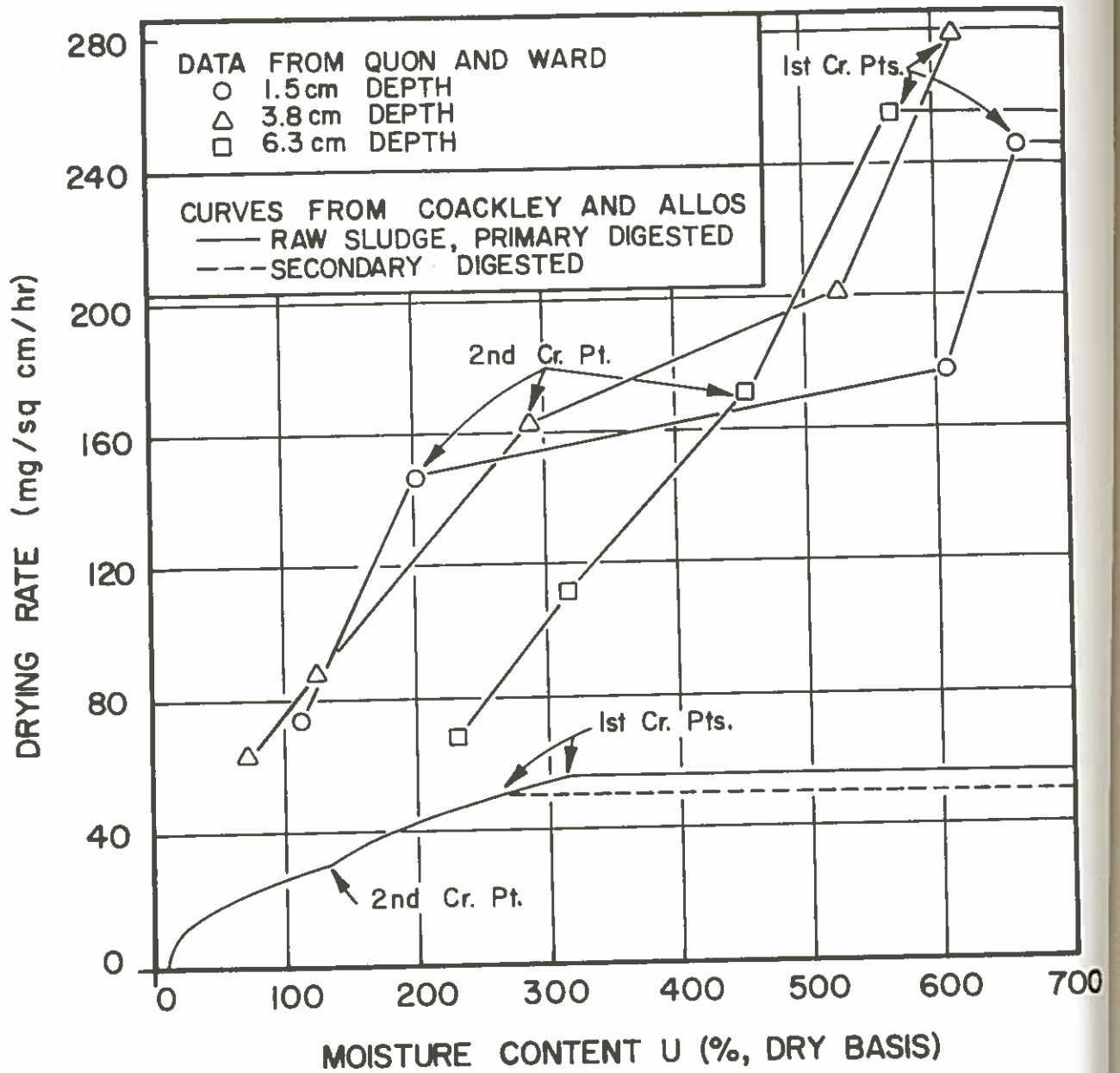$\frac{w}{1-0.01 \cdot w} = U$ or u (both %)

Figure 3-2. Drying rate I_s as a function of moisture content expressed on a dry basis. The samples of Coackley and Allos were approximately 1 cm deep. Curves for raw and secondary digested sludges merged at U = 270%.

at the instant of transition is independent of depth. However, the critical moisture content is an average value calculated from the entire volume of the sample. Thus, samples of identical sludges subjected to identical external drying conditions, but of different depths, will have different critical moisture contents: the greater the depth, the higher the first critical moisture content.

Krischer[7] and Lykow[8,9] have presented a theoretical equation relating the above factors to the first critical moisture content. The formulation is based on the diffusion equation

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{k} \cdot \frac{\partial u}{\partial t}$$

where
u = percent local moisture content

k = mass transfer coefficient $[L^2][T]$

t = time $[T]$

z = vertical distance $[L]$

During the constant rate drying period, the drying rate $I_{S,C}$ on the surface ($z = d$, with d equal to thickness of a flat sample drying on top) will equal the rate of moisture transport to the surface. Thus:

$$-\left(\frac{\partial u}{\partial z}\right)_{z=d} = \text{constant} = \frac{100 \cdot I_{S,C}}{k \, \gamma_o}$$

where γ_o is the weight of skeletal solids per unit initial volume $[F][L^{-3}]$. $I_{S,C}$ is in units of $[F][L^{-2}][T^{-1}]$. A second boundary condition is found at the bottom ($z = 0$):

$$-\left(\frac{\partial u}{\partial z}\right)_{z=0} = 0$$

Assuming at t = 0 that the sample has an even distribution of mois-
ture, the solution for moisture content at any point at time t is

$$u_1 = U_o + 100 \frac{I_{s,c} \cdot Z}{K \gamma_o} \left[\frac{1}{6} - \frac{1}{2} \frac{Z^2}{d^2} - \frac{Kt}{d^2} \right.$$

$$\left. + \frac{2}{\pi^2} \sum \frac{\cos n\pi}{n^2} \cos\left(\frac{n\pi Z}{d}\right) \exp\left(\frac{-n^2 \pi^2 Kt}{d^2}\right) \right.$$

3-1

with U_o as the initial moisture content; u_1 the local moisture content.

The exponential term in Equation 3-1 rapidly approaches zero yielding

$$u_1 = U_o + 100 \frac{I_{s,c} \cdot d}{K \gamma_o} \left(\frac{1}{6} - \frac{1}{2} \frac{Z^2}{d^2} - \frac{Kt}{d^2} \right)$$

3-2

The average moisture content U in the sample at time t is calculated by
integration of Equation 3-2

$$U = \frac{1}{d} \int_o^d u_1 dz = U_o - 100 \frac{I_{s,c} \cdot t}{d \gamma_o}$$

3-3

Assuming that the first critical moisture content occurs at the moment
the surface presents only hygroscopic moisture to evaporation, Equation 3-2
may be used to calculate the drying time available until the first critical
point is reached. This is done by substitution of z = d and $u_1 = u_h$ =
hygroscopic moisture content.

This yields

$$u_h = U_o - 100 \frac{I_{s,c} \cdot d}{k \gamma_o} \left(\frac{1}{3} + \frac{kt}{d^2} \right)$$

3-4

Solving Equation 3-4 for t, and substituting into Equation 3-3:

$$U = u_h + \frac{100}{3} \frac{I_{s,c} \cdot d}{k \gamma_o} = U_{CR}$$

3-5

where U_{CR} is the first critical moisture content.

Equation 3-5 holds strictly for only thin samples of moist solid or plastic materials whose depth d remains constant during the entire drying period. Further, isothermal conditions are assumed.

FALLING RATE DRYING

The falling rate drying curve is often found to possess two distinct drying periods separated by a second critical point. In the case of sludge drying as shown in Figure 3-2, this second point (at a water content of about 50%) may represent the complete absence of non-hygroscopic moisture. The equilibrium moisture content (drying rate = 0) represents hygroscopic moisture which may not be removed under the given drying temperature and humidity.

The second critical moisture content does not appear to be of great significance in sludge drying. This is inferred by Figure 3-2, for the entire falling rate curve can be well approximated by a straight line. Such a straight line calculated statistically will intercept the constant rate line at a point near to the first critical point. This intercept is technically known as the reduced critical point[8] and is designated by

the reduced critical moisture content, with values approximately those predicted by the previously described formulae. This permits the instantaneous falling rate drying intensity $I_{S,F}$ at moisture content U to be expressed as

$$I_{S,F} = I_{s,c} \cdot \frac{U - U_P}{U_{CR} - U_P}$$
3-6

where U_p is the equilibrium moisture content.

The equilibrium moisture contents are generally negligible in comparison to the values for the first (or reduced) critical points. As a result, the straight line falling rate drying curves may be drawn through the origin.

If the assumption is made that Υ_o, the weight of skeletal solids per initial volume may be written

$$\Upsilon_o = \frac{W_{TS}}{A\, d}$$
3-7

then

$$d = \frac{W_{TS}}{\Upsilon_o A}$$
3-8

with W_{TS}/A the weight of total solids per unit surface area $[F][L^{-2}]$. Substitution of Equation 3-8 into Equation 3-5

$$U_{CR} = u_h + \frac{100}{3} \frac{I_{s,c} \cdot d}{K\, \Upsilon_o}$$
3-5

yields

$$U_{CR} = u_h + \frac{100}{3} \frac{I_{s,c}(W_{TS}/A)}{K\, \Upsilon_o^2}$$
3-9

The term u_h, or hygroscopic moisture content, is temperature dependent, and the moisture transfer coefficient varies with temperature, moisture content and depth. Both u_h and k are characteristics of the sludge, as is γ_o. Here the assumption is made that these factors vary little for a given sludge type under natural drying conditions, thus justifying expression of Equation 3-9 as

$$ U_{CR} = f\left(I_{S,c} \cdot W_{TS} / A \right) \qquad \text{3-10} $$

Krischer[3] indicated U_{CR} for capillary-colloidal materials such as water treatment sludge to be a function of $(I_{S,C} \cdot d)^n$, with n < 1. A similar exponent may thus be added to Equation 3-10. The constant rate drying intensity $I_{S,C}$ of Equation 3-10 is obviously not a constant by natural drying. In order to equate the reduced critical moisture contents, however, $I_{S,C}$ is here assumed constant.

DRYING DURATION

The rate of weight loss by drying may be expressed as

$$ -\frac{dW_w}{dt} = A\, I_s \qquad \text{3-11} $$

where
W_w = weight of water in sludge [F]

A = surface or drying area [L^2]

I_s = drying intensity [F][M^{-2}][T^{-1}]

The expression for moisture content

$$ U = 100\, \frac{W_w}{W_{TS}} \qquad \text{3-12} $$

may be rearranged and substituted into Equation 3-11 yielding

$$\frac{-dW_w}{dt} = \frac{-W_{TS}}{100}\frac{dU}{dt} = AI_s$$

3-13

which may be written as

$$dt = \frac{-W_{TS}}{100\,AI_s} \cdot dU$$

3-14

The time necessary to dry the material from the original moisture content U_o to any moisture content U_t is found by integration of Equation 3-14:

$$t = \int_0^t dt = \int_{U_o}^{U_t} \frac{-W_{TS}}{100\,A\,I_s}\,dU$$

3-15

If drying occurs solely in the constant rate period then $I_S = I_{S,C}$. Assuming both $I_{S,C}$ and W_{TS} are constants, Equation 3-15 solves as

$$t = \frac{-W_{TS}}{100\,A\,I_{s,c}} \quad U\Big]_{U_o}^{U_t}$$

$$= \frac{W_{TS}}{100\,A\,I_{s,c}}\,(U_o - U_t) \quad (for\ U_t \geq U_{cR})$$

3-16

Equation 3-6 expressed drying intensity in the falling rate period as

$$I_{s,F} = I_{s,c}\,\frac{U - U_p}{U_{cR} - U_p}$$

3-6

If it is assumed[10] that $U_p \ll U_{CR}$, thus

$$I_{s,F} = I_{s,c}\,\frac{U}{U_{cR}}$$

3-17

Substitution of Equation 3-16 into Equation 3-15 for $I_{S,F} = I_S$ expresses drying time for a solid drying solely in the falling rate period.

$$t = \int_{U_o}^{U_t} \frac{-W_{TS}}{100 A I_S} dU = \frac{-W_{TS} U_{CR}}{100 A I_{S,C}} \int_{U_o}^{U_t} \frac{dU}{U}$$

$$= \frac{-W_{TS} U_{CR}}{100 A I_{S,C}} \left. \ln U \right]_{U_o}^{U_t}$$

$$= \frac{W_{TS} U_{CR}}{100 A I_{S,C}} \ln \left(U_o / U_t \right) \quad (for \ U_o \leq U_{CR}) \quad 3\text{-}18$$

A sludge sample drying in both constant and falling rate periods will have a total drying duration of

$$t = \frac{-W_{TS}}{100 A I_{S,C}} \left(\int_{U_o}^{U_{CR}} dU + \int_{U_{CR}}^{U_t} \frac{U_{CR}}{U} dU \right)$$

$$= \frac{-W_{TS}}{100 A I_{S,C}} \left(\left. U \right]_{U_o}^{U_{CR}} + U_{CR} \left. \ln U \right]_{U_{CR}}^{U_t} \right)$$

$$= \frac{W_{TS}}{100 A I_{S,C}} \left[U_o - U_{CR} + U_{CR} \ln \left(U_{CR} / U_t \right) \right]$$

$$3\text{-}19$$

$$\left(for \ U_o \geq U_{CR} \geq U_t \right)$$

Equations 3-17, 3-18, and 3-19 have been used in conjunction with Equation 3-10 to determine the rate of drying in the absence of filtration of wastewater sludges.[11] There is every indication that similar application can be made with water treatment and advanced wastewater treatment sludges. The complexities of the equations effectively describing drying and filtration simultaneously are now undergoing analysis, with experimental verification and development of applications to engineering design and operation to be stressed.

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IV. APPLICATION

As is increasingly recognized, present day design and operation procedures for non-mechanical dewatering and drying of water treatment sludge leaves much to be desired. Included in the void of knowledge surrounding these sludges are such items as their determinable characteristics which relate to their behavior during dewatering and drying.

The behavior of wastewater sludges on drying beds and in lagoons has been more widely researched than water treatment sludges, and it is here that much that is relevant to water treatment sludges is found.

RELATIONSHIP OF BLUNK

Figure 4-1 represents the classic formula of Blunk determined by many analyses at the Emschergenossenschaft.[1] Here the required drying bed area to dewater 1 m^3 per year of Imhoff sludge is shown to be linearly related to the initial solids content s_o (%). This empirical relation as the result of extensive measurements is

$$A' = 0.8 - 0.02\, s_o$$

4-1

where A' = area required to dewater 1 m^3/year

It is preferred to use

$$\frac{A}{V} = 0.8 - 0.02\, s_o$$

4-1.a

with A = area used during the year $[m^2]$
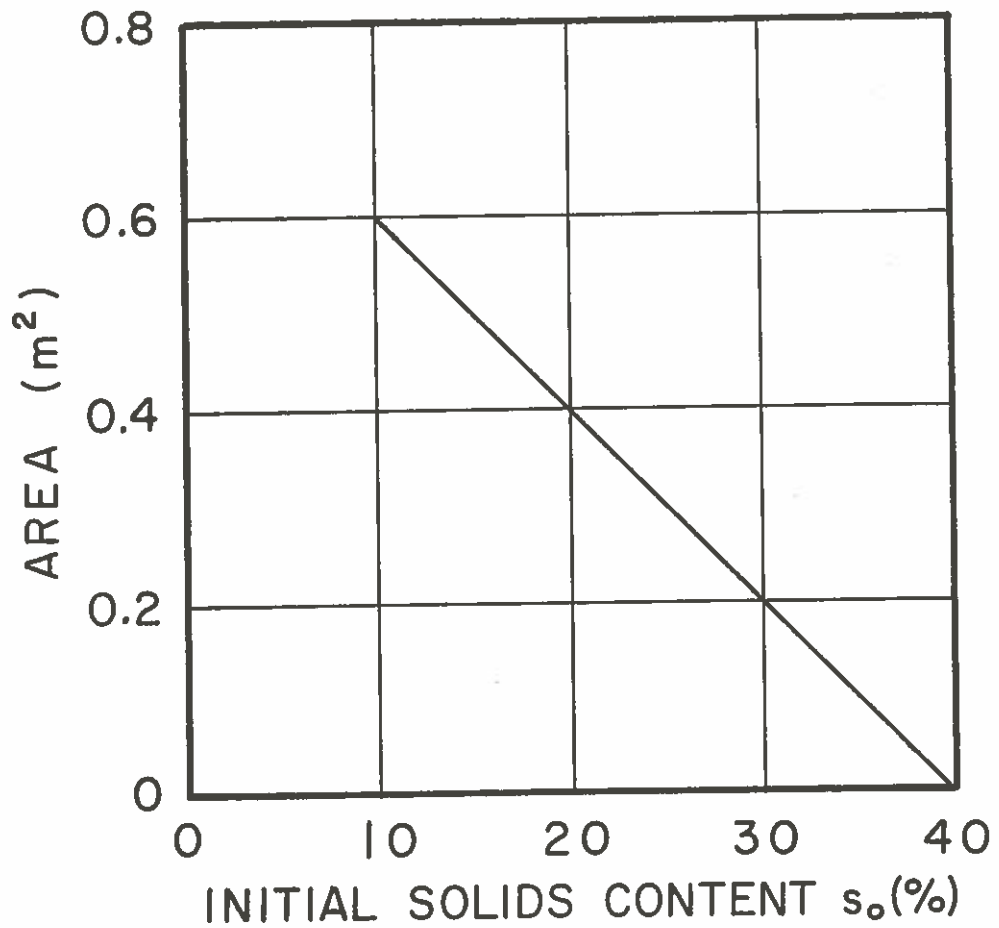
V = yearly sludge volume $[m^3]$

50

Figure 4-1. Area of drying bed required for 1 m^3 of Imhoff sludge per year as a function of initial solids content. Taken from Blunk[1], this linear relationship represents several years of data at the Emschergenossenschaft.

Blunk mentions about 8 months effective use of the drying beds every year. Thus the daily rate of sludge application on the bed area is

$$\frac{V}{365(8/12)}$$

A cubic meter of sludge weighs about 1000 kilograms, allowing the daily sludge load applied on the beds to be written as

$$\frac{W_{TOT}}{t} \simeq \frac{V/1000}{365(8/12)}$$

$$V = \frac{1000\,W_{TOT}/t}{365(8/12)}$$

Referring back to Equation 4-1a, one may write

$$\frac{A}{V} = 0.8 - 0.02\,S_o \qquad\qquad 4\text{-}1a$$

As

$$\frac{A}{\dfrac{1000\,W_{TOT}/t}{365(8/12)}} = 0.8 - 0.02\,S_o$$

yielding ($t = t_f$)

$$4\text{-}2$$

$$t_f = \frac{243\,(0.8 - 0.02\,S_o)\,W_{TOT}}{1000\,A}$$

Recognizing that

$$\frac{W_{TS}}{W_{TOT}} \cdot 100 = S_o$$

with W_{TS} = weight of solids   [kg]

Equation 4-2 may be expressed as

$$t_f = \frac{(19.5 - 0.48 S_o) W_{TS}}{S_o A} \qquad \text{4-2a}$$

RELATIONSHIPS OF HASELTINE

Haseltine[2] derived from much field data an empirical relationship similar to that of Blunk:

$$Y = 0.157 S_o - 0.286 \qquad \text{4-3}$$

Y is called the gross bed loading or solids dewatered per unit area per day; and is in units of $[kg] [m^{-2}] [day^{-1}]$. S_o is in percent.

Equation 4-3 is widely used for the dimensioning of sludge drying beds. It may be rewritten

$$Y = \frac{W_{TS}}{A \cdot t_f} = 0.157 S_o - 0.286$$

Hence,

$$t_f = \frac{6.36 \, W_{TS}}{A \, (S_o - 1.82)} \qquad \text{4-3a}$$

Haseltine developed an additional empirical formula between a so-called "net" bed loading and initial solids content[2]. The
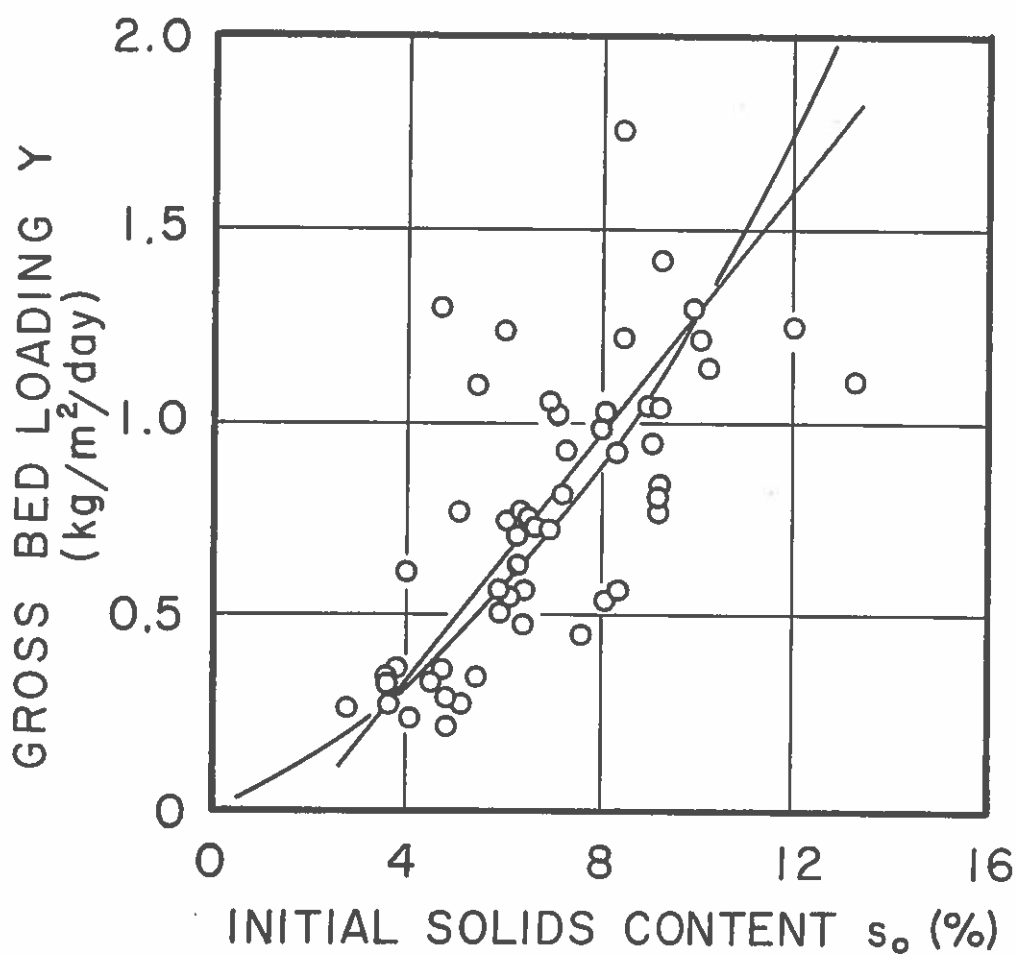
Figure 4-2. Loading rate of wastewater sludge at various treatment plants
in the United States. Shown as a function of the initial solids
content, the points and straight line represent the results from
Haseltine[2]. The curve is from Vater[3].

net bed loading is the gross bed loading times the water content of the dried sludge. The water content w (%) is of course the difference between 100% and the solids content

$$w = 100 - s$$

The net bed loading according to Haseltine is

$$Z = 0.057 \, S_o - 0.082 \qquad \text{4-4}$$

with Z = net bed loading $[kg] \, [m^{-2}] \, [day^{-1}]$

$$Z = \frac{Y \cdot w_f}{100}$$

$$= \frac{Y(100 - S_f)}{100}$$

$$= \frac{W_{TS}}{A \, t_f} \left(\frac{100 - S_f}{100} \right)$$

The solution of the equation for time is

$$t_f = \frac{W_{TS}(17.5 - 0.175 \, S_f)}{A \quad (S_o - 1.44)} \qquad \text{4-4a}$$

RELATIONSHIP OF VATER

Vater[3] reanalyzed the data from Haseltine to develop, again empirically, the equation

$$Y = \frac{W_{TS}}{A \cdot t_f} = 0.033 \cdot S_o^{1.6} \qquad \text{4-5}$$

This may be rewritten

$$t_f = \frac{30 \, W_{TS}}{A \cdot S_o^{1.6}} \qquad \text{4-5a}$$

DRAINAGE RELATIONSHIP OF SWANWICK

Swanwick[4] postulated that the bed loading is a function of specific resistance. His equation from Figure 4-3 approximates

$$Y' = \frac{10^7}{R^{1/2}} \qquad \text{4-6}$$

with Y' = bed loading as solids applied per unit area per year

$$[kg] \ [m^{-2}] \ [year^{-1}]$$

R = specific resistance at 36.9 cm. of mercury $[sec^2] \ [gm^{-1}]$

However Y' is W_{TS}/A per year. The daily loading rate is Y' divided by the days of dewatering during the year. Assume 300 days for England, then

$$Y = \left(\frac{10^7}{R^{1/2}}\right) \frac{1}{300}$$
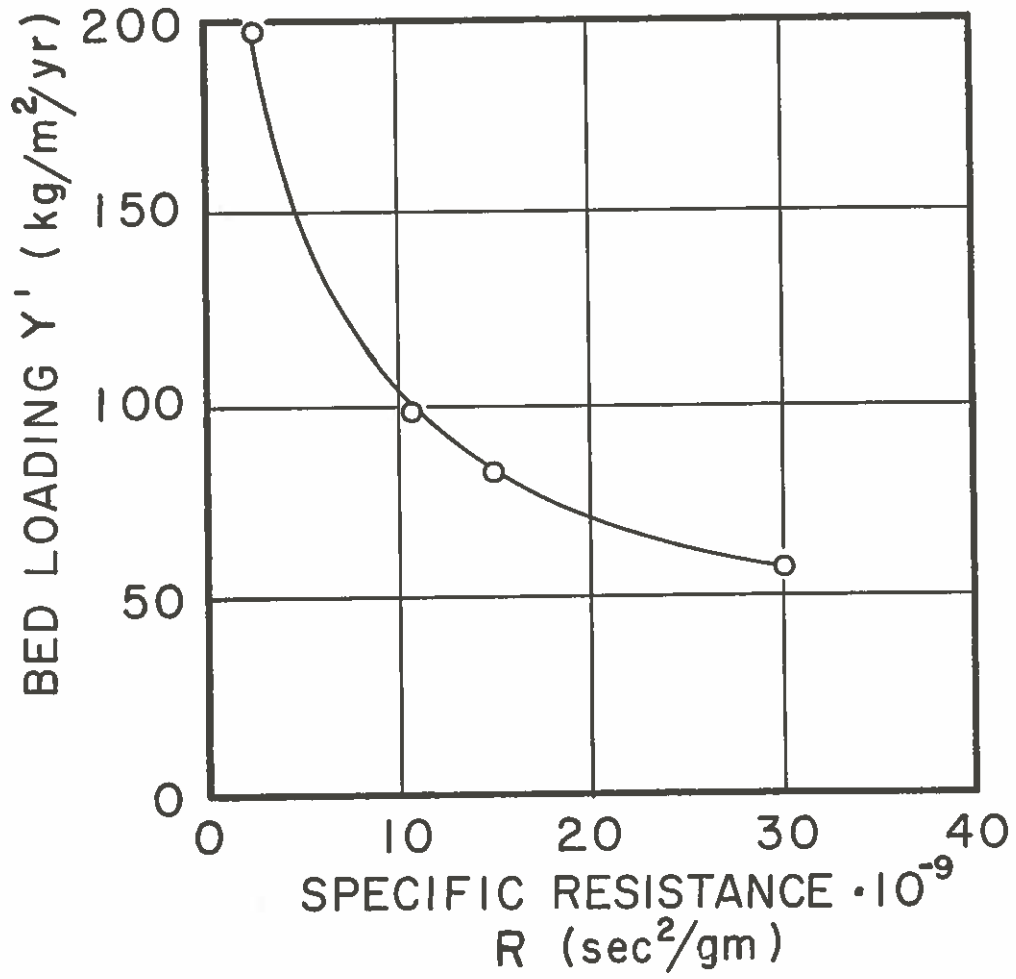
$$= \frac{W_{TS}}{A \cdot t_f}$$

Figure 4-3. The Swanwick relationship of bed loading to specific resistance.[4] Each point represents the mean of a group of loading values.

with t_f in days. It follows that

$$t_f = \frac{3 \cdot W_{TS}\, R^{1/2} \cdot 10^{-5}}{A}$$

4-6a

Unfortunately the coefficient of compressibility of the sludge was not given, thereby allowing Equation 4-6a to be expressed in terms of the specific resistance at unity pressure.

EXPERIMENTS BY VOGLER AND RUDOLFS

Vogler and Rudolfs determined the effect of initial solids content on sand bed dewatering of paper mill white water sludge. Figure 4-4 represents their data and the linear relationship they presented. The high drainage rates of the sludge are apparent from the figure. Expressing t_f in days will yield a relationship

$$t_f = 0.037\, S_o$$

4-7

Also Vogler and Rudolfs plotted a relationship (Figure 4-5) of drainage time versus initial depth for white water sludge. Their relationship is expressed as

$$t_f = 45 \cdot 10^{-5}\, H_o^{1.6}$$

4-8

However

$$H_o = \frac{V}{A \cdot 10}$$
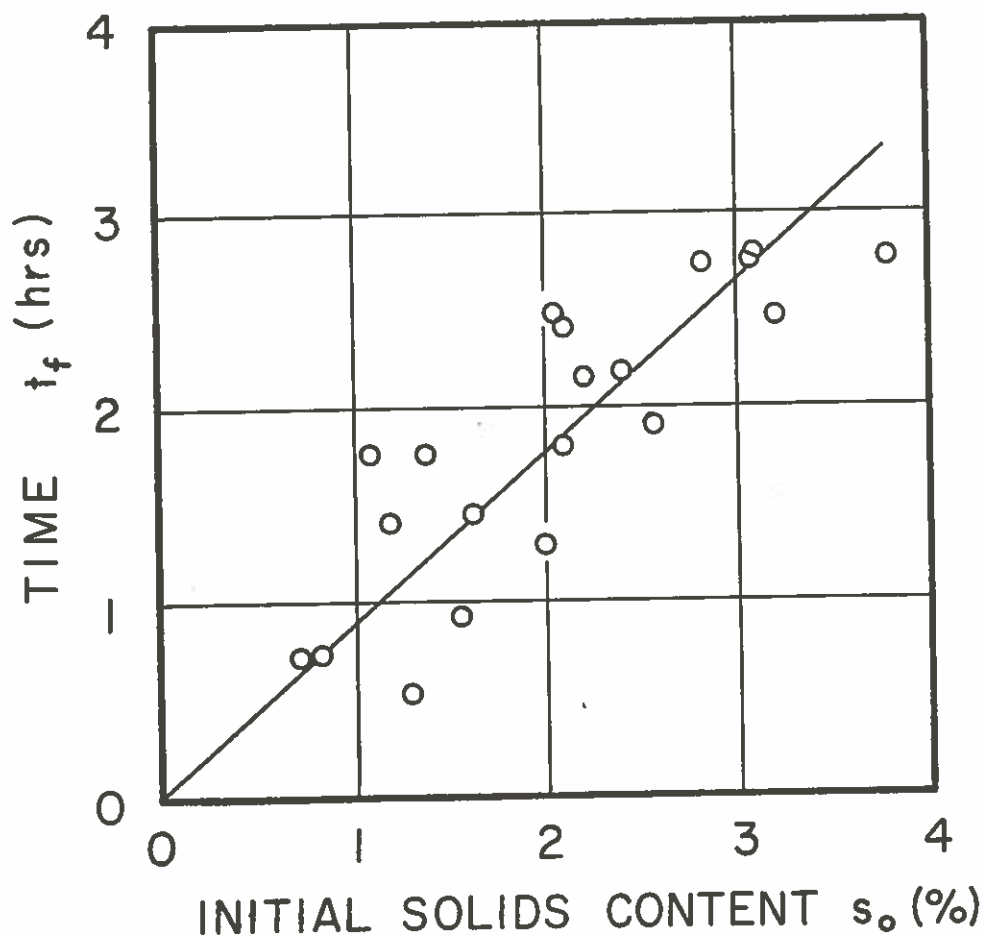
Figure 4-4. Time of drainage of paper mill white water sludge as a
function of the initial solids content. The data and line
are from Vogler and Rudolfs[5].
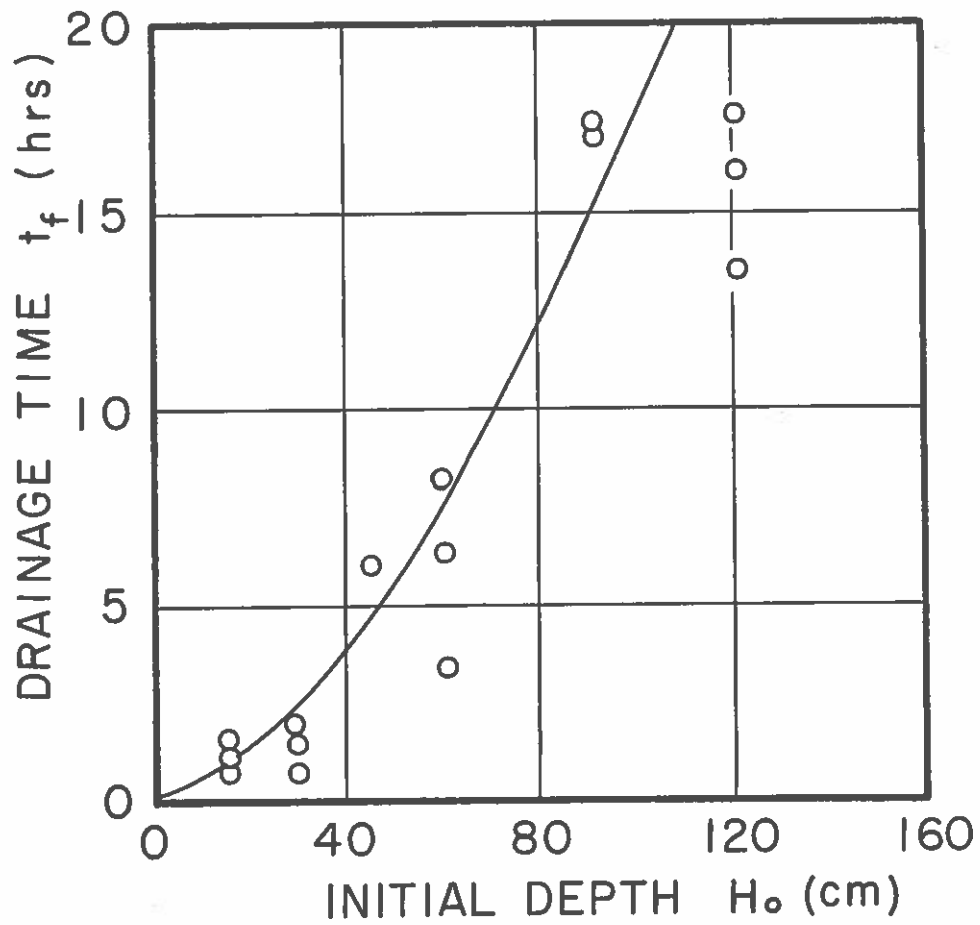
Figure 4-5. Time of drainage of paper mill white water sludge as a function of the initial filling depth. The data points and curve are from Vogler and Rudolfs.[5]

if $\qquad$ A = area $[m^2]$

$\qquad$ V = volume of sludge applied $[1]$

$\qquad$ H_0 = initial depth $[cm]$

V in liters equals W_{TOT} in kilograms. Hence

$$H_o = \frac{W_{TOT}}{A \cdot 10}$$

But

$$W_{TOT} = \frac{W_{TS}}{S_o} \ 100$$

Therefore

$$H_o = \frac{W_{TS} \cdot 10}{S_o A}$$

As a result Equation 4-8 can be expressed

$$t_f = 45 \cdot 10^{-5} \left(\frac{W_{TS} \cdot 10}{S_o A} \right)^{1.6}$$

$$t_f = \left(\frac{0.081 \ W_{TS}}{S_o A} \right)^{1.6} \qquad\qquad 4\text{-}8a$$

DRYING RELATIONSHIP OF SWANWICK

Swanwick[6] suggests a method of calculation of drying time for wastewater sludge whereby an assumption of very rapid drainage is made. After this initial drainage, all weight loss is assumed to occur from evaporation. Swanwick believes the rate of drying averages 75% of the evaporation rate of a free water surface. Further he claims 53% of the rainfall is absorbed by the sludge and must be evaporated. An accumulated mass balance of 75% of the evaporation minus 53% of the rainfall (determined from average meteorological records for the period of observation) allows calculation of the necessary drying time.

DRYING FORMULAE FROM EQUATION

Nebiker and Munz[7] have utilized Equations 3-17, 3-18, and 3-19 to determine drying time for wastewater sludge from various treatment plants in Switzerland. An assumption of eventual drainage to s_f=20% was made; and 25% of rainfall was assumed to be absorbed. Meteorological data were averaged and used applying a 105% drying/evaporation ratio during constant rate drying (as defined in Reference 8 and 9). An equation for critical moisture content

$$U_{CR} = 500 \left(I_{S,K} \cdot \frac{W_{TS}}{A} \right)^{1/2} \qquad 4\text{-}9$$

was used in conjunction with Equations 3-17, 3-18, and 3-19 to calculate total drying durations and bed loads.

SKINNER FORMULATION

Skinner[10] a priori suggested a formulation

$$A = \frac{KNS}{mTv} \qquad \qquad \text{4-10}$$

with K = coefficient dependent on treatment process

N = average yearly precipitation

S = settleable solids in sewage

m = number of months in year available for drying

T = mean yearly temperature

v = mean yearly wind velocity

DRAINAGE FORMULA FROM EQUATION 2-31a

Equation 2-31a may be simplified as follows

$$t = \frac{a' \mu s_o}{100}(\rho g)^\sigma \left(\frac{H_o^{\sigma+1} - H_o H^\sigma}{\sigma} + \frac{H^{\sigma+1} - H_o^{\sigma+1}}{\sigma+1} \right)$$

<div align="right">2-31a</div>

$$= \frac{a' \mu s_o (\rho g)^\sigma}{100\sigma(\sigma+1)} \left[(\sigma+1)(H_o^{\sigma+1} - H_o H^\sigma) + \sigma H^{\sigma+1} - \sigma H_o^{\sigma+1} \right]$$

$$= \frac{a' \mu s_o (\rho g)^\sigma}{100\sigma(\sigma+1)} \left(\sigma H^{\sigma+1} + H_o^{\sigma+1} - \sigma H_o H^\sigma - H_o H^\sigma \right)$$

Note that for a dewatering bed

$$H = \frac{V}{A}$$

with V = volume of sludge

One may write

$$W_{TOT} = \gamma V$$

then

$$H = \frac{W_{TOT}}{\gamma A}$$

As previously

$$W_{TOT} = \frac{W_{TS}}{S} \cdot 100$$

Designate initial conditions with subscript o and final conditions with subscript f. It follows that

$$H_o = \frac{W_{TS_o} \cdot 100}{S_o \gamma_o A_o}$$

$$H_f = \frac{W_{TS_f} \cdot 100}{S_f \gamma_f A_f}$$

The area is obviously a constant, and if the assumption is made that few solids pass into the filter bed, it may be written that

$$W_{TS_o} = W_{TS_f} = W_{TS}$$

(constant for given loading). Hence

$$H_o = \frac{W_{TS} \cdot 100}{S_o \gamma_o A}$$

$$H_f = \frac{W_{TS} \cdot 100}{S_f \gamma_f A}$$

If these values for H_o and $H = H_f$ are inserted into Equation 2-31a for $t = t_f$

$$t_f = \frac{a' \mu S_o (\rho g)^{\sigma}}{100 \sigma (\sigma+1)} \left(\sigma H_f^{\sigma+1} + H_o^{\sigma+1} - \sigma H_o H_f^{\sigma} - H_o H_f^{\sigma} \right)$$
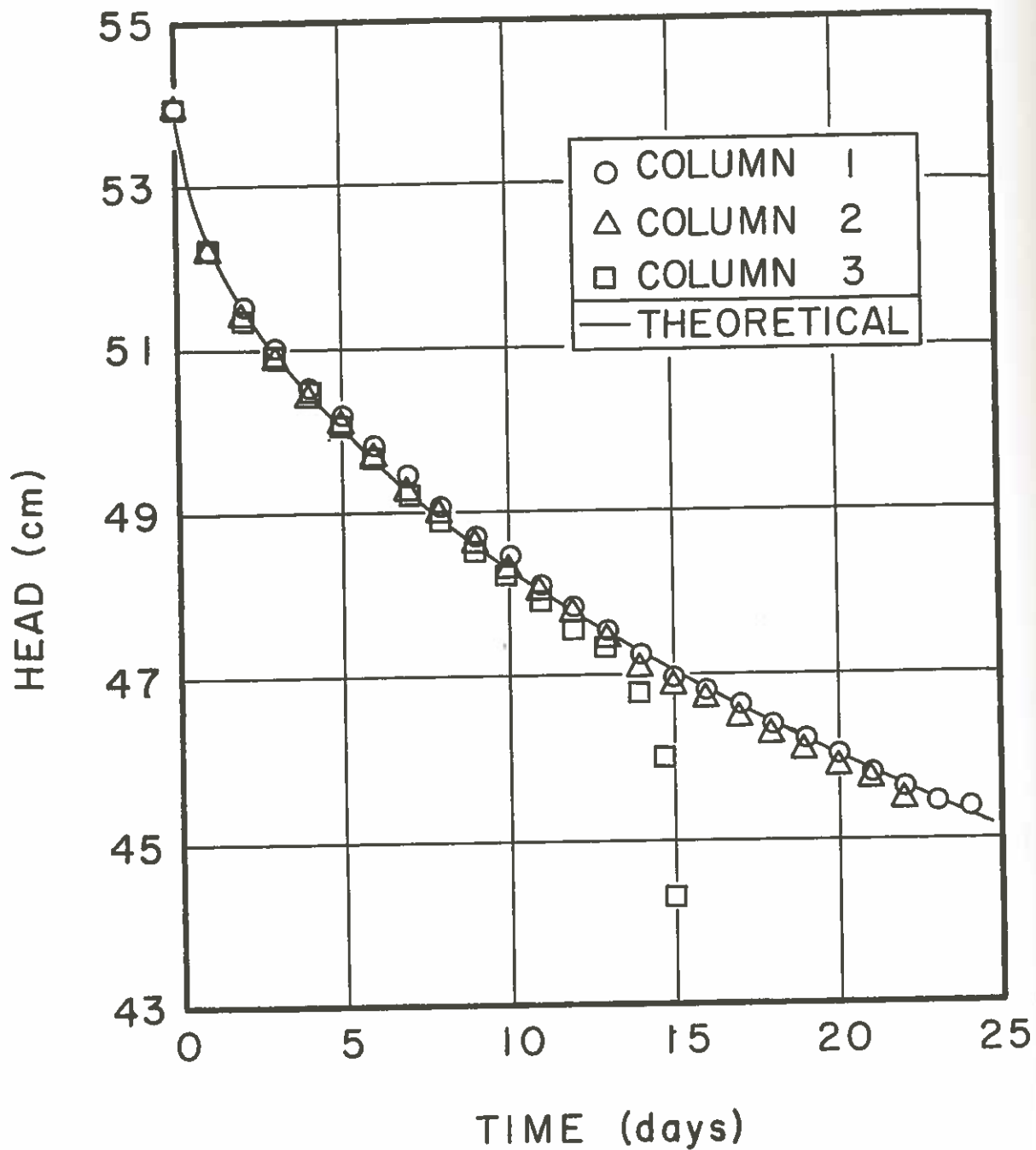
Figure 4-6. Drainage of a wastewater sludge as a function of time.
The theoretical curve refers to Equation 3-31a from
Nebiker, Sanders, and Adrian.[11]

66

$$= \frac{a' \mu s_o (\rho g)^\sigma}{100\,\sigma(\sigma+1)} \left[\sigma \left(\frac{W_{TS} \cdot 100}{s_f \gamma_f A} \right)^{\sigma+1} + \left(\frac{W_{TS} \cdot 100}{s_o \gamma_o A} \right)^{\sigma+1} \right.$$

$$\left. - \sigma \frac{W_{TS} \cdot 100}{s_o \gamma_o A} \left(\frac{W_{TS} \cdot 100}{s_f \gamma_f A} \right)^\sigma - \left(\frac{W_{TS} \cdot 100}{s_o \gamma_o A} \right)\left(\frac{W_{TS} 100}{s_f \gamma_f A} \right) \right]$$

which reduces to

$$t_f = \frac{a' \mu s_o (\rho g)^\sigma}{100\,\sigma(\sigma+1)} \left(\frac{W_{TS} \cdot 100}{A} \right)^{\sigma+1} \left[\frac{\sigma}{(s_f \gamma_f)^{\sigma+1}} \right. \qquad \text{4-11}$$

$$\left. + \frac{1}{(s_o \gamma_o)^{\sigma+1}} - \frac{\sigma}{(s_o \gamma_o)(s_f \gamma_f)^\sigma} - \frac{1}{(s_o \gamma_o)(s_f \gamma_f)} \right]$$

A typical example of the verification of this formula is shown in Figure 4-6 for a wastewater sludge.[11]

COMPARISON OF FORMULAE AND RELATIONSHIPS

Reference to Figure 4-7 indicates the general effect of various water reduction processes on the weight and solids of a wastewater sludge. The process steps include concentration (thickening), dewatering (filtration), and (thermal) drying. Water treatment sludges are, of course, more dilute, and usually can not be thickened beyond 5% solids. The importance of Figure 4-7 is that dewatering of a thickened sludge accounts for 1/2 to 2/3 of the total volume and weight reduction that will occur even if the sludge is left to air dry. If the sludge is not fully dried when removed, the relative effect of dewatering is even greater.
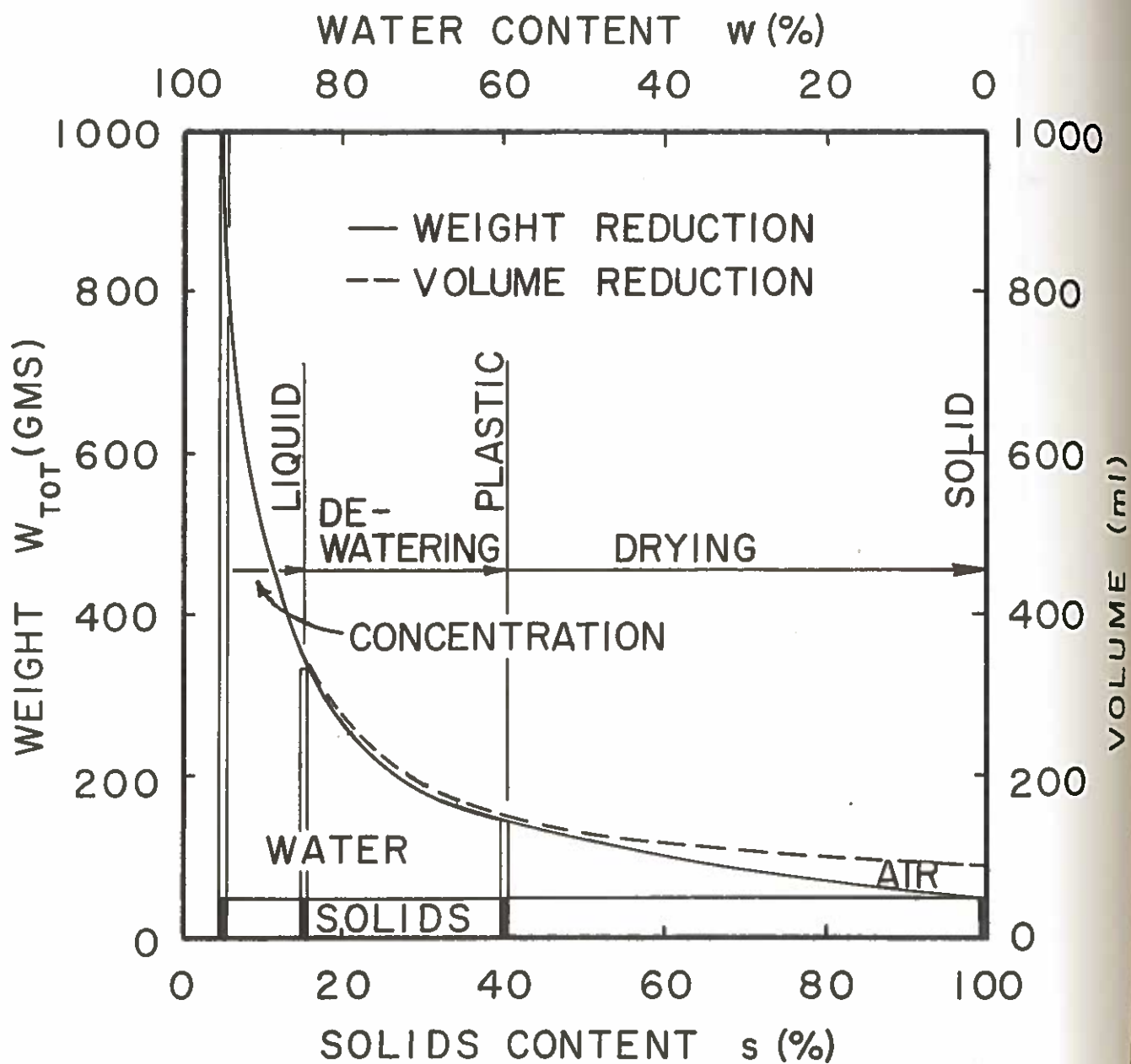
Figure 4-7. Weight and volume reduction of 1 kg of wastewater sludge
at 5% initial solids content. The water reduction processes
indicated are generally effective over the ranges of solids
contents shown.

In a comparison then, of the formulae presented here, it can be seen that only those referring to dewatering by drainage effectively relate to the bed loading relationships. A list summarizing these comparable formulae and relationships follows.

$$t_f = \frac{W_{TS}}{A} \frac{(19.5 - 0.48\,S_o)}{S_o} \qquad \text{4-2a}$$

$$t_f = \frac{W_{TS}}{A} \frac{6.36}{(S_o - 1.82)} \qquad \text{4-3a}$$

$$t_f = \frac{W_{TS}(17.5 - 0.175\,s_f)}{A\ (S_o - 1.44)} \qquad \text{4-4a}$$

$$t_f = \frac{W_{TS}}{A} \frac{30}{S_o^{1.6}} \qquad \text{4-5a}$$

$$t_f = \frac{W_{TS}}{A} \cdot 3 \cdot 10^{-5} R^{1/2} \qquad \text{4-6a}$$

$$t_f = 0.037\,S_o \qquad \text{4-7}$$

$$t_f = \left(\frac{W_{TS}}{A} \frac{0.081}{S_o}\right)^{1.6}$$

$$t_f = \left(\frac{W_{TS}}{A}\right)^{\sigma+1} \frac{s_o\, a'\, \mu}{\sigma(\sigma+1)} (100\,\rho g)^{\sigma} \left[\frac{\sigma}{(s_f \gamma_f)^{\sigma+1}} \right.$$

$$\left. + \frac{1}{(s_o \gamma_o)^{\sigma+1}} - \frac{\sigma+1}{(s_o \gamma_o)(s_f \gamma_f)^{\sigma}} \right] \qquad \text{4-8a}$$

VOLUME (ml)

A comparison of the above proves most interesting. First, however, Equations 4-7 and 4-8a are not numerically comparable with the others, as they apply to white-water sludge, and not digested wastewater sludges. Also, with the exception of Equations 4-6a and 4-11, all are based on widely scattered data from whence arbitrary empirical relationships were established. Both values for coefficients and exponents are tenuous. Nonetheless the following generalization is apparent

$$t_f \propto \left(\frac{W_{TS}}{A}\right)^n$$

with $n \geq 1$. The value of σ in Equation 4-11 appears in the order of 0.8, making the n value of Equation 4-8a and 4-11 very similar. Further, one notes that the dewatering time generally varies approximately as the reciprocal of s_0 . This is not directly seen in Equation 4-11 but closer scrutiny reveals that

t_f varies with $$s_o \left(\frac{1}{(s_o \gamma_o)^{\sigma+1}} - \frac{\sigma+1}{(s_o \gamma_o)(s_f \gamma_f)^{\sigma}}\right)$$

which in turn varies as $s_0^{-\sigma}$. Again a good correlation exists.

The formulation from Swanwick (Equation 4-6a) suggests $t_f \propto R^{1/2}$ Equation 4-11 states $t_f \propto a'$ with a' the specific resistance at unity pressure. Although the exponents differ, there is a general agreement as to sign and magnitude.

Haseltine's formulation (Equation 4-4a) does not yield zero if s_f is set at s_o, an obvious deficiency. Equation 4-11 will, however, again attesting to its rational basis. Also, if s_o is set at s_f, t_f will equal zero. Equation 4-4a will not. The terms involving s_f in Equation 4-11 are

$$\frac{\sigma}{(S_f \gamma_f)^{\sigma+1}} - \frac{\sigma+1}{(S_o \gamma_o)(S_f \gamma_f)^{\sigma}}$$

which are clearly a net negative number since $s_o > s_f$. Hence an increase in s_f (a more thorough dewatering) yields an increase in dewatering time.

Equation 4-11 is seen to be dimensionally correct, theoretically sound, experimentally proven, and -- by comparison to existing formulae -- consistent with actual field results. The emphasis here on the justification of Equation 4-11 should not detract from its vast significance. Note the following:

1) For the first time laboratory tests can determine dewatering rates on drying beds in the same manner that vacuum filter performance can be predicted, thus allowing a direct comparison of the two.

2) The effect of sludge conditioners on drying bed dewatering rates can be determined by ordinary specific resistance tests. Large amounts of existing conditioner data may now be re-evaluated for drying beds, as most conditioning data were obtained in terms of specific resistance.

3) As an approximation, Equation 4-11a may be used to determine bed loadings, resulting in possibly significant changes in design and operation.

4) Prediction of dewatering in lagoons and on wedge wire filters can hopefully be derived from this work.

5) The coefficient of compressibility is as important a sludge parameter as specific resistance. The profession should take immediate note of this.

The drying formulae; Equations 3-17, 3-18 and 3-19, can also be very favorably compared to the empirical relationships previously discussed. Examination of Equation 3-19 at $t = t_f$

$$t_f = \frac{W_{TS}}{100 \, A \, I_{s,c}} \left[U_o - U_{cR} + U_{cR} \, \ln \left(\frac{U_{cR}}{U_f} \right) \right] \qquad \text{3-19}$$

reveals again that $t_f \propto W_{TS} / A$. If one reformulates the brackets in terms of s , with $U = \frac{100 \, (100 - s)}{s}$:

$$t_f \propto \frac{100 - S_o}{S_o} - \frac{100 - S_{cR}}{S_{cR}} \left(1 - \ln \frac{(100 - S_{cR})}{(100 - S_f)} \frac{S_f}{S_{cR}} \right)$$

It may be noted t_f falls to zero if $S_f = S_{cR} = S_o$, as would be expected. Also seen is that increase in s_o or a decrease in s_f will increase the drying time.

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V.   SOLIDS MEASUREMENT BY GAMMA RAY ATTENUATION

ATTENUATION THEORY

Investigation of rational filtration models, such as those developed in Chapter II, requires measurements of the head and solids content in situ, in addition to parameters of specific resistance and coefficient of compressibility. Piezometers normally suffice for measurements of head. Because filtration on porous beds is a transient phenomenon, and also one in which the sludge cake is deforming, it is desirable to perform repeated measurements of the solids content at fixed locations. Gravimetric methods of solids determination are clearly unsuitable, as each sample removed for gravimetric analysis destroys part of the flow system. An alternative method which sounds promising is to record electrical resistance measurements taken from within the sludge. Work done on this idea by Coakley[1] in 1962 ended in failure due to ionic migration, especially with high solids content. Similar problems of determining the water content distribution are encountered in studying drying. Gravimetric analysis will destroy the flow system and electrical conductivity measurements produce ionic migration of the non hygroscopic water.

Gamma ray attenuation provides a convenient, rapid, non-destructive method for repeatedly measuring the solids content and water content of a sludge. The theoretical basis for the method and its practical application has been discussed by a number of writers. [2,3,4,5,6,7] Statistical aspects of radiation are discussed in APPENDIX B. The intensity of monoenergetic gamma rays after passing through a thickness x of mass is related to their incident intensity through the equation

$$N_1 = N_0\, e^{-\mu \rho x}$$

5-1

where the terms are defined as

ρ = mass density, gm/cm^3

N_1 = intensity of emitted beam, counts/min.

N_0 = intensity of incident beam, counts/min.

μ = mass absorption coefficient, cm^2/gm

X = thickness of mass, cm.

If there are several layers of material through which the gamma ray beam is passing then the intensity of the emitted beam, N_n, is related to the intensity of the incident beam, N_0, through the equation

$$N_n = N_0 e^{-\left[\mu_1 \rho_1 X_1 + \mu_2 \rho_2 X_2 + \cdots + \mu_n \rho_n X_n\right]} \qquad 5\text{-}2$$

where the subscripts refer to the properties of the particular mass.

Reference to Figure 5-1 will be helpful in applying the above results to use of gamma ray attenuation to determine the solids content of sludges. In Figure 5-1a the gamma ray beam traverses the column wall of thickness L_p, then the sludge of thickness L_{SL}, then the other column wall of thickness L_p. According to Equation 5-2 the applicable attenuation relation would be

$$N_{WTS} = N_0 e^{-\left[\mu_p \rho_p L_p + \mu_{SL} \rho_{SL} L_{SL} + \mu_p \rho_p L_p\right]} \qquad 5\text{-}3$$

Where the subscripts designate the plastic column walls and the sludge. However, the sludge is made up of solids and water. This permits idealization of the system to that shown in Figure 5-1b. Application of Equation 5-2 results in the expansion of Equation 5-3 to

$$N_{WTS} = N_0 e^{-\left[2\mu_p \rho_p L_p + \mu_w \rho_w L_w + \mu_s \rho_s L_s\right]} \qquad 5\text{-}4$$

where the subscripts refer to plastic, water and solids. If the column
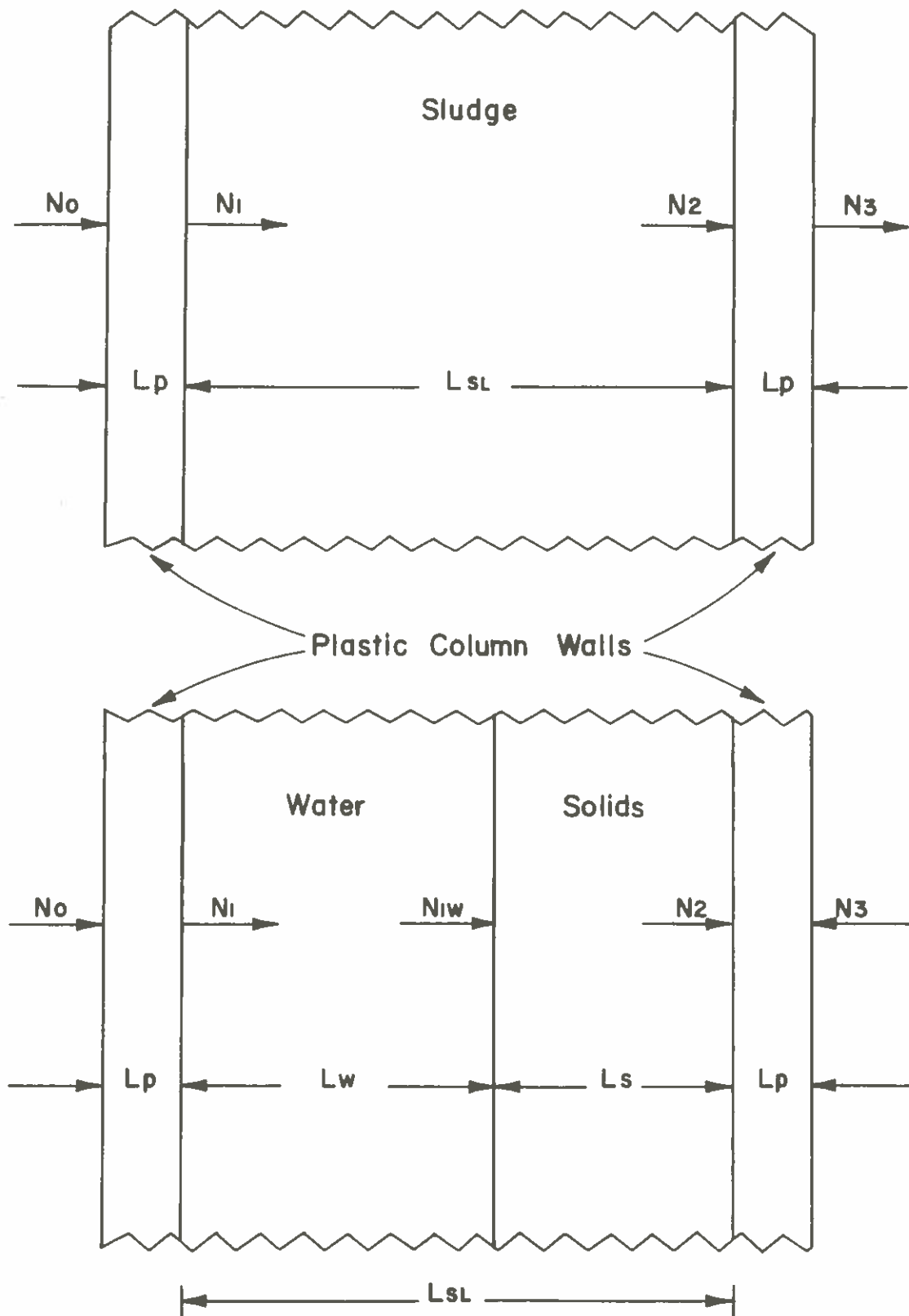
Figures 5-1a and 5-1b. Attenuation Through a Column of Sludge. Figure
5-1a denotes a homogeneous mixture of solids and water,
whereas in Figure 5-1b the solids and water are
viewed separately.

were empty, attenuation would be due to the plastic according to the equation

$$N_{mo} = N_o e^{-2\mu_p \rho_p L_p}$$

5-5

where N_{mo} is the emitted beam intensity. Division of Equation 5-4 by Equation 5-5 results in the simplified form

$$N_{W_{TS}} = N_{mo} e^{-[\mu_w \rho_w L_w + \mu_s \rho_s L_s]}$$

5-6

MENSURATION

A relation between L_w, L_s, L_{sL} and the water content w is needed (the water content and the solids content are related through the equation w = 100-S). Reference to Figure 5-2 will be helpful in deriving such a relation. The distance through the sludge L_{SL} is related to L_w and L_s by the equation.

$$L_{sL} = L_w + L_s$$

5-7

as can be seen from Figure 5-1. The solids content is defined as

$$\frac{S}{100} = \frac{W_{TS}}{W_w + W_{TS}} = \frac{\rho_s g V_{TS}}{\rho_w g V_w + \rho_s g V_{TS}}$$

5-8

where the terms not previously defined are

W_{TS} = weight of total solids

W_W = weight of water

ρ_s = density of total solids

V_{TS} = volume of total solids

V_W = volume of water

g = acceleration of gravity.

Solving Equation 5-8 for V_{TS} results in

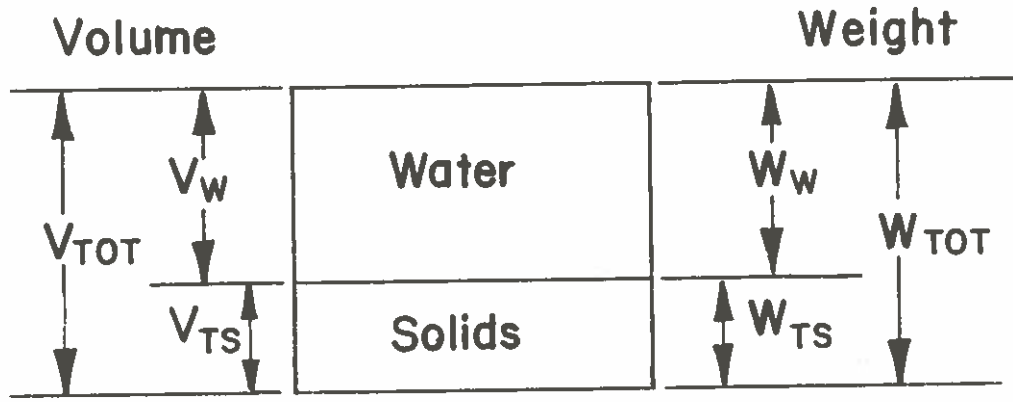$$V_{TS} = \frac{S \rho_w V_w}{100 \rho_s (1 - \frac{S}{100})}$$

Figure 5-2. Relation Between the Weight and Volume of an Element of Sludge.

which combines with equation $V_{TOT} = V_{TS} + V_W$ to yield

$$V_{TOT} = V_W \left[\frac{\frac{s}{100} \rho_w + \rho_s (1 - \frac{s}{100})}{\rho_s (1 - \frac{s}{100})} \right]$$

which becomes on rearranging

$$\frac{V_W}{V_{TOT}} = \frac{\rho_s (1 - \frac{s}{100})}{\frac{s}{100} \rho_w + \rho_s (1 - \frac{s}{100})} \qquad 5\text{-}9$$

The respective volumes are related through the area A and Equation 5-7 by the relation

$$V_{TOT} = A \, L_{SL} = A \, (L_W + L_S) = V_W + V_{TS}$$

so that Equation 5-9 becomes

$$L_W = L_{SL} \left[\frac{\rho_s (1 - \frac{s}{100})}{\frac{s}{100} \rho_w + \rho_s (1 - \frac{s}{100})} \right] \qquad 5\text{-}10$$

and the distance through the solids becomes from Equation 5-7

$$L_S = L_{SL} \left[\frac{\frac{s}{100} \rho_w}{\frac{s}{100} \rho_w + \rho_s (1 - \frac{s}{100})} \right] \qquad 5\text{-}11$$

APPLICATION TO SOLIDS MEASUREMENT

For any given experimental column the distance L_{SL} will be known, or it can be measured. Hence, if Equations 5-10 and 5-11 are combined with Equation 5-6 the solids content will be directly related to the counting measurements N_{WTS} and N_{mo} through the equation

$$N_{W_{TS}} = N_{m_o} e^{-\rho_s \rho_w L_{SL} \left[\frac{\mu_w - \frac{s}{100}(\mu_s - \mu_w)}{\rho_s + \frac{s}{100}(\rho_w - \rho_s)} \right]} \qquad 5\text{-}12$$

Equation 5-12 is shown in Figure 5-3 for various distances, called column thickness in the figure, through which the gamma rays travel. Figure 5-3 was drawn up for a solid having a mass absorption coefficient of $\mu_s = 0.0777$ cm^2/gm. Figure 5-4 represents the water content of a sludge as a function of the radiation attenuation.

The figure shows a graph with the vertical axis labeled $\dfrac{Nw_{TS}}{Nm_0}$ ranging from 0.1 to 1.0, and horizontal axis labeled S/100 ranging from 0.0 to 1.0. Four lines are labeled X = 5 cm, X = 10 cm, X = 15 cm, and X = 20 cm.

Relative Gamma Radiation Attenuation as a Function of Water Content

X = Column Thickness
Nw_{TS} = Transmitted Radiation Intensity
Nm_0 = Incident Radiation Intensity
μ_w = 0.0839 cm^2/gm
μ_s = 0.0777 cm^2/gm
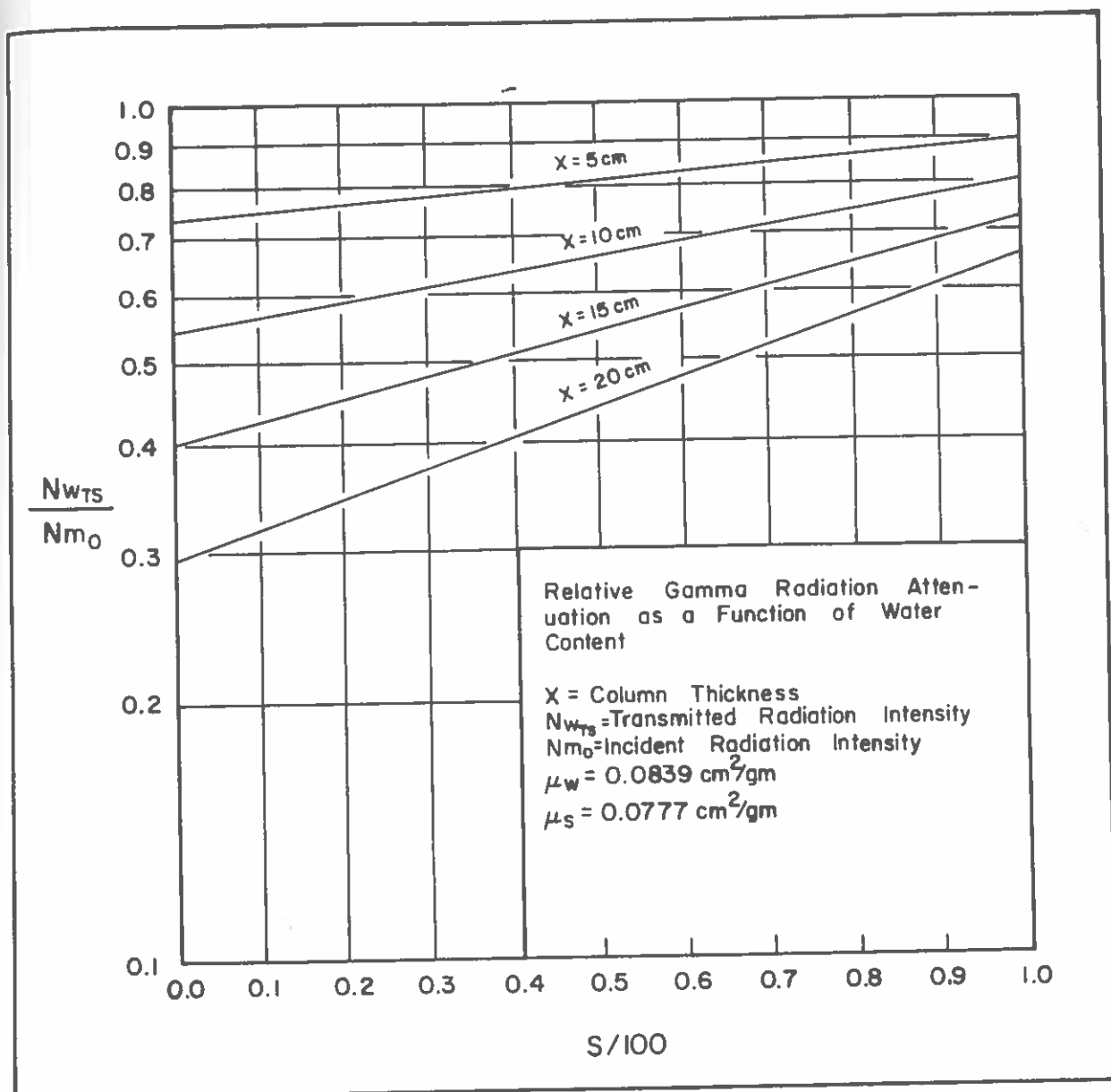
Figure 5-3. Relative Gamma Radiation Attenuation as a Function of Solids Content. The attenuation when the solids concentration is zero is due to the water in the column; the attenuation when s = 100 is that which would be obtained with the column filled with solids. The denominator represents the radiation intensity passing through an empty column.
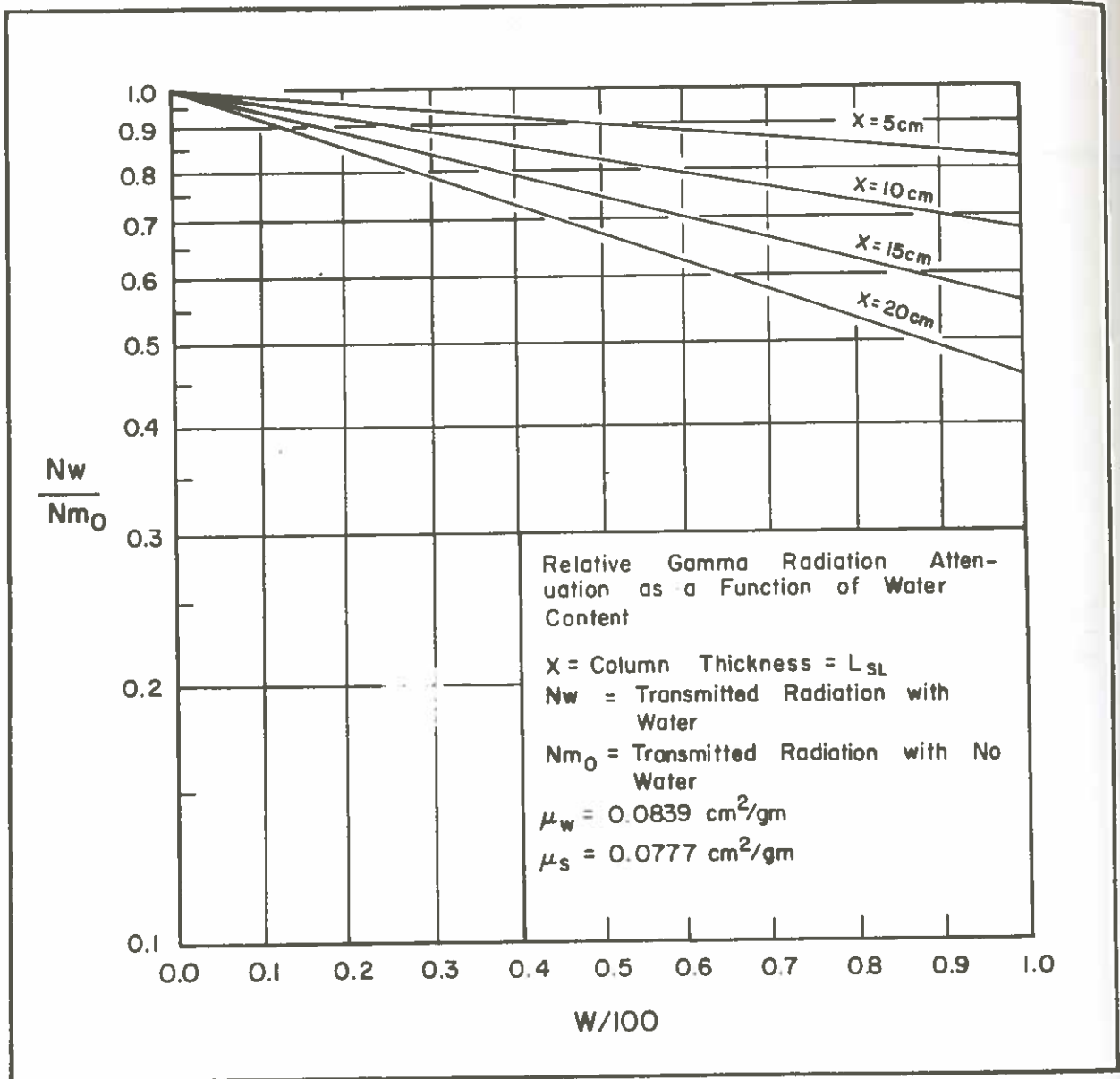
Figure 5-4. Relative Gamma Ray Attenuation as a Function of Water Content. The denominator N_{m_0} represents the counting rate through the column which has only solids present.

Equation 5-12 may be rearranged to express the solids content

directly as a function of the radiation attenuation. Equation 5-12 becomes

$$\frac{S}{100} = \frac{\rho_s \left[\mu_w \, \rho_w \, L_{SL} - \ln \left(\frac{N_{m_o}}{N_{WTS}} \right) \right]}{(\rho_w - \rho_s) \, \ln \left(\frac{N_{m_o}}{N_{WTS}} \right) + \rho_w \, \rho_s \, L_{SL} \, (\mu_w - \mu_s)} \quad 5\text{-}13$$

EXPERIMENTAL EQUIPMENT

The precision electronic radiation counting equipment for measuring

the solids content is shown in Figure 5-5. A schematic diagram is shown

in Figure 5-6. The 3-inch crystal scintillation detector was manufactured

by Harshaw-Hammer Co., while the 400 channel analyzer, resolver-integrator,

and the print-punch-type control unit were all manufactured by the Technical

Measurement Corp., the typewritten by IBM, and the paper tape punch by

Tally Corp. Detailed drawings of the source shielding and collimating

slits are shown in Figures 5-7 and 5-8. The shielding was designed to

achieve maximum personnel protection against radiation hazards while

readily permitting the collimation openings to be changed.
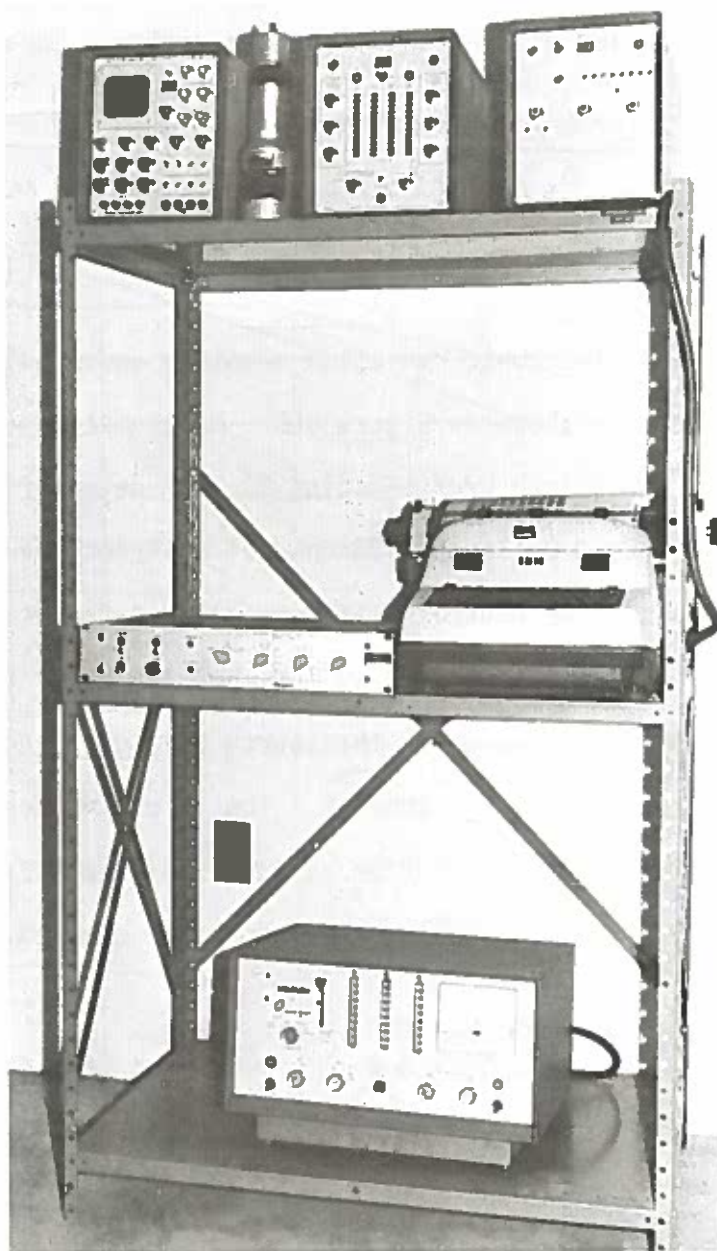
Figure 5-5. Electronic Pulse Height Analysis Instrumentation. On the
top shelf are the 400 channel pulse height analyzer, detector,
resolver-integrator, and the print-punch-type control
unit; on the middle shelf are the high voltage supply
and the typewriter output unit; and on the bottom shelf
is the scaler.

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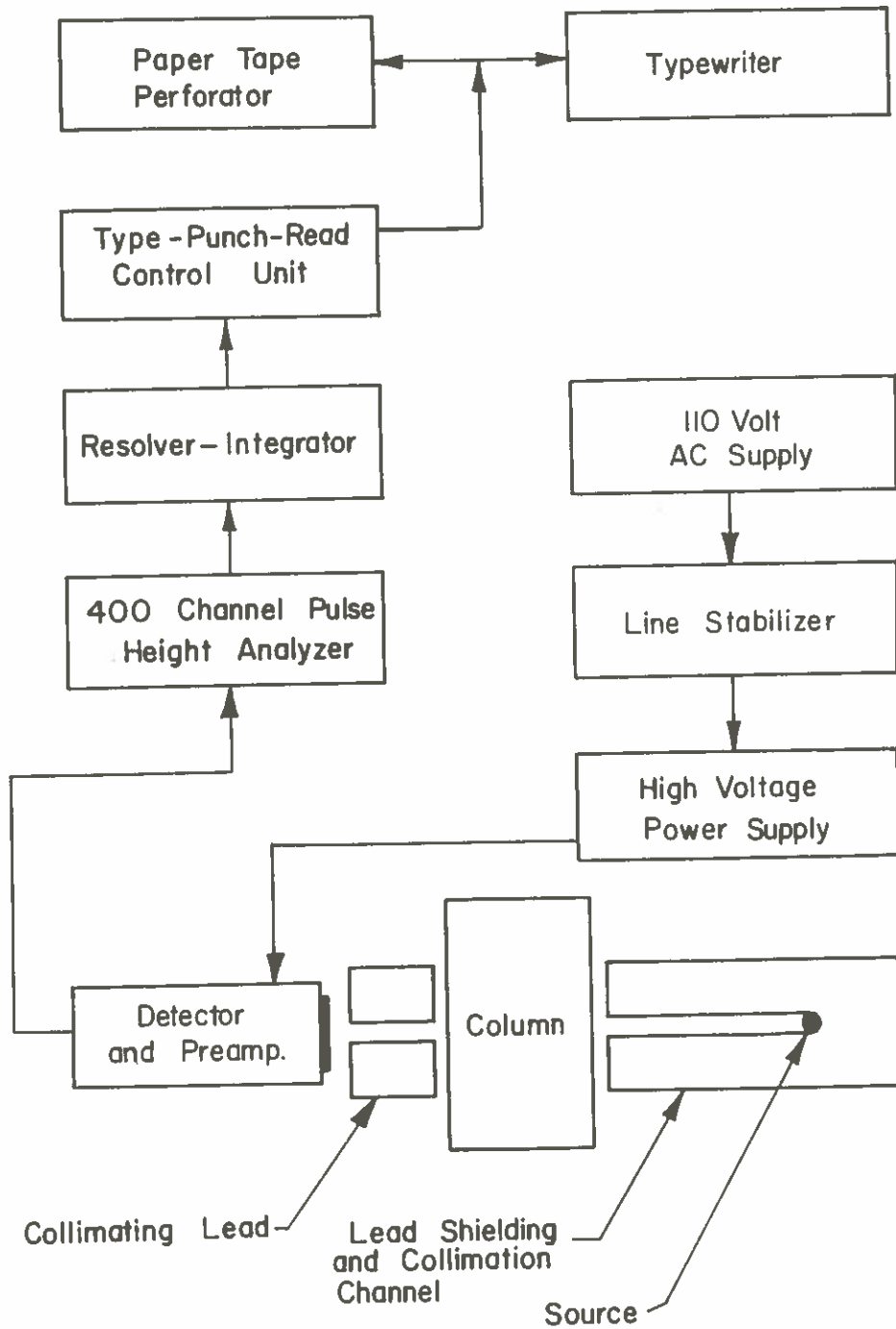
Figure 5-6. Schematic Diagram of the Gamma Ray Attenuation Equipment.

Figure 5-7.   Exploded View of the Source Shielding.

NO.	PART NAME	MAT.
1	DOOR	LEAD
2	COLLIMATION BLOCKS	LEAD
3	SLEEVE	STEEL
4	SHIELD	LEAD
5	SOURCE HOLDER	LEAD

Figure 5-8. Assembled View of the Source Shielding.

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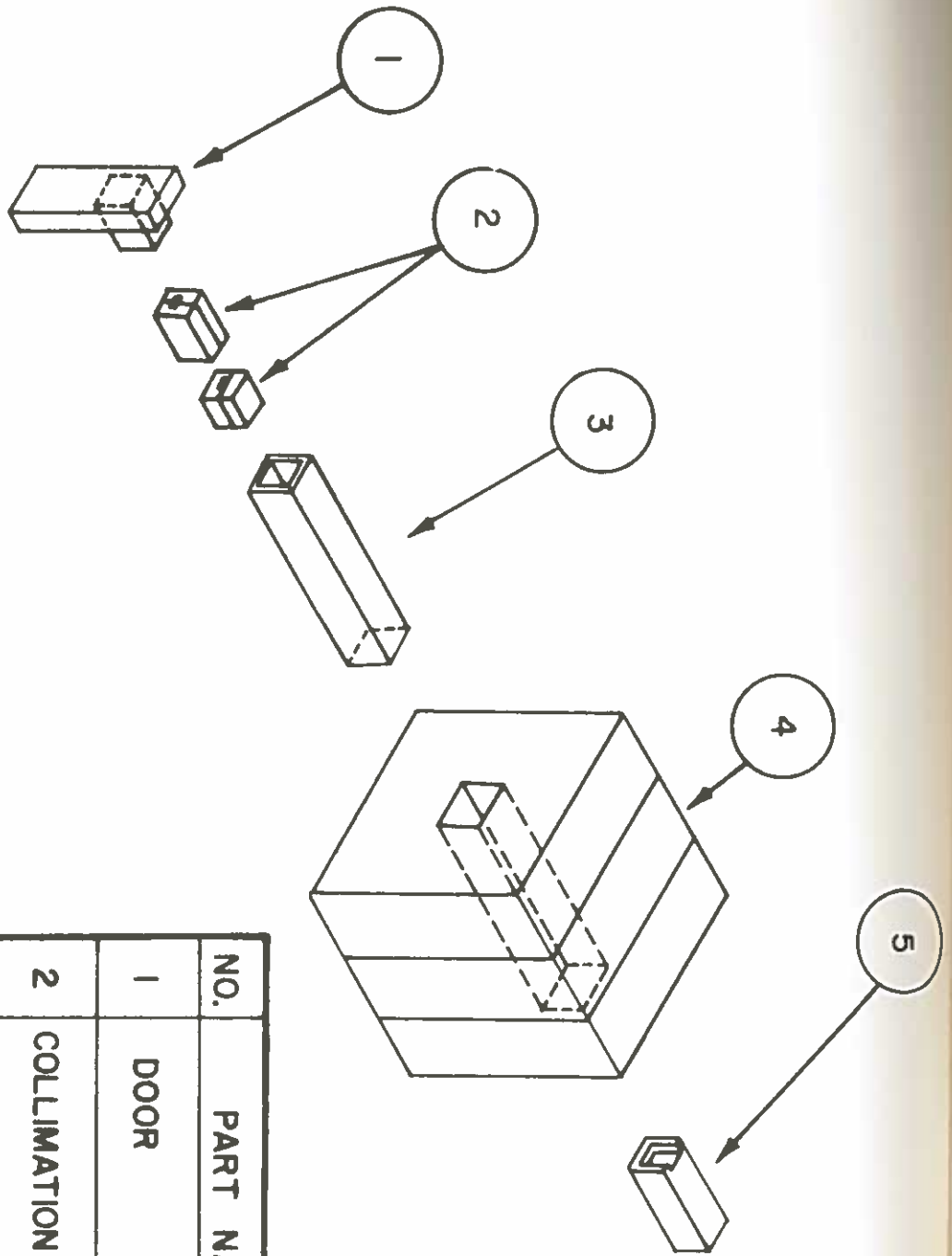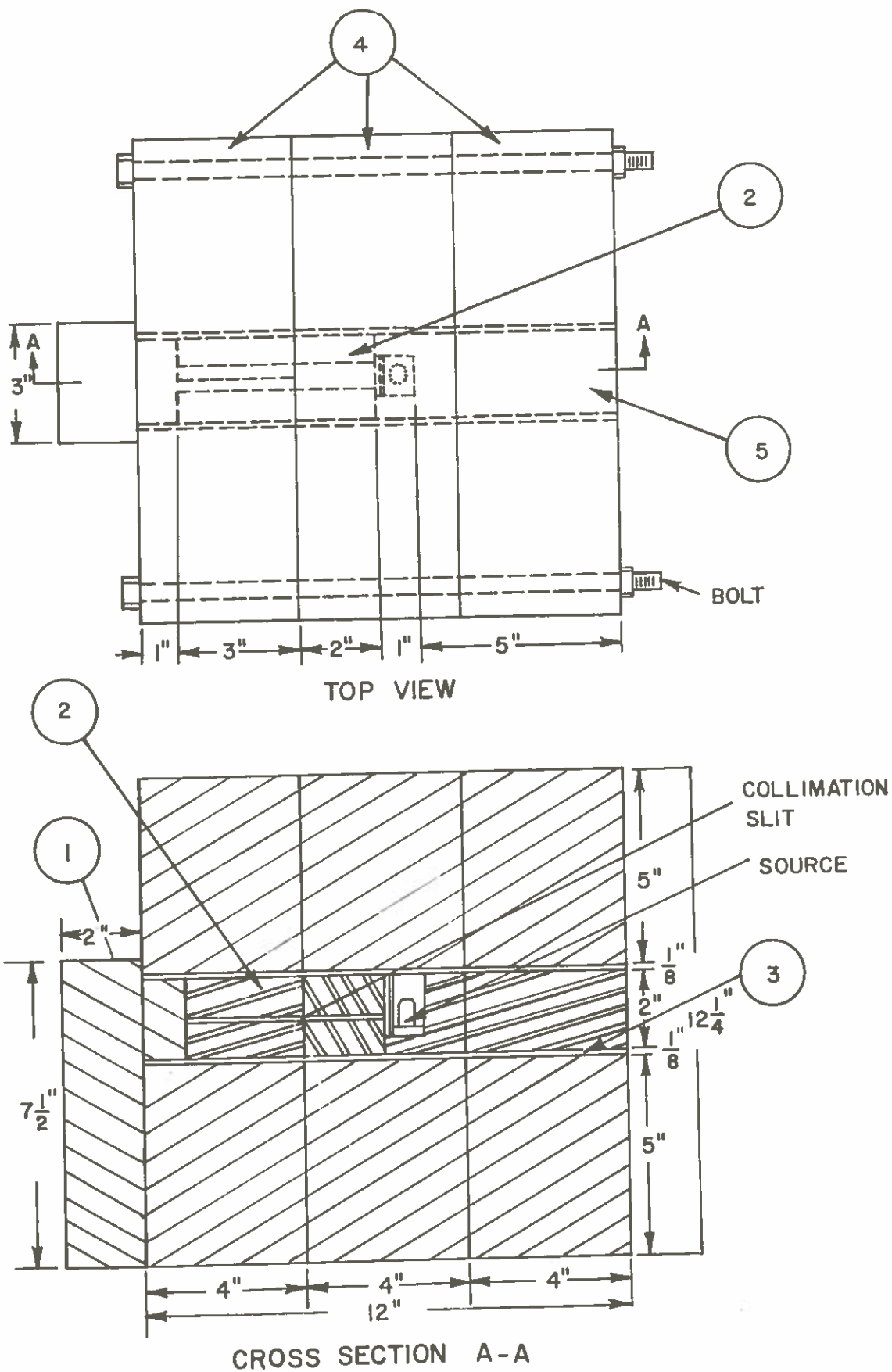
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VI. INVESTIGATION OF WATER TREATMENT SLUDGES

SLUDGE SOURCES

Sludges from water treatment plants throughout the Commonwealth of Massachusetts were collected and brought to the laboratory for analysis. Figure 6-1 shows the geographical location of these facilities. A brief resumé of each plant follows.

Amesbury The raw water supply for the Town of Amesbury comes from approximately 300 - 3 to 4 inch shallow artesian wells in the general vicinity of the treatment works. Because of the abundant natural iron which occurs in the well supply, treatment includes aeration followed by manually cleaned slow sand filters; mechanical addition of alum and caustic soda; settling in a four-compartment manually cleaned basin; and chlorination. The sedimentation basins are emptied of the accumulated sludge twice annually.

The settled sludge is decanted with the supernatant being wasted into a peripheral stream leading ultimately to the Merrimac River. The concentrated sludge is then discharged into a nearby lagoon where it dewaters and dries for approximately 6 months. The dewatering lagoon is surrounded by several supply wells. Therefore, owing to iron carried by infiltration and percolation, there may be some recycling of iron into the supply system.

During an observation of the emptying and cleaning operation, samples of supernatant were collected from the mouth of the pipe through which the supernatant discharged to the stream. Owing to the unusual iron concent of the water, the stream became a bright reddish-brown color as the supernatant entered. A density current with a sharp supernatant-stream water interface was observed.
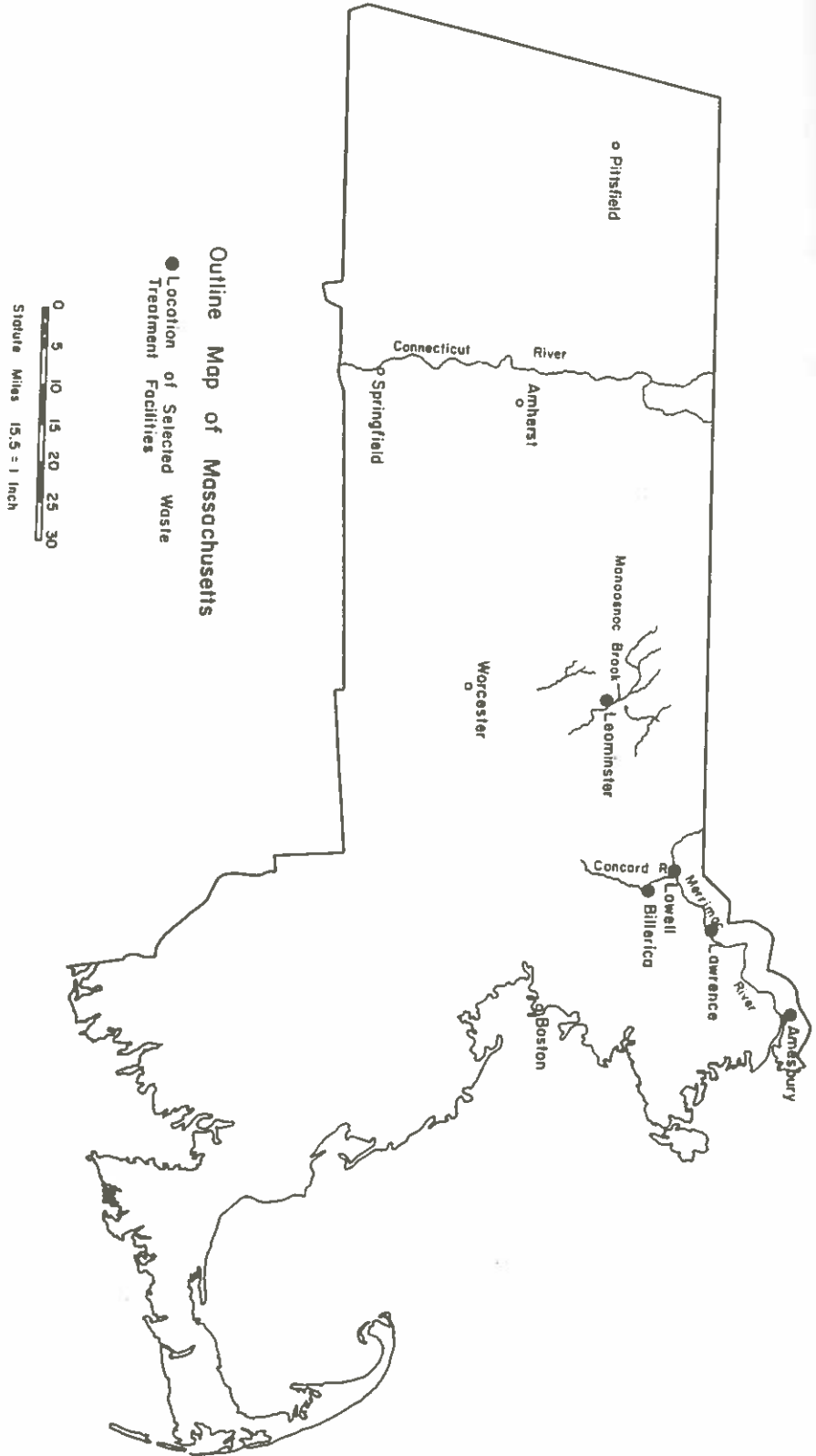
87

Figure 6-1. Water Treatment Plants whose Sludges Were Selected for Study

Having been informed of the approximate time necessary to drain the decanted sludge into the lagoon, a 50 gal composite sludge sample was taken from the mouth of the discharge pipe which filled the lagoon. When the level in the sedimentation basin was such that a man could safely wade, a 50-gal sludge sample was taken from within the basin.

Billerica Billerica draws its raw water supply from the Concord River which flows in a northerly direction through the town with ultimate discharge into the Merrimac River. The Concord River receives a high suspended solids loading, and is very turbid. Industrial wastes of various characteristics are freely discharged within its banks. In warmer weather, copious algal growths and vascular aquatic plants choke the shores.

To cope with these problems, relatively modern, well operated treatment facilities are located on the west bank of the river. Treatment includes rapid sand filtration, chemical coagulation included with activated carbon, followed by sedimentation in a continuous mechanically cleaned basin and chlorination. The accumulated sludge is drawn off daily for ultimate disposal in a neighboring lagoon. Since the lagoon's capacity is exceeded by the sludge produced and very little land is locally available, overflowing sludge enters directly into the Concord River through a drainage depression some 100 feet behind the lagoon. The addition of activated carbon for color and odor removal easily identifies Billerica sludge by its jet-black color. The solids content is high.

Sludge samples were taken in 5-gal polyethylene carboys from the orifice of a telescoping valve which drains the accumulated sludge from the sedimentation basin. Smaller samples were collected at random from the lagoon. Samples of partially dried sludge cake were taken from the overflow which had remained in a shallow local depression.

<u>Lawrence</u>   The highly industrialized Merrimac River serves as the raw

water supply for Lawrence. High in solids, color, turbidity, industrial

waste, and (in warmer months) algae, the river water is pumped for treat-

ment into one of the nation's first* rapid sand filtration facilities.

Owing to the nature of the raw water, additional treatment includes chemical

coagulation, activated carbon addition and odor control by post aeration

just prior to distribution.

Periodically, one sedimentation basin at a time is shut down and the

contents are flushed directly into the Merrimac River. Sludge samples,

taken from the sedimentation basins with a bailer sampler, appeared grey-

black from activated carbon and gelatinous.

<u>Leominster</u>   A series of impoundments on Monoosnoc Brook constitute the

raw water supply for Leominster. Algal populations in the summer months,

slight turbidity, and consumer need for an elevated pH require treatment

facilities including rapid sand filtration, chemical coagulation, activated

carbon addition, chlorination and post treatment liquid alum injection.

Sludge is accumulated in manually cleaned sedimentation basins which are

periodically flushed directly into nearby Monoosnoc Brook downstream from

the impoundments.

Sludge samples were bailed from the sedimentation basins. Owing to a

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*<u>Note</u>: Begun in 1874, the Lawrence waterworks, modified in 1938 by the
WPA, lies across the street from the original plant which overlooks the
Merrimac River. A visitor's log-book, begun with the initiation of the
water treatment in Lawrence, has been kept intact and up to date by the   .
staff of the treatment works. Signatures of the plant's visitors form
the entries for a large volume of "Who's-Who in Environmental Engineering."

recent clean-out, the sludge floc was very low in solids and was considered nonrepresentative of the periodic effluent from the basins.

LOWELL Another raw water supply from the Merrimac River undergoes treatment at Lowell. Flowing in an easterly direction through Lowell, the Merrimac is but slightly less polluted than downstream when it reaches the Town of Lawrence. The modern treatment facilities include pre-chlorination at the intake followed by rapid sand filtration, chemical coagulation and sedimentation in a manually cleaned basin. The sand filter is overlain by a thin layer of activated charcoal for algae taste and odor control. Cleaning is accomplished by a continuously traversing vacuum which eliminates the necessity for compartmentalizing filters for backwashing. During one periodic cleaning of the sedimentation basins, sludge was sampled at the mouth of the pipe which drained the accumulated sludge directly into the Merrimac River. Since activated carbon is not added in granular form, Lowell sludge is grey (rather than black) and quite gelatinous.

LABORATORY ANALYSES

Solids Content

The solids content of each sludge was determined as outlined in APPENDIX A. Standard Methods (12th Edition) Part III, "Residue on Evaporation" was used as a guide for laboratory work. It is interesting to note, however, that within the text of Standard Methods, oven drying times of from 1 hour to an unspecified upper limit of time at 103° C and from 10 min to an unspecified upper limit of time at 600° C are recommended for both "Total Residue" and

"Fixed Residue Determinations." Several experiments were performed to aid in deciding upon an adequate drying time. Replicate samples were heated in the oven for different durations (both at 103°C- 105°C and 600°C) and their weight loss was determined. The results of these tests indicated that samples heated for oven durations of 8 hr. at 103°C and 20 min. at 600°C maintained a constant weight.

Specific Resistance

The specific resistance test represents the rate at which water may be drained from a given volume of sludge when subjected to a driving force such as gravity or vacuum. APPENDIX A contains a discussion of the materials and methods for determination of the specific resistance.

The filtrate density and dynamic viscosity (classically assumed equal to water) play an important role in specific resistance calculations. A series of viscosity tests on the filtrate were performed yielding results which indicated that the values of filtrate density and dynamic viscosity for water may be employed in the calculations after appropriate adjustment for the test temperature.

Multiple sets of specific resistance tests at three different pressures were performed on several sludges using a volume of 100 ml in the Buchner funnel. The test aliquots for each sludge all came from the same bulk sample and were apportioned as outlined in APPENDIX A.

Filtering 100 ml samples of sludges with low solids content resulted in early cake cracking which yielded inconsistent specific resistance data.

However, further investigation has shown that consistent specific resistance data for the same sludges may be obtained with 250 ml samples are filtered. Results of multiple tests performed with 100 ml samples and tests performed with 250 ml samples are presented in RESULTS AND DISCUSSION.

Coefficient of Compressibility

The specific resistance for any particular sludge exhibits a unique variation with pressure according to the sludge's compression characteristics as described by Equation 2-28a. The graph of specific resistance vs. pressure (as outlined in APPENDIX A) exhibits the compression characteristics of the sludge and permits calculation of the coefficient of compressibility.

RESULTS AND DISCUSSION

Units

As pointed out by Swanwick and Davidson[1], early investigators reported specific resistance in units of $\sec^2$/gm. These units resulted from interpreting pressure in units of gm/cm^2 rather than $dynes/cm^2$. The laboratory data presented in Table 6-1 is in a form for ready comparison to results reported by previous investigators. As outlined in APPENDIX A, these units are dimensionally correct when considering c as the WEIGHT of sludge solids per unit volume of filtrate (rather than the MASS of sludge solids per unit volume of filtrate). Simple multiplication of specific resistance (in $\sec^2$/gm) by g (980 cm/sec^2) results in units of cm/gm and implies that c is considered as the mass of sludge solids per unit volume of filtrate.

Solids Content

Water treatment sludges generally contained a lower solids content than wastewater sludges reported in the literature. Wastewater sludges, for the most part, undergo a thickening operation prior to final treatment and are not usually discharged into a stream. Common practice has been to return water treatment sludges to the raw water source, so sludge thickeners are not used. This may have minimized the effect of the sludge on the stream. The lower solids content does not, however, alleviate the tremendous total solids load transported to streams daily by the mounting volumes of water treatment wastes.

Specific Resistance

It is evident from the laboratory data and the results from O'Brian and Gere[6] presented in Table 6-1 that water treatment sludges have, for the most part, lower specific resistance values than wastewater sludges. The water treatment sludges tested were homogeneous, inorganic, and void of filamentous binders. With this in mind, it is not unreasonable to expect that water treatment sludges will lend themselves to a variety of dewatering techniques.

The specific resistances of the pulp and paper activated sludge (2% $FeCl_3$) reported by Eckenfelder and O'Connor[4] and the lime treated sludges reported by O'Brien and Gere[6] point out the fact that chemical conditioning may be of significant value in lessening the resistance of sludge to filtration.

Although the water treatment sludges tested resulted in a wide range of values for specific resistance, they were all within one order of magnitude. The specific resistances of Billerica and Lawrence sludge determined using 100 ml filter samples compare favorably with those obtained using 250 ml filter samples.

Sludge	Solids Content %	Sample Volume (ml)	Coefficient of Compressibility	$R @ \Delta p =$ 38.1 cm Hg (sec^2/gm)	Reference
Digested Wastewater	15.0	NA	0.63	23.8 X 10^9	Coackley[2]
Digested Wastewater	15.0	NA	0.56	18.7 X 10^9	Coackley[2]
Digested Wastewater	4.9	NA	0.76	4.07 X 10^9	Niemitz[3]
Raw Wastewater	NA	NA	0.54	4.70 X 10^9	Eckenfelder[4] & O'Connor
Pulp & Paper Activated Sludge 2% FeCl$_3$	NA	NA	0.80	0.165 X 10^9	Eckenfelder[4] & O'Connor
Digested Wastewater	6.35	100	0.51	72.4 X 10^9	Nebiker, Sanders, & Adrian[5]
Digested Wastewater	4.89	100	0.74	14.0 X 10^9	Nebiker, Sanders, & Adrian[5]
Digested Wastewater	3.70	100	0.66	48.0 X 10^9	Nebiker, Sanders, & Adrian[5]

Table 6-1 (Continued on next page)

Physical Properties of Water Treatment and Wastewater Sludges

Sludge	Solids Content %	Sample Volume (ml)	Coefficient of Compressibility	$R @ \Delta p = 38.1$ cm Hg (sec^2/gm)	Reference
Water Treatment 3.0 mg/l Lime	1.00	NA	NA	0.098×10^9	O'Brien & Gere[6]
Water Treatment 2.0 mg/l Lime	1.00	NA	NA	1.29×10^9	O'Brien & Gere[6]
Water Treatment	1.00	NA	NA	1.85×10^9	O'Brien & Gere[6]
Water Treatment Billerica (x)	4.65	100	1.21	2.49×10^9	Laboratory Data
Water Treatment Lawrence (y)	0.944	100	1.02	10.4×10^9	Laboratory Data
Water Treatment Lowell (z)	3.81	100	0.886	4.23×10^9	Laboratory Data
Water Treatment Amesbury	2.06	250	0.802	1.04×10^9	Laboratory Data
Water Treatment Billerica	4.65	250	0.831	3.48×10^9	Laboratory Data
Water Treatment Lawrence	0.944	250	1.32	9.85×10^9	Laboratory Data

x=average of 3 sets, y=average of 5 sets, z=average of 4 sets

Table 6-1

Physical Properties of Water Treatment and Wastewater Sludges

Note, however, that the 100 ml Billerica and Lawrence values are the average of 3 and 5 sets respectively while the comparable 250 ml values are from one set of 3 tests. The current program of testing with 250 ml filter volumes is designed to determine the effect of initial sample volume on specific resistance.

Future studies of gravity sludge dewatering on filter beds will be carried out with the recently completed apparatus shown in Figures 6-2 and 6-3. Note that the filter columns are spiraled with piezometer taps which can be coupled directly to the piezometer boards in the rear.

Coefficient of Compressibility

O'Brian and Gere[6] do not report values for the coefficient of compressibility of water treatment sludges tested, so no comparable data are available with which to compare values obtained with Massachusetts water treatment sludges. Somewhat higher coefficients of compressibility were obtained for the water treatment sludges than are cited in the literature for wastewater sludges. This indicates that water treatment sludges produce a more compressible cake than wastewater sludges.
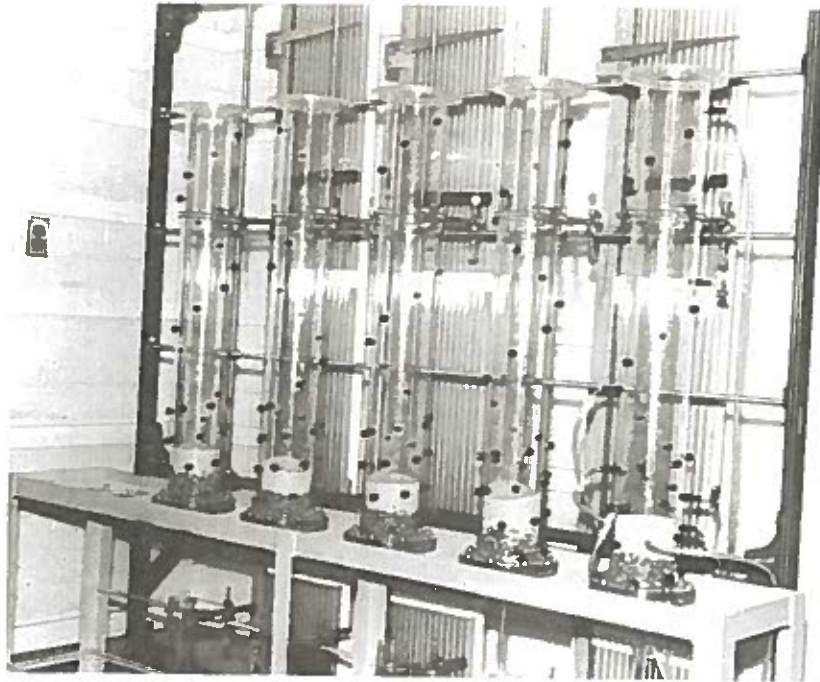
Figure 6-2.   Plexiglas Columns for The Study of Gravity Dewatering on Sand Filters.
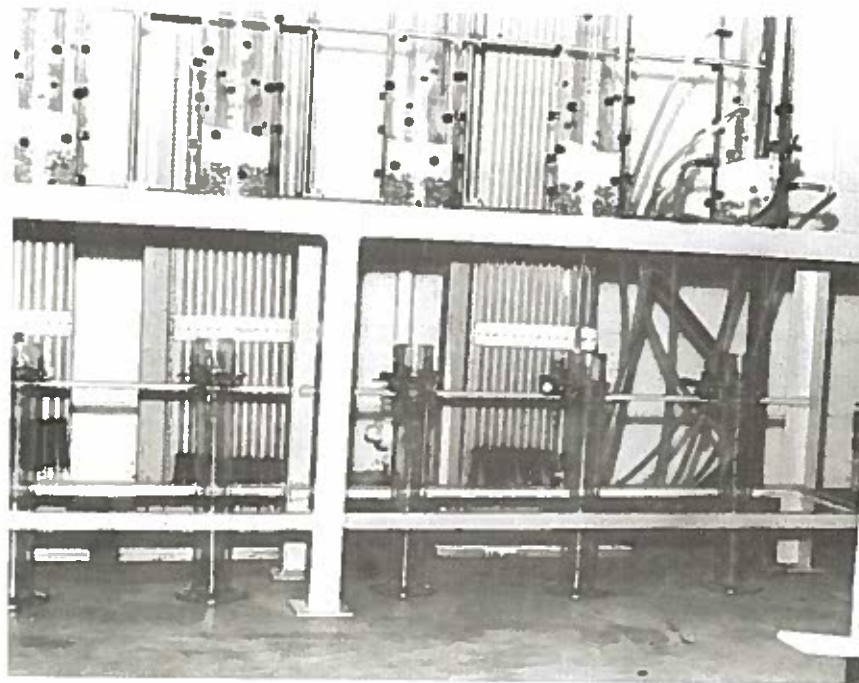

Figure 6-3.   View Showing Piezometers Attached to Manometers. (Note the graduated cylinders for filtrate capture)

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6. O'Brien and Gere, "Waste Alum Sludge Characteristics and Treatment," Research Report No. 15, New York State Department of Health, 65 (1966).

APPENDIX A. LABORATORY PROCEDURES

A. Storage of sludge sample.

The bulk samples are stored in a $20^{\circ}C$ constant temperature room in closed containers.

B. Preparation of the test samples.

1. Bulk samples are thoroughly mixed by gentle stirring to provide homogeneity.

2. A smaller 32-ounce polyethylene bottle is filled and its contents used throughout the testing. This aliquot is sufficient to perform triplicate solids analysis as well as three specific resistance tests.

C. Laboratory analysis.

1. Solids Determinations.

Solids determinations are run in triplicate 50 ml samples according to Standard Methods (12th Edition) part III, Residue on Evaporation. See Figure A-1 for data format.

2. Volatile and Fixed Solids.

Volatile and fixed solids determinations follow Standard Methods, "Residue of Volatile and Fixed Suspended Matter" using dried samples from the solids determination tests above.

3. Specific Resistance.

The lack of a standard procedure and equipment for determination of specific resistance requires a detailed outline of equipment assembly and utilization.

a. The following custom glass equipment is fabricated:

(1) Join the female portion of a 24/40 ℥ ground glass joint to the open end of a 250-ml buret (1.0 ml

100

Date Collected _____

Solids Determination

Sludge _____   Date of Analysis _____

Sample Number				
A	Wet Dish & Sludge _____ Dry Dish _____	Wet Dish & Sludge _____ Dry Dish & Sludge _____	Dry Dish & Sludge _____ Dry Dish _____	$\dfrac{\text{Residue}}{\text{Wet Sludge}} \times 100 = \%\ \text{Solids}$
	Wet Sludge _____	Wgt. Water _____	Residue (total) _____	
	Dry Dish & Sludge _____ Ig. Dish & Sludge _____	Ig. Dish & Sludge _____ Dry Dish _____	Fixed Residue _____ Vol. Residue _____	
	Vol. Residue _____	Fixed Residue _____	Residue (total) _____	____ × 100 = ____ %
B	Wet Dish & Sludge _____ Dry Dish _____	Wet Dish & Sludge _____ Dry Dish & Sludge _____	Dry Dish & Sludge _____ Dry Dish _____	$\dfrac{\text{Residue}}{\text{Wet Sludge}} \times 100 = \%\ \text{Solids}$
	Wet Sludge _____	Wgt. Water _____	Residue _____	
	Dry Dish & Sludge _____ Ig. Dish & Sludge _____	Ig. Dish & Sludge _____ Dry Dish _____	Fixed Residue _____ Vol. Residue _____	
	Vol. Residue _____	Fixed Residue _____	Residue _____	____ × 100 = ____ %
C	Wet Dish & Sludge _____ Dry Dish _____	Wet Dish & Sludge _____ Dry Dish & Sludge _____	Dry Dish & Sludge _____ Dry Dish _____	$\dfrac{\text{Residue}}{\text{Wet Sludge}} \times 100 = \%\ \text{Solids}$
	Wet Sludge _____	Wgt. Water _____	Residue _____	
	Dry Dish & Sludge _____ Ig. Dish & Sludge _____	Ig. Dish & Sludge _____ Dry Dish _____	Fixed Residue _____ Vol. Residue _____	
	Vol. Residue _____	Fixed Residue _____	Residue _____	____ × 100 = ____ %

Figure A-1. Data Format for Solids Determinations

subdivisions).

(2) Construct a vacuum adaptor as shown in Figure A-2 taking care that the glass in the narrow neck above the ringseal is not weakened. Anneal the fixture. A small drip extension (now shown) on the effluent end will be helpful.

(3) Use a short male section of a 29/26 $\overline{\overline{5}}$ joint, flair the funnel adaptor as shown in Figure A-3a. Anneal the fixture. Join the adaptor to the Buchner funnel with epoxy cement as shown in Figure A-3b.

(4) Although not essential, the vacuum plug shown in Figure A-4 is easily constructed from the male portion of a 29/42 $\overline{\overline{5}}$ joint (a 29/26 $\overline{\overline{5}}$ male portion will substitute).

b. Assembly of equipment.

(1) A photograph of the apparatus set up for specific resistance testing is shown in Figure A-5. Note that owing to the special funnel adaptor, the Buchner funnel may be removed and replaced in exactly the same alignment without skewing.

(2) The components are assembled according to the schematic diagram of Figure A-6. Two screw clamps (not shown) may be substituted in series for the pressure control valve. The nearmost clamp may be used to bring the manometer slightly below the desired pressure setting and the foremost clamp should be used to fine - adjust the level.
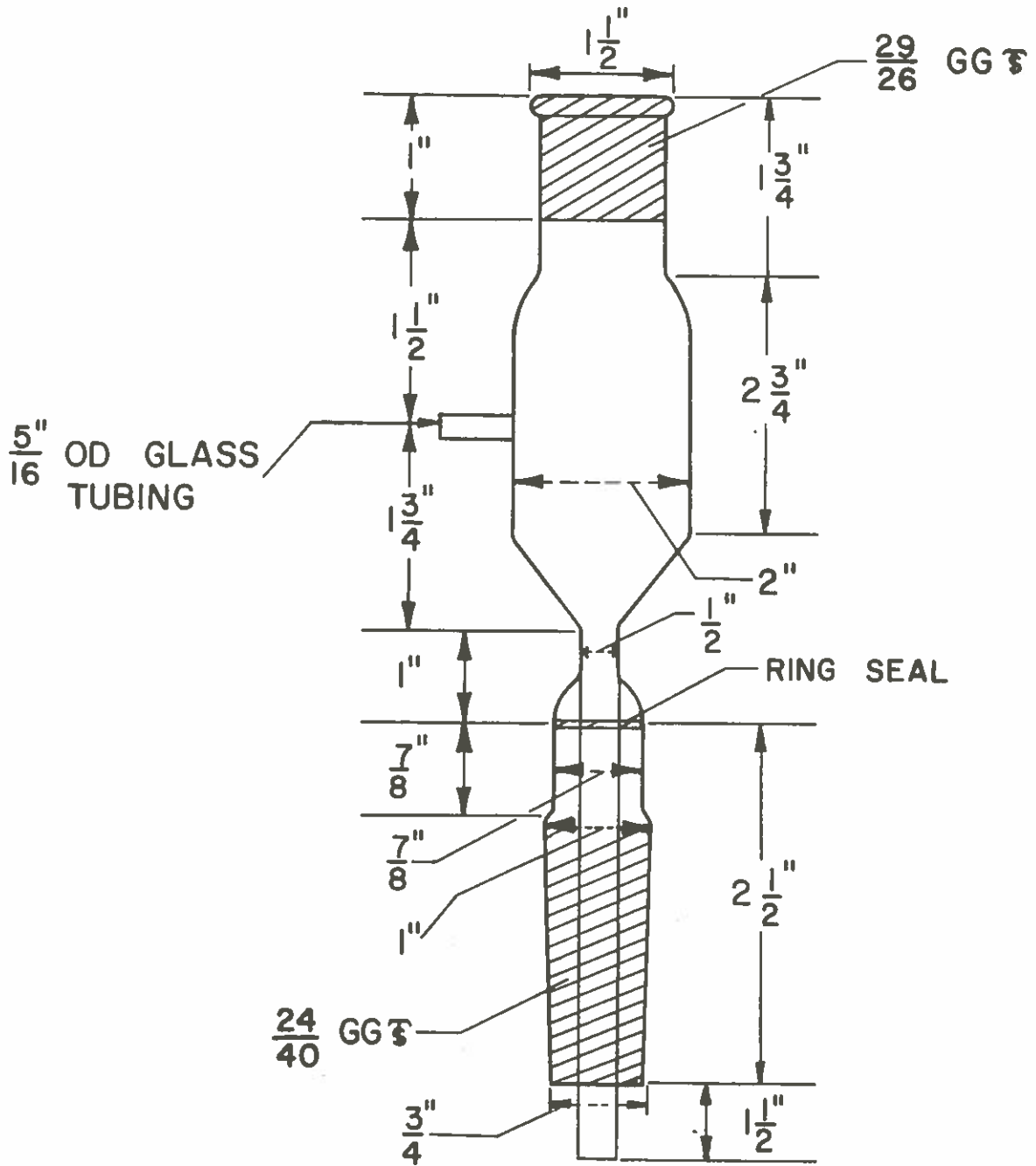
Figure A-2.   Vacuum Adaptor

104

Figure A-3a. Funnel Adaptor

2"

$\frac{1}{8}$"

1"

$1\frac{1}{2}$"

$\frac{29}{26}$ GG ⊺⌴

5"

$2\frac{1}{2}$"

$2\frac{1}{4}$"

$3\frac{1}{2}$"

1"

$\frac{1}{2}$"
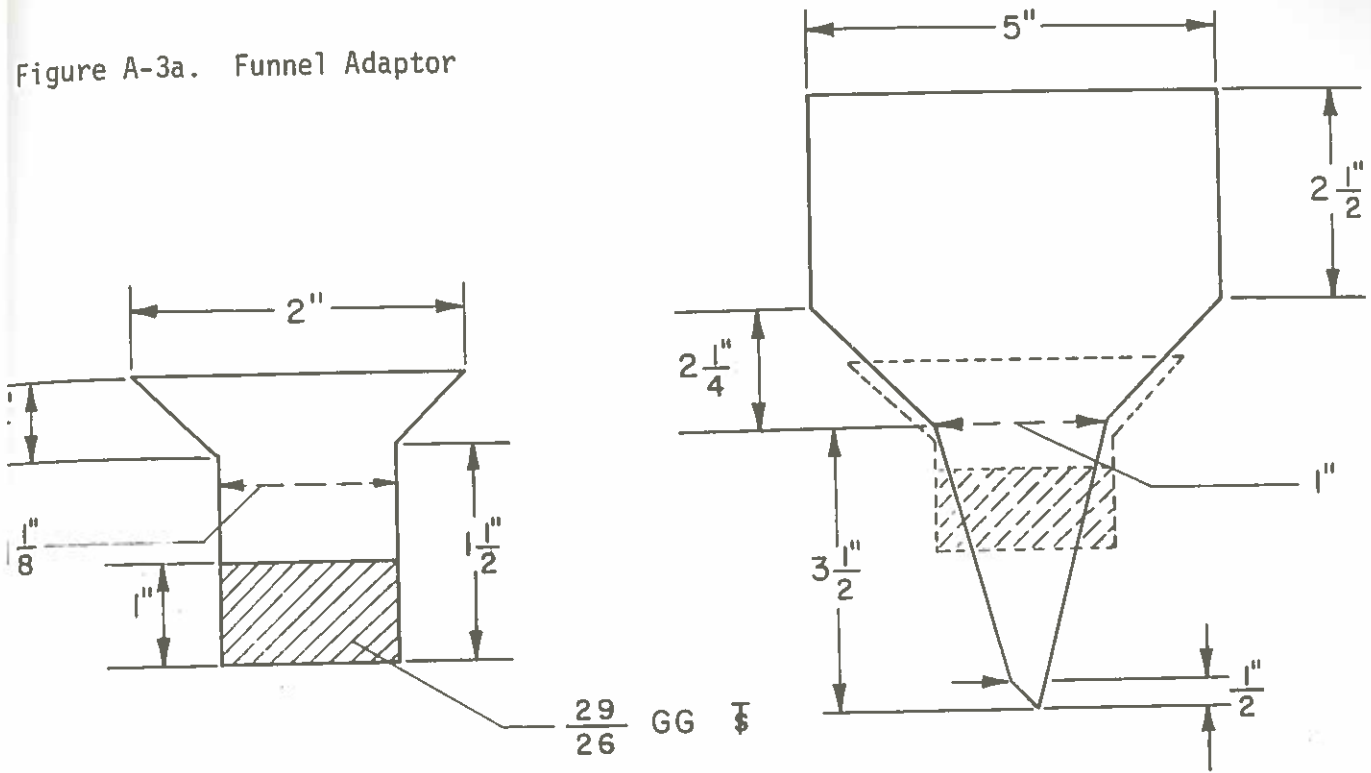
Figure A-3b. Adaptor in Place on Buchner
Funnel (seal with epoxy cement)

$\frac{7}{8}$"

$\frac{7}{8}$"

$\frac{1}{2}$"

$1\frac{1}{2}$"

$1\frac{1}{8}$"
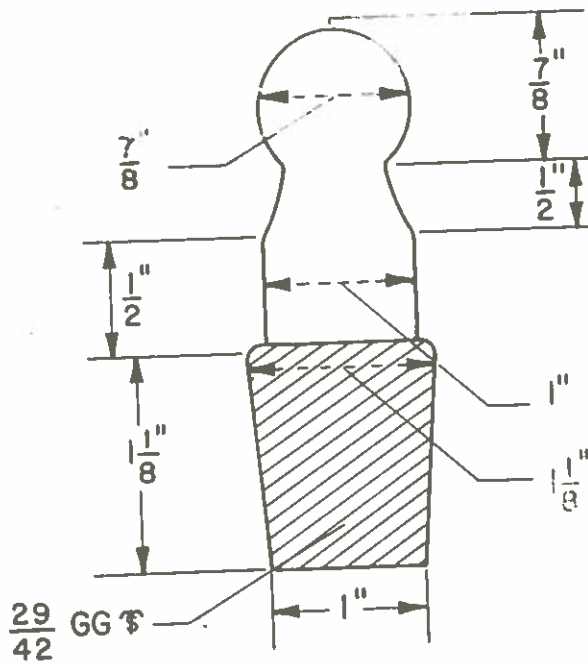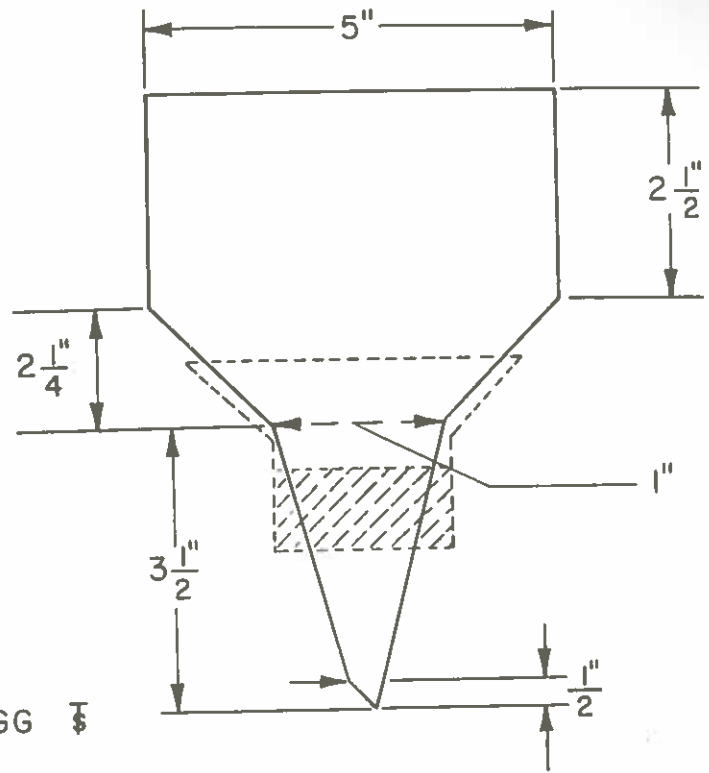
1"

$1\frac{1}{8}$"

$\frac{29}{42}$ GG ⌴

1"

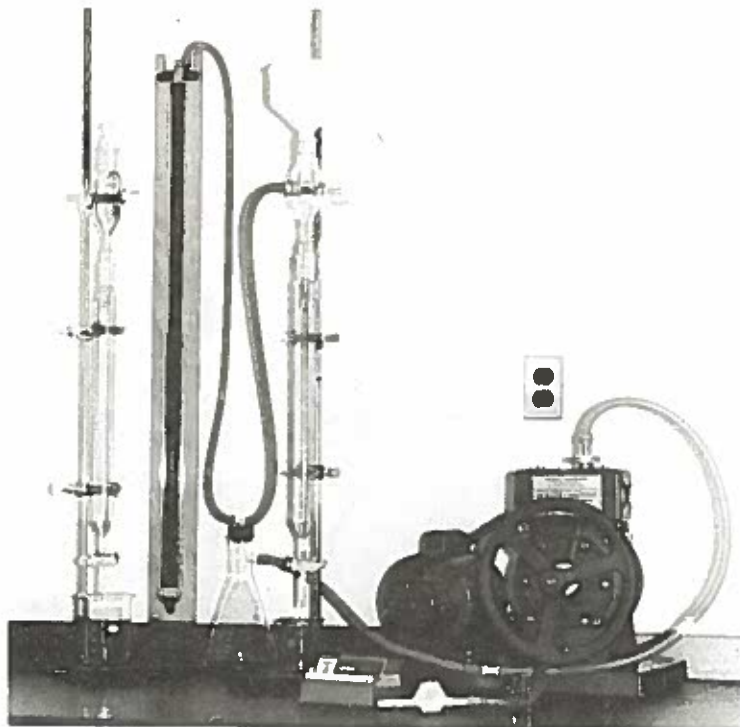Figure A-4. Vacuum Plug (optional)

Figure A-5. Buchner Funnel Apparatus set up for Testing (note the vacuum plug in place on the alternate equipment)
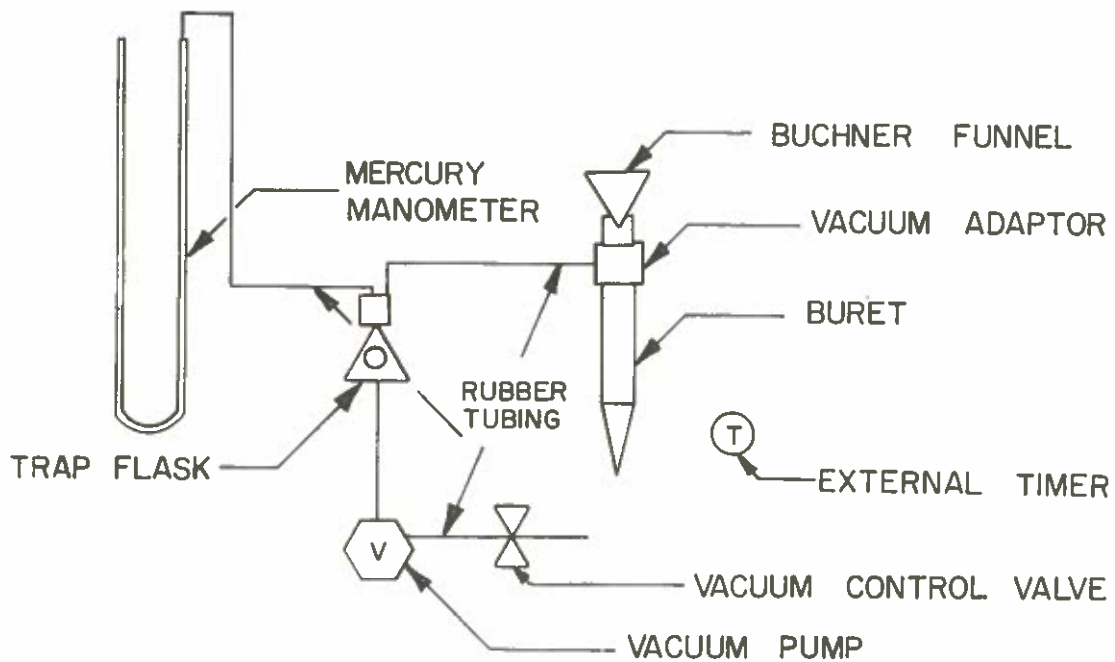

Figure A-6. Schematic Flow Diagram for Specific Resistance Testing.

c. Test Procedure.

(1) Determine the solids content in triplicate of the sludge prior to testing (see item A-1).

(2) Remove the Buchner funnel from the vacuum adaptor and insert the vacuum plug. By means of the pressure control selected, adjust the manometer to a constant pressure level. Since three separate tests will be run in the range from 10 cm Hg to 50 cm Hg, it is suggested that a low pressure test be run first to become familiar with the equipment.

(3) Wet the porous surface of the Buchner funnel and cover with Whatman No. 5 filter paper. Smooth out wrinkles and air bubbles, replace the funnel in the vacuum adaptor, and apply a vacuum to void the paper of excess water.

(4) Measure 100 ml of wet sludge into a graduated cylinder and apply to the filter surface of the Buchner funnel.

(5) Apply a vacuum to the system. Since there will be a time lag between the inception of the vacuum and the actual preset value, the timer is not actuated until the preset pressure value is reached. Record the initial volume on the data sheet shown in Figure A-7 when the timer is started. Record the pressure and room temperature at this time.

(6) Remove the Buchner funnel at the end of each test, record the depth of sludge cake, and perform triplicate residue (total) tests on the cake as outlined.

Date __2/1/68__

Specific Resistance
Sludge __Billerica__
Volume __100__ (ml)

Analysis No __3__
$T = 23\,^{\circ}C$
$P_{TOT} = 48.9\,Cm\,Hg$

Left __24.5__   Pressure __24.4__   Right

Time (Sec)	Volume (ml) Buret Read.	Cum. Filt. Vol	Time / Volume	Remarks
0	250.0	0	0	24.4 P 24.3 T= 23°C
30	245	5	6	
60	240	10	6	
90	237	13	6.9	24.5 P 24.4 Cake Glazed
120	234	16	7.5	
150	231	19	7.9	
180	229	21	8.6	
210	227	23	9.1	24.5 P 24.5
240	225	25	9.6	
270	223	27	10.0	Cake showing Lumps
300	221	29	10.3	Pres. Holding
330	219	31	10.6	
360	218	32	11.2	
390	216.5	33.5	11.6	Cake becoming more lumpy
420	215	35	12.0	
450	213.5	36.5	12.3	Pres. Holding
480	212	38	12.6	
510	211	39	13.1	Cake more lumpy
540	209	41	13.2	
570	209	41	13.9	
600	207.5	42.5	14.1	Bubbles forming on top
630	206	44	14.3	
660	205	45	14.7	End Test
				Cake depth = 0.4 cm

(right margin, vertical) Filtrate Golden Colored

$S_o = 4.65\,\%$ Cake Solids $\overline{AVE.}\ S_f = 8.25\,\%$

	A		B		C
wet pan & sludge	7.68310		8.86273		
dry pan (-)	1.44627		1.45186		
wet sludge	6.23683		7.41087		NOT RUN
dry pan & sludge	1.91531	%=	2.11810	%=	
dry pan (-)	1.44627	7.52	1.45186	8.96	
solids	0.46904		0.66624		

Figure A-7.   Sample Data and Data Format for Specific Resistance Test

d. Data Analysis.

(1) Subtract the succeeding buret readings from the
    reading at t=0   and record in the cumulative
    volume column (See Figure A-7).

(2) At each reading time, divide the time in seconds by
    the cumulative volume in ml.

(3) On Cartesian paper plot t/V as the ordinate versus V
    as the abcissa. The slope "b" may be directly
    determined from the line of best fit. Figure A-8
    shows the plot for the sample specific resistance
    data provided.

e. Calculation of Specific Resistance.

(1) Specific resistance may be calculated according
    to following equation (see text)

$$R = \frac{2bA^2 \Delta p}{\mu c}$$

where: R = specific resistance (sec^2/gm)

b = slope of t/V   vs. V   (sec/ml^2)

A = area of filter paper (cm^2)

Δp = test pressure (dynes/cm^2); Δp is taken as
the average test pressure in cm Hg X 13333.22 to
convert   to dynes/cm^2.

μ = dynamic viscosity (poise) or dyne·sec/cm^2

μ may be found in various handbooks or calculated
by the formula:

$$\mu = \frac{1.0}{2.1482[(t-8.435)+\sqrt{8078.4+(t-8.435)^2}]-120}$$

where t = test temperature (^{o}C)

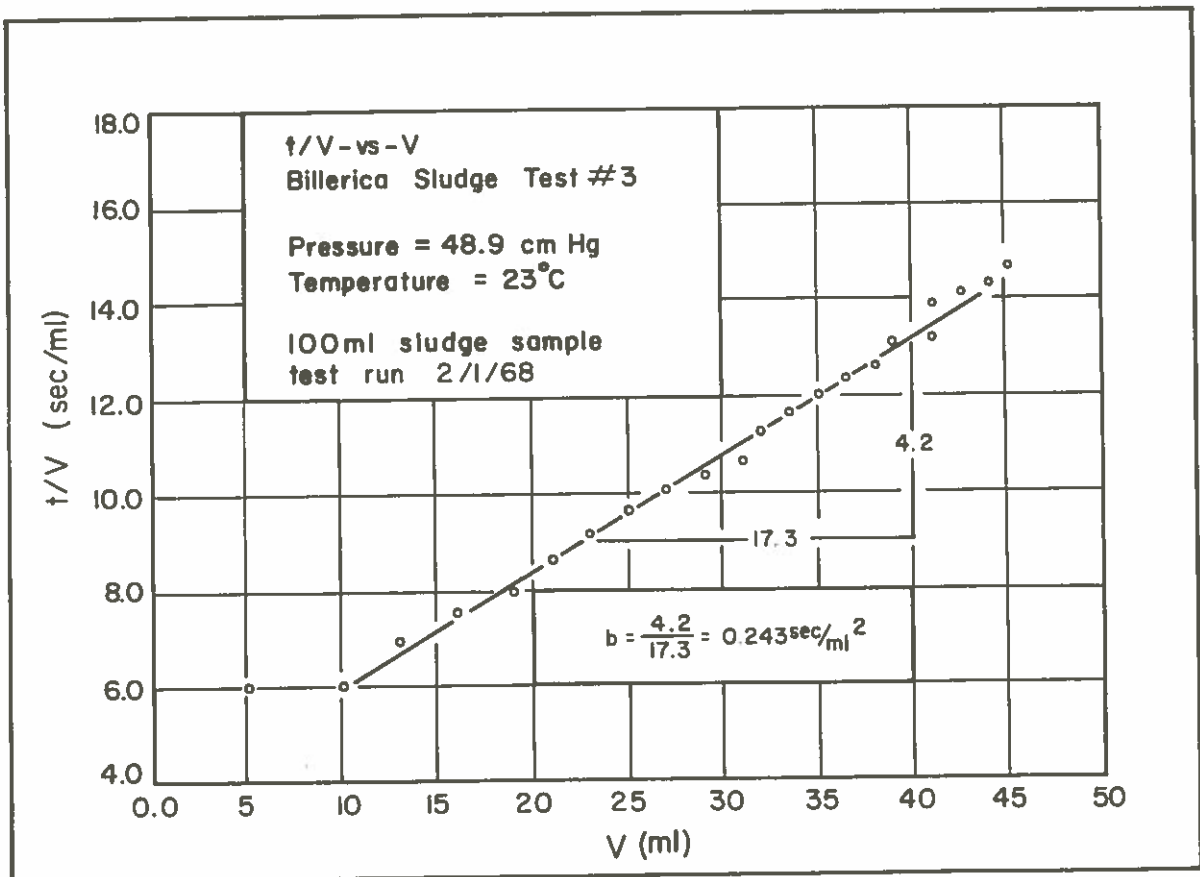Figure A-8.   Plot of Sample Specific Resistance Data.

c = weight of solids per unit volume of filtrate ($gm/cm^2 \cdot sec^2$), given by the formula:

$$c = \cfrac{\rho g}{\cfrac{100}{S_o} - \cfrac{100}{S_f}}$$

where: ρ = density of filtrate (assumed water) at the test temperature (gm/cm^3)

g = gravitational constant ($980.0 \ cm/sec^2$)

s_o = initial sludge solids content

s_f = sludge solids content after testing

(2) Sample calculation

b = 0.243 (see figure A-8)

diameter of filter paper = 11.1 cm

pressure = 48.9 cm. hg.

temperature = $23^0 C$

s_o = 4.65%

s_f = 8.25%

$$R = \cfrac{2(0.243)(9.364 \times 10^3)(48.9 \times 1.33 \times 10^4)}{(9.358 \times 10^{-3})(0.997538) \cdot (980)}$$
$$\left(\frac{100}{0.0465}\right) - \left(\frac{100}{0.0825}\right)$$

$$R = 3.05 \times 10^9 \ sec^2/gm$$

(3) The calculations of section F.1 easily lend themselves to a "Fast Fortran" computer solution as shown here.

```
5 PROGRAM SR
10 READ 100, RHO
15 READ 101, SO, SF
20 G=980.0
```

```
25 C=RHO*G/((1.0-S)/S-(1.0-SF)/SF)

30 READ 102, T

35 Z=1.0/(2.1482*((T-8.435)+SQRT(8078.4+(T-8078.4)**2))_
   120.0)

40 READ 103, B, P

45 A=96.77

50 Y=13332.2

55 R=(2.0*A**2*B*P*Y)/(Z*C)

60 PRINT 104, R

100 FORMAT (F10.7)

101 FORMAT (2F9.7)

102 FORMAT (F9.5)

103 FORMAT (2F8.4)

104 FORMAT (E13.6)

105 END

110 ENDPROG
```

The nomenclature is as follows: (see section F.1)

RHO*G = ϱ g

SO = s_o

ST = s_f

C = c

T = test temperature

Z = μ = dynamic viscosity

B = b

P = Δp

R = R

A = A

Y = conversion of Δp in cm Hg to Δp in dynes/cm^2

In this program, B is pre-calculated. However, least

squares calculation of B from the data in

Figure A-7 may be included.

f. Calculation of Coefficient of Compressibility.

It has been experimentally determined that specific

resistance is related to pressure by the following

equation:

$$R = a' (\Delta p)^{\sigma}$$

where R = specific resistance (sec^2/gm)

a' = cake constant (R@ Δp = 1)

Δp = test pressure (cm Hg)

σ = coefficient of compressibility (dimensionless)

(1) Before determining the coefficient of compressibility,

three or more specific resistance tests must be run,

as previously outlined, at pressures of about 10, 20,

and 50 cm Hg.

(2) Plot on bi-logarithmic paper each resistance ordinate

versus the pressure at which the resistance was run.

Determine the slope σ which is the coefficient of

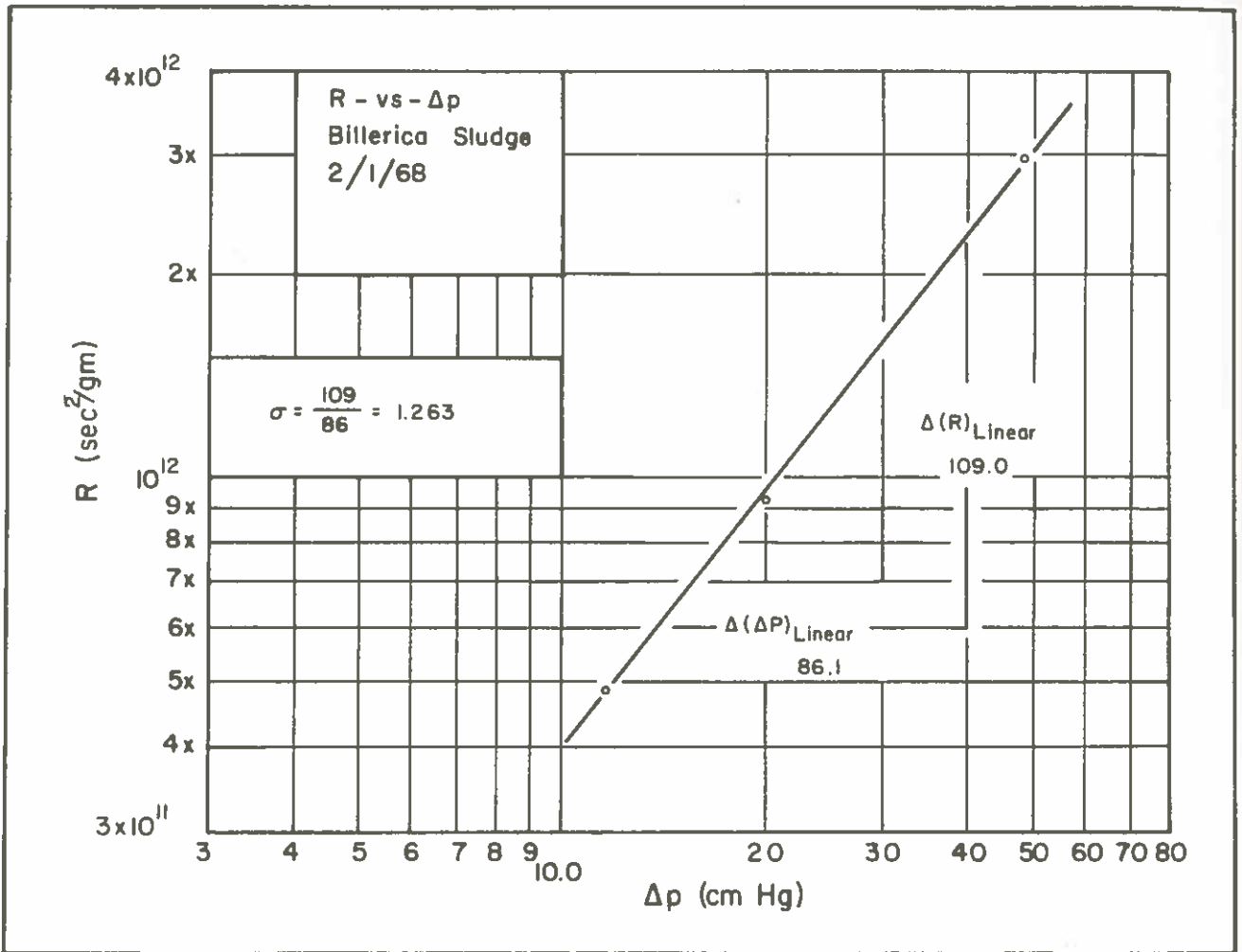compressibility. a' is not shown in Figure A-9.

Figure A-9.   Graph for Calculating Coefficient of Compressibility.

APPENDIX B.   RADIATION STATISTICS

STATISTICAL DESCRIPTION OF RADIOACTIVITY

From a statistical viewpoint some interesting properties of radio-activity can be described. Let the number N_0 be the population of atoms in a radioactive sample at the beginning of an observation period. Let m represent the number of atoms in the sample that disintegrate during the observation period t . Then the problem may be formulated as, "What is the probability of drawing exactly m radioactive atoms from a sample of N_0 during a period t , given that the probability of an atom being radioactive is p ?" Let W_m be the desired probability. The answer is given by the binomial probability law[1] as

$$W(m) = \frac{N_0!}{(N_0 - m)!\,m!}\; p^m\,(1-p)^{N_0 - m} \qquad \text{B-1}$$

The probability is 1-p that an atom will not decay during the observation period t . The probability of an atom surviving is determined from the observation that the decrease in the number of atoms of a radioactive substance during an observation period is proportional to the number of atoms present, $dN/dt = -\lambda N$ which integrates to

$$\frac{N}{N_0} = e^{-\lambda t} \qquad \text{B-2}$$

Where λ is a constant of proportionality having dimensions of time.$^{-1}$

Hence, the probability p of an atom decaying during the observation period is

$$p = 1 - \frac{N}{N_0} = 1 - e^{-\lambda t} \qquad \text{B-3}$$

Combination of Equations B-2 and B-3 with Equation B-1 produces the binomial probability law expressed in terms of decay properties.

$$W(m) = \frac{N_o!}{(N_o-m)!\,m!}\,(1-e^{-\lambda t})^m (e^{-\lambda t})^{N_o-m} \quad \text{B-4}$$

The average number of disintegrations during the observation period is given by the binomial probability law as $N_o p$. That is, the probability that any atom will decay during the observation period multiplied by the total number of atoms gives the average number of disintegrations. From this result and Equation B-3 the average number of disintegrations is

$$Average\ Number = N_o\,(1-e^{-\lambda t}) \quad \text{B-5}$$

For small values of λt it is convenient to express $e^{-\lambda t}$ as a Maclaurin series

$$e^{-\lambda t} = e^{-\lambda o} - \lambda t\, e^{-\lambda o} + \frac{(\lambda t)^2}{2!} e^{-\lambda o} - \frac{(\lambda t)^3}{3!} e^{-\lambda o} + \cdots$$

which reduces to

$$e^{-\lambda t} = 1 - \lambda t \quad \text{B-6}$$

for $\lambda t \ll 1$. Equation B-5 then becomes

$$Average\ Number = N_o\,\lambda t$$

and the average disintegration rate is

$$Disintegration\ Rate = \frac{Average\ Number}{t}$$
$$= N_o\,\lambda$$

However, the disintegration rate may also be expressed as $-\frac{dN}{dt}$ so $\frac{dN}{dt} = -\lambda N_0$ which agrees with Equation B-2.

In addition to knowing the average disintegration rate it is useful to know its variance. Consider the mean disintegration rate from Equation B-1

$$\sum_{m=0}^{m=N_0} m \, W(m) = \sum_{m=0}^{m=N_0} \frac{N_0!}{(N_0-m)!m!} \, m \, p^m (1-p)^{N_0-m}$$

This expression is similar to that obtained from the binomial theorem when expanding $(pX + [1-p])^{N_0}$ which results in

$$(pX + [1-p])^{N_0} = \sum_{m=0}^{m=N_0} \frac{N_0!}{(N_0-m)!m!} \, p^m X^m (1-p)^{N_0-m} \qquad \text{B-7}$$

Equation B-7 may be compared to Equation B-1. Recognize that

$$\sum_{m=0}^{m=N_0} \frac{N_0!}{(N_0-m)!m!} \, p^m X^m (1-p)^{N_0-m} = \sum_{m=0}^{m=N_0} X^m \, W(m) \qquad \text{B-8}$$

Hence:

$$(pX + [1-p])^{N_0} = \sum_{m=0}^{m=N_0} X^m \, W(m) \qquad \text{B-9}$$

The derivative of this expression is

$$N_0 \, p \, (pX + [1-p])^{N_0-1} = \sum_{m=0}^{m=N_0} m X^{m-1} \, W(m) \qquad \text{B-10}$$

It is interesting to note that when x=1 this result constitutes proof of
the statement that the average number of disintegrations during the observation
period is $N_0 p$.

The second derivative of Equation B-9 is

$$N_o (N_o-1) p^2 (pX+[1-p])^{N_o-2} = \sum_{m=0}^{m=N_o} m(m-1)X^{m-2} W(m) \quad \text{B-10 a}$$

This may be evaluated when x=1 to obtain

$$N_o(N_o-1) p^2 = \sum_{m=0}^{m=N_o} m(m-1) W(m) \quad \text{B-11}$$

which becomes

$$N_o(N_o-1)p^2 = \sum_{m=0}^{m=N_o} m^2 W(m) - \sum_{m=0}^{m=N_o} m W(m) \quad \text{B-12}$$

The two terms on the right bear closer scrutiny. The second term is the
average value of m while the first expression is the average value of m^2 .
Equation B-12 may then be written

$$N_o (N_o-1) p^2 = \overline{m}^2 - \overline{m} \quad \text{B-13}$$

This expression is similar to that obtained by expanding the expression
for the variance of statistical data ^{2}

$$\sigma_x^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2 = \frac{n}{n-1} \left\{ \frac{1}{n} \sum_{i=1}^{n} (X_i - \overline{X})^2 \right\} \quad \text{B-14}$$

$$= \frac{n}{n-1} \left\{ \frac{1}{n} \sum_{i=1}^{n} (X_i^2 - 2X_i\overline{X} + \overline{X}^2) \right\}$$

$$= \frac{n}{n-1} \left\{ \overline{X_i^2} - 2\overline{X}\,\overline{X} + \overline{X}^2 \right\}$$

$$\sigma_x^2 = \frac{n}{n-1} \left\{ \overline{X_i^2} - \overline{X}^2 \right\} \quad \text{B-15}$$

Let $n = N_0$ and $x=m$ so

$$\sigma_m^2 = \frac{N_0}{N_0-1}\left(\overline{m_i^2} - \overline{m}^2\right)$$

and for $N_0 \gg 1$

$$\sigma_m^2 = \overline{m_i}^2 - \overline{m}^2 \qquad\qquad \text{B-16}$$

which results in

$$\overline{m}^2 = \sigma_m^2 + \overline{m}^2$$

This may be substituted into Equation B-13 to obtain

$$N_0(N_0-1)\,p^2 = \sigma_m^2 + \overline{m}^2 - \overline{m} \qquad\qquad \text{B-17}$$

From Equation B-10a $\bar{m} = N_0 p$ so

$$N_0(N_0-1)p^2 = \sigma_m^2 + N_0^2 p^2 - N_0 p$$

which gives

$$\sigma_m^2 = N_0\, p\,(1-p) \qquad\qquad \text{B-18}$$

Combination of Equations B-3 and B-18 yields

$$\sigma_m^2 = N_0\,(1-e^{-\lambda t})\,e^{-\lambda t}$$

which can be simplified through Equation B-6 to

$$\sigma_m^2 = N_0\,(\lambda t)(1-\lambda t) \qquad\qquad \text{B-19}$$

Now $N_0 \lambda t$ was shown previously to be the average number of disintegrations taking place during the observation period so Equation B-19 becomes

$$\sigma_m^2 = Average\ Number\ of\ disintegrations\ (1-\lambda t)$$

and when $\lambda t \ll 1$ this becomes σ_m^2 = Avg. No. of disintegrations during observation period. In terms of the counting rate this is,

$$\sigma = \sqrt{Counting\ rate} \qquad \text{B-20}$$

Under some quite general assumptions the binomial frequency distribution of radioactive decay, given by Equation B-1 can be reduced to the much more easily handled Poisson frequency distribution. The restrictions are (1) $\lambda t \ll 1$ (2) $N_0 \gg 1$ and (3) $N_0 \gg m$. For 250 mc of C_s^{137} observed during a one minute period the values of the terms are:

(1) $\lambda t = 4.00 \times 10^{-8}$ (2) $N_0 = 1.39 \times 10^{18}$ and (3) $\frac{m}{N_0} = 3.99 \times 10^7$

so the restrictions are well satisfied. Under these restrictions the binomial distribution given by Equation B-1 can be reduced to the Poisson distribution [3]

$$W(m) = \frac{M^m e^{-M}}{m!} \qquad \text{B-21}$$

where M is the expected number of disintegrations during the counting period. The Poisson distribution[4] has the property that its mean is M and its variance is M . For situations in which m is greater than about 10, M can be approximated by m with a negligible error.

Then the standard deviation is

$$\sigma = \sqrt{m} \qquad \qquad \text{B-22}$$

which agrees with the result obtained from the binomial distribution and expressed in Equation B-20.

The fractional standard deviation has a particularly simple form for the Poisson distribution. By definition

$$F.\,S.\,D. = \frac{\sigma}{m} = \frac{\sqrt{m}}{m} = \frac{1}{\sqrt{m}} \qquad \qquad \text{B-23}$$

This result indicates that if m were 100 random events the fractional standard deviation would be $\frac{1}{10}$, or 10%. In order to reduce the fractional standard deviation to 1% it would be necessary to count 10,000 random events. The consequence of dealing with random processes and extracting information from data obtained from observations of the process was pointed out by Evans[5] who said, "No mere method of treating the same total data can ever reduce the magnitude of the fractional uncertainty due purely to randomicity." Thus, if a series of repeated readings of some event are taken, the expectation of the mean, M, can be better approximated the more readings there are, but the fractional standard deviation of the expectation of the mean will still be $1/\sqrt{M}$.

Frequently it is necessary to work with the quotient of counting rates. The question arises of what to consider as the variance of the quotient.

If the expected value of one reading is M_1 and its variance is $\sigma_1{}^2$, while the second reading has an expected value of M_2 and a variance of $\sigma_2{}^2$, then the corresponding fractional standard deviations are σ_1 / M_1 and σ_2 / M_2, respectively. The quotient of M_1 / M_2 has a standard deviation σ_Q. The root mean square of the fractional standard deviations of M_1 and M_2 can be shown to be the fractional standard deviation of the quotient[6]. That is,

$$\frac{\sigma_Q}{M_1 / M_2} = \sqrt{\left(\frac{\sigma_1}{M_1}\right)^2 + \left(\frac{\sigma_2}{M_2}\right)^2} \qquad \text{B-24}$$

and as the measured counting rates closely approximate the expected counting rates, the standard deviation of the quotient may be expressed as

$$\sigma_Q = \frac{m_1}{m_2} \sqrt{\left(\frac{\sigma_1}{m_1}\right)^2 + \left(\frac{\sigma_2}{m_2}\right)^2}$$

In addition, the standard deviations of m_1 and m_2 are given by Equation B-22 for a Poisson distribution so that

$$\sigma_Q = \frac{m_1}{m_2} \sqrt{\frac{1}{m_1} + \frac{1}{m_2}} = \frac{m_1}{m_2} \sqrt{\frac{m_1 + m_2}{m_1 m_2}} \qquad \text{B-25}$$

APPLICATION TO THE SCALER

In the absence of dead time losses in the detector, it may be assumed that the scaler records a fixed percentage of the radioactive disintegrations which take place. The reading obtained from the scaler can then be treated as the observed rate m and the statistics which have been developed apply directly to the scaler readings. In particular, for each reading of the scaler the expected standard deviation is given as

$$\sigma = \sqrt{m} \qquad \text{B-22}$$

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