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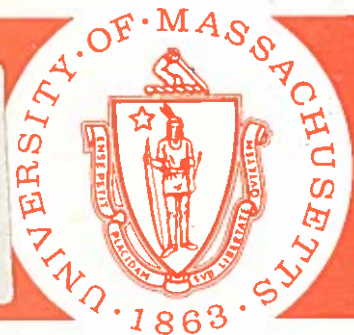
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# SLUDGE DEWATERING BY CENTRIFUGATION

Donald L. Ray  
John H. Nebiker  
Donald Dean Adrian

Research Report No. 2  
for the  
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ENVIRONMENTAL ENGINEERING  
DEPARTMENT OF CIVIL ENGINEERING  
UNIVERSITY OF MASSACHUSETTS  
AMHERST, MASSACHUSETTS

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**By**

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## LIST OF SYMBOLS

The following symbols are in the order in which they are used in the subject presentation.

- $F_c$  = centrifugal force    M    L    T<sup>-2</sup>  
 $M$  = mass M  
 $u_i$  = instantaneous velocity    L    T<sup>-1</sup>  
 $r$  = radius or distance from center of curvature    L  
 $\omega$  = angular velocity T<sup>-1</sup>  
 $(m_p - m_l)$  = mass of particle minus mass of liquid M  
 $D_p$  = diameter of Particle L  
 $(\rho_p - \rho_l)$  = density of particle minus density of liquid    M L<sup>-3</sup>  
 $F_s$  = centripetal force    M    L    T<sup>-2</sup>  
 $u_t$  = terminal velocity    L    T<sup>-1</sup>  
 $\mu$  = viscosity    M    L<sup>-1</sup>    T<sup>-1</sup>  
 $s$  = depth of pool liquid    L  
 $t$  = detention time    T  
 $\bar{V}$  = volume    L<sup>3</sup>  
 $Q$  = flow rate    L<sup>3</sup> T<sup>-1</sup>  
 $g$  = acceleration of gravity    L    T<sup>-2</sup>  
 $C$  = clarification capacity    M    L    T<sup>-1</sup>  
 $g_s$  = slippage force    M    L    T<sup>-2</sup>  
 $r_2$  = radius to bowl wall    L  
 $r_1$  = radius to surface of pool liquid    L  
 $r_m$  = radius to mean pool depth    L



$L$  = length of bowl       $L$

$L_1$  = length of cylindrical portion of bowl       $L$

$L_2$  = length of conical portion of bowl       $L$

$u_H$  = horizontal velocity of flow through the centrifuge       $L \ T^{-1}$

$A$  = cross-sectional area of pool volume       $L^2$

$b$  = slope of a particle's path when the particle flows and settles simultaneously

$R_s$  = residence time of solids on the beach       $T$

$C_s$  = relative conveyor speed       $L \ T^{-1}$

$B$  = bowl speed       $T^{-1}$

$P$  = conveyor pitch       $L$

$G$  = gear ratio

$T$  = thickness of cake at discharge       $L$

$P$  = rate of wet cake discharge       $M \ T^{-1}$

$D_c$  = density of wet cake       $M \ L^{-3}$

## CHAPTER I

## INTRODUCTION

Problems of Sludge Disposal

A major role of wastewater treatment plants is the separation of solids from their liquid environment. This separation is accomplished by sedimentation or filtration in various treatment operations. The solids removed have a loose structural form with large pore spaces which contain liquid. This combination of solids and liquid is referred to as sludge. It constitutes a much larger volume than that of the solids alone.

The problem of how to dispose of this resulting sludge is increasing in complexity. An increasing population is being sewered, and a higher degree of treatment is projected; both creating larger amounts of sludge requiring disposal (1). Simultaneously, population increase has resulted in higher land values on which sewage treatment plants are located, and in many cases, land is no longer economically available to use for land disposal of sludge by filling or spreading.

The disposal of sludge is also complicated by inadequacies of past research in producing guidelines to be used by plant designers and operators. Hence, trial and error methods have usually determined the optimum procedure to be used (2).

A major factor which adds to the problem is the large variation in type of sludge. This variation is due to the type and efficiency of waste treatment employed, the geographical area in which the waste originates, and the presence or absence of industrial wastes and storm drainage.

Efforts to reduce the costs of sludge disposal center on volume reduction of the sludge by such processes as gravity thickening, vacuum filtration, lagooning, centrifugation, drying, and incineration. The choice of process depends on the desired end product - whether a moist or dry cake, or ash. Increasingly, engineers are seeking an ash product, as it is biologically stable and minimal in volume. It is to be noted that sludges cannot self-sustain their combustion unless first dewatered to about 15-20% solids, thus, vacuum filtration or centrifugation is required.

The use of centrifuges is gaining in popularity. From all indications, this is due to the availability of polyelectrolytic coagulants which reduce the solids lost in the centrate. In spite of greater popularity, centrifuge selection and operation remains an art and not a science, and it appears at the present time that consulting engineers and regulatory officials are totally dependent in selection of centrifuges on the advice of the centrifuge manufacturers.

The purpose of this paper is to review the state of the art regarding centrifugal dewatering of wastewater sludges, and to indicate avenues of approach for rationalization of selection and operation of centrifuges.

## CHAPTER II

## CONCEPTS OF CENTRIFUGATION

Centrifugal Force

Centrifuges makes use of the familiar principle that an object traveling a curved path is acted on by a force. This force is referred to as the centrifugal force and is exerted in the direction away from the center of curvature of the path. The centripetal force is the force applied to the object in the direction of the center of curvature which causes the object to travel in a circular path.

As indicated in Figure 1, the centrifugal force is:

$$F_c = \frac{m u_i^2}{r} \quad (\text{Eq.1})$$

Where:  $F_c$  = centrifugal force      M      L      T<sup>-2</sup>  
 $m$  = mass      M  
 $u_i$  = instantaneous velocity      L      T<sup>-1</sup>  
 $r$  = radius or distance to center of curvature      L

Equation 1 is frequently expressed in terms of angular velocity. Noting that:

$$\omega = \frac{u_i}{r} \quad (\text{Eq.2})$$

Where:  $F_c = mr\omega^2$   
 $\omega$  = angular velocity      T<sup>-1</sup>

Stokes' Law

The laws controlling the movement of discrete particles through a continuous fluid phase have been defined by Newton, Stokes, Svedberg and others. A single solid particle settling due to the attraction of gravity, accelerates until a constant terminal velocity  $u_t$  is reached. At this point, the two forces acting on the particle, namely that force

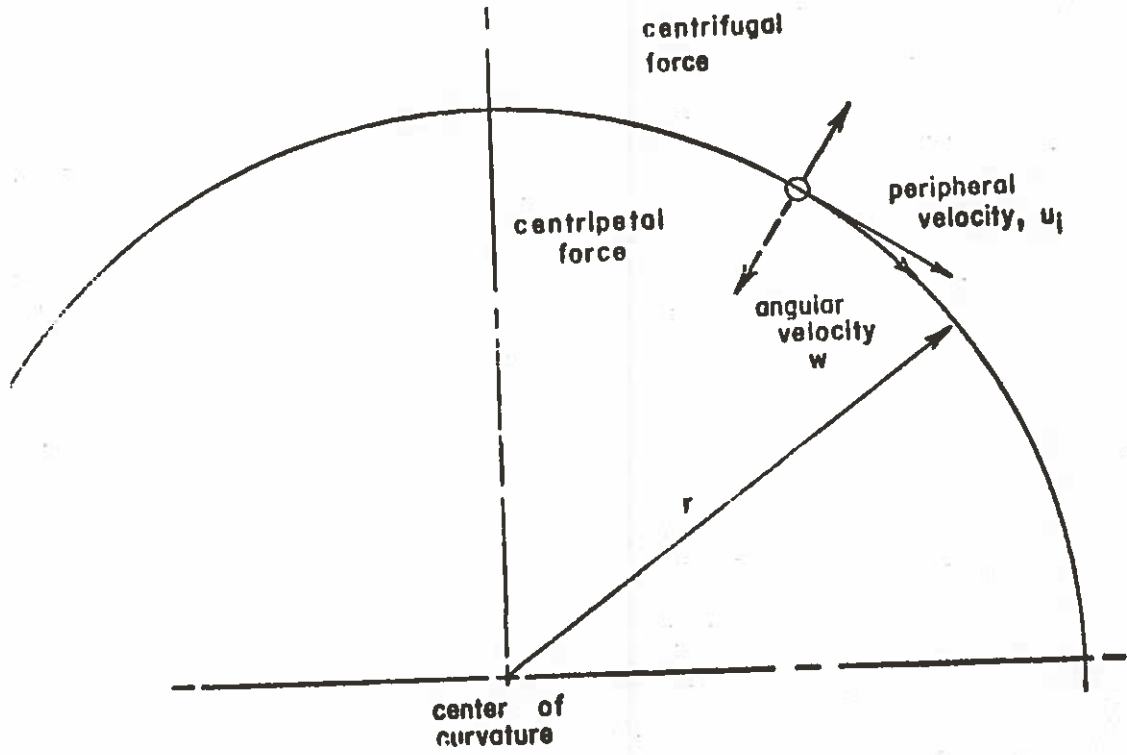


Figure 1: Model for the analysis of centrifugal force.

resulting from gravitational acceleration, and the force resulting from frictional drag of the surrounding medium, are equal in magnitude.

In a centrifuge, the particle may be considered to settle under the acceleration of a centrifugal force ( $m\omega^2 r$ ) rather than under the gravity force ( $mg$ ) because the centrifugal force is many times larger than the gravity force. Because the centrifugal force is always increasing as the particle moves away from the center of curvature, the particle never reaches a true terminal velocity. However, at any given radial distance ( $r$ ) its settling velocity is very nearly given by the Stokes' Law relation. The effective force acting on a particle is:

$$F_c = (m_p - m_f)\omega^2 r \quad (\text{Eq.3})$$

Here ( $m_p - m_f$ ) is the mass difference between particle and fluid or buoyant mass.

For a sphere:

$$F_c = \frac{\pi D_p^3 (\rho_p - \rho_f)\omega^2 r}{6} \quad (\text{Eq.4})$$

$\rho_p$  is the mass density of the particle of diameter  $D_p$ , and  $\rho_f$  is the mass density of fluid. The drag force opposing sedimentation by Stokes' Law is:

$$F_s = 3\pi\mu D_p u_t \quad (\text{Eq.5})$$

So at equilibrium:

$$F_c = F_s$$

and

$$u_t = \frac{(\rho_p - \rho_f) D_p^2 \omega^2 r}{18\mu} \quad (\text{Eq.6})$$

where  $u_t$  is the velocity of the particle outward at equilibrium or the terminal velocity.

Consider the case where the settling takes place in a rotating cylindrical tube of radius,  $r$ . If the liquid has a depth,  $s$ , around the circumference of the cylinder, the distance,  $x$ , which a particle will settle is;

$$x = u_t \cdot t$$

With a constant inflow and outflow of fluid,  $t$ , is the detention time:

$$t = \bar{V}/Q$$

Where:  $\bar{V}$  = the volume of liquid in the bowl

$Q$  = flow rate  $L^3 T^{-1}$

$$x = u_t \cdot t = \frac{(\rho_p - \rho_f) D_p^2 \omega^2 r}{18\mu} \cdot \frac{\bar{V}}{Q} \quad (\text{Eq.7})$$

Now the flow rate at which one-half of the solid particles will be removed ( $Q_c$ ) is given for:

$$x = s/2 \quad (\text{if } s \ll r)$$

$$\text{by: } Q_c = \frac{(\rho_p - \rho_f) D_p^2 \omega^2 r \bar{V}}{9\mu \cdot s} \quad (\text{Eq.8})$$

Also with a given flow rate,  $Q$ , the critical diameter of particles is given by:

$$D_{pc} = \sqrt{\frac{9Q \mu \cdot s}{(\rho_p - \rho_f) \bar{V} \omega^2 r}} \quad (\text{Eq.9})$$

This is interpreted to say that most particles with diameters larger than  $D_{pc}$  will be removed by the centrifuge, and most particles with smaller diameters will appear in the effluent.

In each of these equations (8 and 9), the calculations are based on the behavior of a single discrete particle under conditions of unhindered settling, and the assumption that the particle is always in equilibrium with the force field of the centrifuge under conditions defined by Stokes' Law.

In practice the results predicted by Stokes' Law are seldom achieved for the following reasons (3):

\*In most waste systems, the migration of any single particle is interfered with by the smaller particles ahead of it, so hindered settling occurs.

\*A particle moving through the centrifuge is constantly passing through a zone of higher force by which it is being accelerated and its velocity is something less than that calculated for equilibrium conditions.

\*The law of conservation of momentum predicts that the rotational velocity of the particle, if it is relatively heavy, will lag behind the rotational velocity of the centrifuge and the fluid in it.

\*Wastewater particles flocculate when brought into close proximity with each other; thus, the diameter of the particle is variable with time.

\*A portion of the energy imparted to the feed stream in bringing it up to the rotational velocity of the centrifuge may create further subdivision of the particles to be separated, reducing the effective value of the diameter of the particle, particularly, if these particles are of low mechanical stability such as activated sludge.



## CHAPTER III

## HISTORY OF CENTRIFUGES

Early History (1902-1934)

Herman Schaefer was the first to conduct tests on the possibility of using centrifugal force to dewater sludge in Cologne, Germany, as early as 1902 (4). Dr. Gustav Ter Mer joined forces with Schaefer in 1907, and they began testing a perforated basket machine. After improvements had been made, the machine was sold to a number of European cities such as Hamburg, Frankfurt, and Moscow. These machines all operated on a batch basis.

The first machine to be used in the United States was an imperforated basket type installed in 1920 at Milwaukee, Wisconsin (4). The unit thickened activated sludge, but was found to be incapable of removing a sufficiently high per cent of solids to be acceptable. Baltimore experimented with the imperforated basket machine during the years 1921-1925, using primary digested sludge, and this resulted in a solids recovery of 65%, and a cake solids content of 29%.

It was not until 1934 that any additional testing of centrifuges was reported. The American Centrifugal Company began testing an improved imperforated basket type at Collingsworth, New Jersey (5). The results were similar to previous attempts with solid cake concentration excellent and solid recovery still poor, being 75% of the total incoming solids. The performance of each of these units was unacceptable due to the low solids recovery, but the concentration of solids in the cake held promise for the future.

Intermediate History (1935-1954)

Two new types of centrifuges were introduced during the 1930's. One was a Bird solid bowl unit which could be fed continuously and which discharged the solids without

stopping the rotation of the bowl. Dorr Company ran tests on this machine using it to dewater raw and digested sludge at Rahway, New Jersey and New Haven, Connecticut, and also for dewatering water softening sludge at Cedar Rapids, Iowa (6). Again, it was found that solids concentration was good, but centrate solids were too high to permit economically feasible operation.

The other type machine was produced by DeLaval in 1937. This unit was a disc-type valve centrifuge and was used to thicken activated sludge in Peoria, Illinois (7). Sludge concentration was increased from 1% to 4.2% with 75% solids recovery. The problem with this type of machine was plugging of the openings and low capacity. In 1950, DeLaval introduced an improved disc nozzle machine at Sioux Falls, South Dakota (8), to again thicken activated sludge. This unit concentrated the waste activated sludge to 5.2% solids while recovering 97% of the feed solids. Several more of these units were installed in Sioux Falls, but due to high maintenance costs, use of this type centrifuge for thickening activated sludge was temporarily stopped.

In 1959, Sharples tested a nozzle type centrifuge on a thickened activated sludge waste containing approximately 2% suspended solids (9). Plugging of the nozzles was an early problem, but a successful procedure to screen the influent was devised and several week-long runs were achieved without difficulty. Solids concentration in the underflow was 5 to 10%, and solids recovery amounted to 75 to 90% without the use of chemicals. Woodruff, et al (10), reported one application of this type of centrifuge to thicken activated sludge prior to the addition of primary clarifier underflow, with final dewatering in a solid bowl centrifuge.

#### Recent History (1954-1967)

It is generally agreed that the modern age of dewatering municipal sewage sludge by centrifugation began in 1954, with the installation of a Bird solid

bowl type unit at Daly City in San Mateo County, California (11). Centrifugation was used because of concern about odors, for the plant was located in a densely populated area and in close proximity to a fully developed residential zone and city playground. Thus, a housed centrifuge was preferred rather than sludge drying beds.

In the ensuing year a large number of these units were used to dewater digested sludge, but tests revealed that solids recovery was still low, being 50 to 70%. Difficulties with solids build-up arose in some cases where the centrate was recycled to the head of the treatment plant (11). The cake concentration was found to be very good with ranges from 20% to 35%.

Although the use of centrifuges was increasing, its applicability was still limited. Not until improved machine design and advanced technology resulted in a clearer centrate did usage become widespread. These improvements were in the form of a shallower bowl angle and larger length to diameter ratios. Another aspect which greatly improved the performance of centrifuges was the development of organic conditioning agents such as polyelectrolytes or polymers. Further development in each of these areas will provide even greater opportunities for centrifuges to be used.

As of December, 1966, the major manufacturers of centrifugal equipment reported a total of approximately 233 installations of centrifuges processing sludge originating from municipal and industrial waste treatment plants. Of these 78 were used by municipal plants (9).

## CHAPTER IV

## DESCRIPTION OF CENTRIFUGES

Types of Centrifuges

A wide variety of centrifuges have been developed for different applications in the process industries. These centrifuges are utilized according to their operating characteristics which best fit the process needs of washing, dewatering, classification or clarification, or a combination of these factors. Fig. 2 shows the three general types of centrifuges employed and their field of application. These units are classified according to the type of revolving container employed, such as "basket type" with imperforated or perforated walls, a "disc-type" with nozzle discharge, and a "solid bowl" unit.

Basket Centrifuges: The basket type centrifuges with perforated and imperforated walls were used initially. The perforated basket centrifuge is used mainly where the process requires washing or drying of uniform coarse solids such as sugar crystals. The perforated unit is not used where classification or clarification is required (6). The spin dryer of a domestic washing machine is a perforated basketed centrifuge.

The imperforate wall unit performs in just the opposite manner and can produce a highly clarified or classified product by skimming, but its dewatering capability is only fair. It is often used with dilute concentrations of fines, such as removal of biological cultures from pharmaceutical process streams (6). Both perforated and imperforated type units operate on a semi-continuous basis.

Disc Centrifuge: This type of centrifuge is widely used in the processing industries to make liquid-liquid, liquid-solid, and liquid-liquid-solid separations. Typical applications would be cream-milk separation, oil-water-solids separation, thickening of starch, and clay-silt classification. The high centrifugal forces developed make this an excellent clarifying unit, and permit it to capture and to


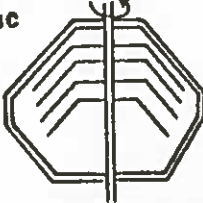

Centrifuge	Dewatering	Classification	Clarification	Application
Basket  perf. imperf.	very good	poor	poor	coarse solids
Disc 	poor	good	very good	fine solids
Solid Bowl 	good	fair	fair to good	fine & coarse solids

Figure 2: General Types of centrifuges (6).

polish slurries containing very fine solids (6).

The slurry is fed to the center of the centrifuge and is subjected to a high centrifugal force. The material with the higher specific gravity is forced to the surface of the disc and slides down the underside of the disc to the bowl surface. Here the solids are compacted by centrifugal force and finally discharged through nozzles located around the bowl. The clarified effluent which has a lower specific gravity is drawn upward through the disc stack until it is discharged over a weir at the top of the bowl (10).

This type of centrifuge has been used to thicken activated sludge in some cases (12). It is not suitable for use with primary or digested sludge because the solids will plug the discharge nozzles. It does not perform satisfactorily as a dewatering unit, but it can be operated on a continuous basis.

Solid Bowl-Conveyor Centrifuges: This solid bowl unit combines the best features of the other types of centrifuges. It is an excellent dewatering unit and classifier, and a good clarifier. It can also operate on a continuous basis. Some of its typical applications are: drilling mud classification, coal dewatering, antibiotic clarification, lime mud classification, dewatering of pulp and paper mill wastes, and dewatering sewage sludge (6).

#### Types of Solid-Bowl Centrifuges

There are three general shapes of the bowl for solid bowl centrifuges: conical, cylindrical, and conical-cylindrical, as shown in Figure 3. The type used in industrial processes depends on the characteristics of the slurry and solids, and the specific process needs.

The conical solid-bowl conveyor is an excellent classifying and dewatering device, but it cannot efficiently handle fine hydrous solids. For example, the conical solid-bowl conveyor may be used for classifying magnesium hydroxide from carbonate sludges, and to dewater the carbonate to 50-70% total solids (6).

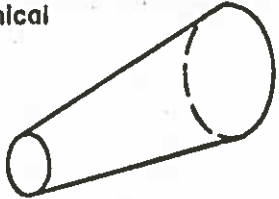
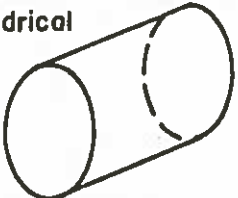
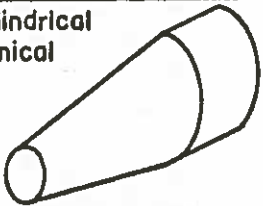
Centrifugal Type	Dewatering	Classification	Clarification
<b>Conical</b> 	good	good	poor
<b>Cylindrical</b> 	poor	fair	very good
<b>Cylindrical Conical</b> 	good	fair to good	good

Figure 3: Types of solid bowl centrifuges (6).



The cylindrical solid-bowl unit is used where good clarification is desired and cake dryness is not too important. It will remove most of the fine solids, especially those which are quite hydrous (6).

The cylindrical-conical unit combines the good aspects of both cylindrical and conical units, permitting efficient dewatering and effective clarification in the same unit. It is this shape with a long "bowl length" and shallow "bowl angle" which has proved to be the most efficient unit for use in sludge dewatering (6).

#### Description of Solid-Bowl Centrifuges

The solid bowl centrifuge (figures 4 & 5) consist of a rotating cylindrical-conical shell, supported between bearings, and an internally contoured screw conveyor supported by trunnions at each end. The screw conveyor rotates at a slightly different speed from the bowl by means of an external planetary gear assembly, also supported by one of the trunnions. At the cylindrical end of the bowl is located a dam or weir to regulate the level of liquid in the bowl. Other essential components include a steel base plate, rugged pillow block bearings, and an induction type motor connected through a V-belt drive assembly (9).

The feed slurry enters the bowl through a hollow shaft which is part of the screw conveyor, and is distributed through ports to the bowl pond. The pond takes the form of a concentric annular ring of liquid on the inner wall of the bowl. The depth of the pool is adjusted by weirs at the large end of the bowl. The centrifugal force causes the heavier particles to settle toward the surface of the bowl wall. As the liquid flows toward the centrate discharge, progressively smaller particles are removed (9).

The solids are then pushed by the screw conveyor, due to friction developed between the conveyor and the solids, up the conical section where the solids are dewatered with the liquid draining back into the pool. The solids are discharged first into a hopper attached to the centrifuge, then they are dropped onto a



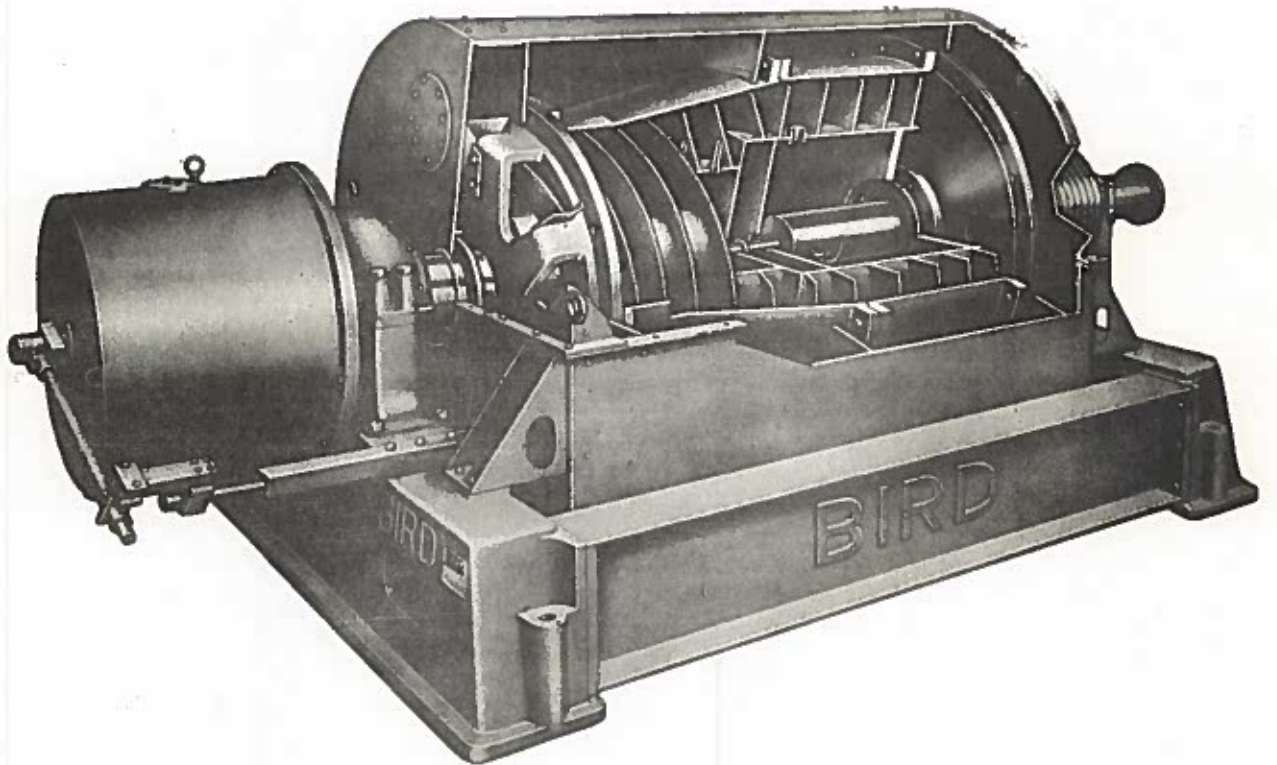


Figure 4: Illustration of cylindrical-conical, solid bowl centrifuge (14).

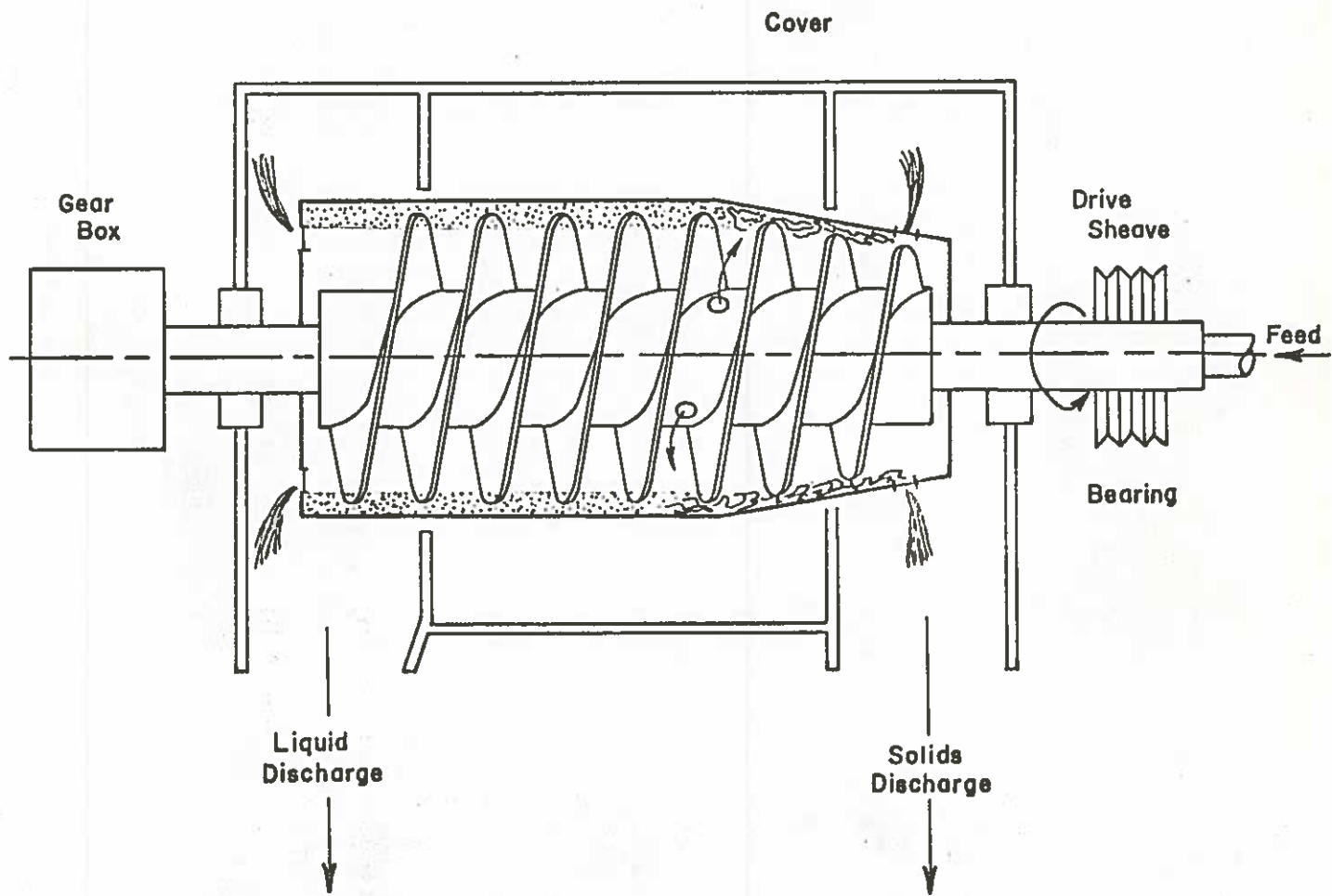


Figure 5: Schematic illustration of solid bowl, conveyor type centrifuge (9).

transporting vehicle to be carried away. The liquid which is discharged over the weir is usually recirculated by pumps to the head of the treatment plant or to the thickener (9).

The material used to construct the bowl and conveyor varies depending on the application. Mild steel is normally used where abrasion and corrosion are not a problem. Where this is a consideration, stainless steel can be used throughout or just applied to surface where wearing would occur (9).

## CHAPTER V

## APPLICATION OF CENTRIFUGES

Wastewater Applications

The use of centrifugal force to dewater waste sludge is no longer an untried or yet to be proven process in waste disposal technology. The process can dewater all types of sludge with the aid of synthetic polyelectrolytes. As manufactures expand their marketing efforts, a wider range of application will result (13).

The applications for which centrifuges are best suited in wastewater disposal can be classified into three general types. The first classification is the use of a centrifuge as a thickener. The second class is its use as a dewatering unit. Third is a combination of the first two uses where the centrifuge is used to dewater sludge part of the time and thicken sludge the remaining time.

Most early applications of centrifuges were to thicken sludge. Although there were numerous problems concerning the operation and maintenance of the unit yet to be solved, the fact remained that some dilute sludges, such as activated sludge, which are difficult to concentrate, could be thickened. This resulted in hope for a more economical disposal process utilizing centrifuges.

Thickening: The Chicago Sanitary District has conducted extensive tests on thickening waste activated sludge with both solid bowl and disc-type centrifuges. Reports by Ettelt and Kennedy (14), have shown that a disc-type centrifuge would thicken activated sludge to about 7.0%. Clogging of the unit by oversize particles

reduced the through-put, and required considerable maintenance. Attempts to remove the oversize particles by passing the influent through vibrating and rotating screens prior to the unit did not improve the situation. The disc-type unit was able to recover 87 to 98% of the solids fed at a feed rate of 3600 gph.

Tests were also conducted on two types of solid bowl centrifuges, using the same type of activated sludge. One type was a concurrent flow centrifuge and the other was a countercurrent flow centrifuge, i.e., flow in the same or opposite direction as the movement of the screw conveyor. With activated sludge alone, the concurrent machine thickened sludge to a range of 7 to 7.5%, while the countercurrent unit was able to thicken the sludge to 6.6 to 7.0%. When the activated sludge was mixed with primary sludge (ratio 50:50), the centrifuges were able to thicken the sludge to 9.8%. Other adjustments were made such as varying the pool depth, but because of poor solids capture, this was not a suitable application without the use of chemicals.

Thickening operations using a centrifuge are usually applied to the problem of concentration of activated waste sludge. Kraus and Longley (7) and Bradney and Bragstad (8) have conducted studies dealing with this aspect of centrifugation. Activated waste sludge, with approximately 1% solids, was fed to a centrifuge which concentrated it to approximately 5% solids. The amount of solids in the centrate varied, probably due to the particular type of activated sludge used, but both studies indicated that the centrate when returned to the head of the treatment plant did not present any difficulties.

The Yonkers Sewage Treatment Plant of Westchester County, New York is

currently engaged in a very successful thickening operation (15). A settled primary sludge with 8% solids is fed to a digester which produces an effluent solids concentration of 4 to 5%. This sludge is then thickened by a solid bowl centrifuge to 10 to 11% and delivered to a barge which transports the sludge to the ocean for final disposal. In the two years of 1961 and 1962, the sludge barge averaged 79 trips a year. In 1964 and 1965, after centrifuge thickening was adopted, the average number of trips decreased to 32.5 per year. The centrate is returned to the preaeration tank, and there has not developed any problem due to solids buildup in the system.

Dewatering: Dewatering of sludge to a relatively dry cake is the second application of centrifuges. The horizontal-solid-bowl type unit is the most efficient. Basket centrifuges, which dewater sludge very effectively, do not have high solids removal characteristics, while disc-type units have high solids removal, but poor dewatering characteristics.

Soon after the solid bowl centrifuge was installed at Daly City, the City of San Leandro, California used a centrifuge to dewater sludge from 3 sour digesters (16). During this time, the centrifuge was also used to handle the raw sludge from an 8 MGD plant in order that repairs could be made on the digesters. The operation was so successful that the centrifuge continued to be used as the only dewatering method. Raw domestic and industrial sludge was fed to the unit at a concentration of 3.5%, and was dewatered to 26%. The centrate contained 1.9% solids which, when recycled to the plant, caused some problems.

Los Angeles County Sanitary District conducted an extensive study to determine an economical method of dewatering sludge from their 300 MGD plant (17).

Their conclusion was to use 5-40" diameter solid bowl centrifuges which would handle 1000 gpm of digested sludge with approximately 4% solids concentration, and produce a cake containing 30 to 35% solids. It was found that a solids recovery of 66% was attainable, and the centrate could be screened and discharged directly to the ocean. The cake was sold as an ingredient to make a mulch for soil conditioning.

The cake from a centrifuge can be disposed of in any manner normally used for sludge disposal. New Orleans utilizes two centrifuges to dewater raw sludge from a 24 MGD plant prior to incineration (11). This results in a reduction in the amount of heat required for incineration because of the decrease in moisture contents.

The centrifuge cake can be placed on sand drying beds, if necessary. This combination might be utilized in case of excessive overloading of sand beds due to an increase in population, in which case the centrifuge would reduce the volume of sludge, speed up the drying process and eliminate the excessive moisture which causes odors. The centrifuge cannot economically replace sand drying beds except in plants where land area is scarce or the beds would be a nuisance to the surrounding areas.

Thickening and Dewatering: Another application of centrifuges is to use the machine for both thickening and dewatering the same plant. Treasure Island, Florida utilizes a centrifuge daily to thicken waste activated sludge to 5 to 7% prior to pumping the sludge into a primary digester (18). Then for approximately 3 days per month, the same machine is used 6 hours per day to dewater the digested sludge. The digested sludge has 3 to 5% solids and is dewatered



to 14-26% cake solids. Chemicals are added to increase solids recovery from 85 to near 100%.

This application is of particular interest to small plants where it may not be necessary to use the centrifuge continuously. It is apparent that having a high capital cost machine idle 24 days a month would be very uneconomical. By using the machine more, higher economical levels could be obtained, plus thickening of the sludge before digestion would permit a smaller digester volume.

Waste activated sludge is probably the hardest sludge to dewater. Present practice is to combine this sludge with other types, such as raw primary, which results in a mixed sludge which is easier to dewater. This combination is not always possible when for instance, the raw sludge has some usefulness as a by-product.

Woodruff, et al, reported on a case similar to this (10). Their recommendation was to utilize two different types of centrifuges in series. First, a disc-type unit was used to thicken the underflow from a clarifier. Then, this prethickened sludge was dewatered by a solid-bowl unit. The resulting cake contained 22% solids.

#### Industrial Application

Centrifugal separators began finding applications in the process industries about 80 years ago. The earliest users were in sugar manufacture and in separating cream from milk. From this, the number of applications has grown to a larger and ever increasing value.



Industrial centrifuges are used in many different ways, but almost all of them fall into one of two methods of operation. In one method centrifuges which separate by sedimentation are used, and the other method uses centrifuges which force the liquid through a filter medium by centrifugal action.

The sedimentation centrifuges are the types which will be discussed in this paper. They consist of many different types of machines from slow speed large diameter batch operated units to high speed continuous units. A brief discussion of the various process applications of these type units is given by Smith (19).

These machines are widely used to dewater industrial waste sludge. Bird Machine Company lists some of these applications (20) as shown in Table I.

The paper industry has used centrifuges to a large extent in treatment of their waste. The main reason for this is the ability of the centrifuge to handle different concentrations of waste sludge without excessive adjustment. One plant, which produces high quality hardboard wall paneling and plastic-coated decorating paneling, used a solid bowl centrifuge to dewater a 1% underflow from a clarifier-floculator (20). A cake with 30% dry solids was produced at the rate of 15,000 lbs. daily, and was then blended with saw dust and fed continuously to a fire box.

Another plant which produced 1300 tons of bleached paper and board per day with a waste water flow of 40 MGD, utilized three solid bowl centrifuges to dewater the underflow from a system of clarifiers (20). The sludge was a by-product of waste-stock reduction, and fiber recovery operations. Feed ranged from 3 to 5% solids, and the discharged cake had 29% solids with recovery of 94% of the solids.

## TABLE I

## INDUSTRIAL WASTE TREATMENT APPLICATIONS (20)

## Dewatering of:

Settled Paper Mill Waste

Tanning Waste

Packing House Waste

Foundry Sludge

Calcium Carbonate from Water Treatment Plants, with and  
without Reburning

Refinery Wastes

Cannery Wastes

Chemical Process Wastes

Neutralized Waste Pickle Liquor from the Steel Industry

Textile Dyeing Wastes

Activated Industrial Waste Sludge

Coal Preparation Refuse

Dusts from Basic Oxygen Furnace in Steel Industry

Foundry waste water contains large amounts of bentonite clay and fly ash as well as cereal binder. A machine tool manufacturer dewatered sludge from a clariflocculator with underflow concentration of 7-8% solids to 63% solids and disposed the solids as a dry cake (20).

Chemical manufactures have made use of solid bowl centrifuges to dewater underflow from primary clarifiers in activated sludge and phenolic waste treatment plants. A 2 to 3% feed was dewatered to approximately 24-26% solids which was then placed in a landfill (20).

Many industries use water softening treatment on incoming water supplies. The sludge from these plants contains 10 to 20% suspended solids and is dewatered to yield cakes containing 65% solids prior to calcination and reuse or disposal. Also, a well-clarified centrate can be obtained (20).

Centrifuges are widely used for oil refinery wastes, removing the bulk of suspended solids from feeds containing solids-stabilized oil-water emulsions. Cake containing 50-60% solids, 30-40% water and 10-20% oil is fed directly to incinerators. Liquid effluent is further processed for ultimate separation of the water and reclamation of the oil (20).

#### Cost of Centrifugation (2)

The cost to dewater sludge will vary considerably depending on the type of sludge, the volume and consistency of the sludge, and the percentage of solids which must be removed. The total annual cost will usually be in the range of \$5 to \$35 a ton, with the average cost of approximately \$12 per ton.

The capital cost for centrifuges is usually 30% less than the capital cost of vacuum filters. The operating cost is approximately the same for a given application. Chemicals sometimes may not be required for centrifuges when necessary for vacuum filters resulting in a large cost saving.

## CHAPTER VI

### OPERATING CHARACTERISTICS OF CENTRIFUGES

As discussed previously (Chapter II), the sedimentation characteristics of a centrifuge can be related to gravitational sedimentation by use of Stokes' Law. The variables which are important in both operations include the following:

1. Particle size and shape
2. Differential density between particle and liquid
3. Particle concentration
4. Viscosity - temperature relationship

Wastewater sludge, however, is composed of solids which are combinations of granular, fibrous, and flocculent particles (21). As such, the solids tend to cluster during settling with resulting changes in their size, shape, and relative density.

Because of the difficulty in determining some of these variables for wastewater sludge, accurate mathematical models are difficult to construct for centrifuges. It can be observed, however, that the greater the density differential between solids and liquids, the faster the rate of settling. The viscosity-temperature relationship illustrates that as the temperature of the water increases, the viscosity decreases, and the rate of settling increases.

The primary reason that centrifuges can be used in so many different applications is the numerous modifications which can be made in their operation.

These modifications can be divided into two categories, machine variables and process variables (6). Table II list these variables, and it is observed that machine variables are modifications which can be made in the operation or design of the centrifuge, while process variables are modifications of the liquid waste. All of these variables are inter-related and usually compromises must be made in order to achieve overall efficiency in solids removal and dryness of the cake.

#### Machine Variables (6, 13, 22)

Bowl and Conveyor Design: Bowl and conveyor design are variable only to a certain extent. Prior to installation of the unit, the size and shape of the bowl, along with the pitch and lead of the conveyor, can be adjusted in manufacturing the centrifuge. The differential conveyor speed is usually non-varying, however, manual adjustments or equipment modification can be made where necessary. This leaves only the speed of the bowl and the pool volume which can be adjusted in the field.

The pool volume can be increased when designing the bowl by increasing the length and diameter. If the flow rate remained constant, this would result in an increase in detention time. The vertical distance through which the particles must settle is regulated by the overflow weirs and can be held constant. Under these conditions, the removal efficiency would be improved by increasing the pool volume. An increase in the diameter of the bowl would also cause an increase in the centrifugal force, which would also aid in improving solids recovery.

The ratio of length-to-diameter is limited by mechanical parameters. This

TABLE II

## MACHINE AND PROCESS VARIABLES (6)

## MACHINE VARIABLES

1. Bowl Design
  - (a) Length-to-diameter ratio
  - (b) Bowl Angle
2. Bowl Speed
3. Pool Volume (Depth)
4. Conveyor Design
5. Relative Conveyor Speed

## PROCESS VARIABLES

1. Feed Rate
2. Solids Characteristics
  - (a) Particle Size
  - (b) Density
3. Feed Consistency
4. Temperature
5. Chemical Aids

ratio has been increased to the range of 2.5 - 3.5 for present units as compared with ratios of 1.5 - 1.8 for earlier units. In general, the higher the ratio, the greater the clarification capacity.

The angle which the conical portion makes with the cylindrical portion is another important consideration in the design of the bowl. As the solids are forced from the pool up the drainage deck, or beach, water is removed due to the centrifugal force and flows countercurrently back into the pool. If the slippage force is too great, the solids will slip around the conveyor and return to the pool. The slippage force,  $g_s$ , (Fig. 6) is a function of the sine of the angle  $\theta$  times the centrifugal force,  $F_c$ . By reducing the angle of the bowl, the slippage force is reduced and hydrous materials with low structural strength can be conveyed up the beach. The introduction of long bowl machines permits shallower angles without significantly sacrificing the pool volume needed for clarification.

Bowl Speed: The centrifugal force " $F_c$ " as shown in Fig. 6 is given by the following relation:

$$F_c = m\omega^2 r \quad (\text{Eq. 10})$$

$\omega$  is the bowl speed and its significance in increasing the centrifugal force and thus, the clarification capacity is apparent. This speed is easily adjusted in the field to achieve optimum results.

The clarification capacity (C) of a centrifuge is proportional to the equivalent applied gravitational force and the total time which the solids would be subjected to this force, and is expressed in terms of gravity-minutes of retention. The equivalent applied gravitational force (G) can be determined

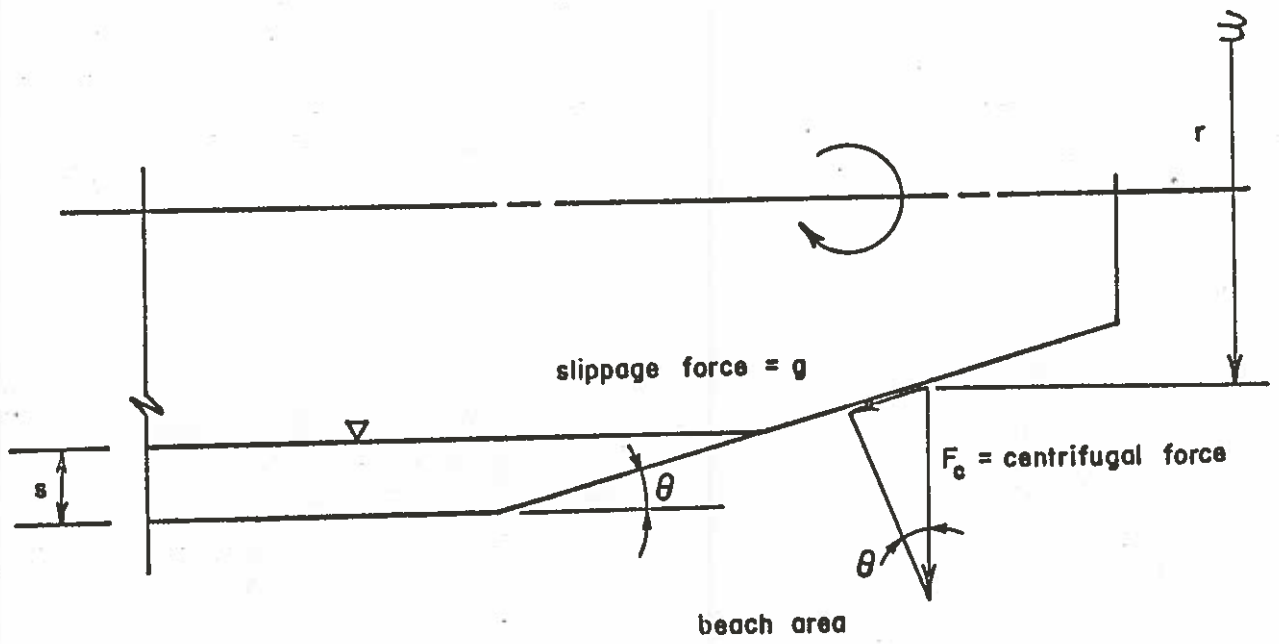


Figure 6: Model for the analysis of the slippage force acting on a particle being conveyed up the beach area.



by dividing the centrifugal force ( $F_c$ ) by the gravitational force ( $F_g$ ).

$$F_g = m g = 1 \text{ gravity force}$$

so

$$G = \frac{F_c}{F_g} = \frac{m \omega^2 r}{m g} \text{ equivalent gravity force}$$

The total time would be the detention time of the centrifuge.

$$t = \frac{V}{Q}$$

so

$$C = \frac{F_c}{F_g} \cdot \frac{V}{Q} \quad (\text{Eq.11})$$

$$C = \frac{m \omega^2 r}{m g} \cdot \frac{V}{Q} \quad C = \frac{\omega^2 r}{g} \cdot \frac{V}{Q} \quad (\text{Eq.12})$$

$$C = G \cdot t \text{ (equivalent gravity-minutes)}$$

A sedimentation basin with a two hour detention time would provide 120 gravity-minutes of clarification, a solid bowl centrifuge with a detention time of 0.25 to 1.25 minutes can develop 930 to 4600 gravity-minutes of clarification. Thus, a centrifuge can provide better clarification than a sedimentation basin with a reduced detention time.

Since the centrifugal force increases as the square of the bowl speed, it is noted from Equation 12 that the feed rate can be increased without reducing the clarifying efficiency, if the bowl speed is proportionally increased. If the feed rate remains constant with an increase in centrifugal force, higher capture of feed solids will result.

The increase in centrifugal force will also aid in squeezing water out of the sludge on the beach. This results in a much dryer cake. Thus, it would seem that machines should be designed for a maximum speed which would result in the maximum centrifugal force.

However, this is not the case. The higher speed and consequent higher forces, result in certain disadvantages. First, the higher forces result in large pressures being exerted on the solids, forcing them against the surface of the conveyor and the bowl wall, tending to lock these two parts together. If the solids are abrasive in nature, high friction and costly wear will result. Second, particles which are not abrasive usually resist conveying. This makes the removal of the solid up the beach much more difficult because of the increase in slippage force which has been previously noted.

The relationships between bowl speed and solids recovery, and bowl speed and cake dryness are shown in Figs. 7 and 8, respectively. This illustrates that with an increase in bowl speed, the solids recovery and cake dryness both increase.

Pool Depth: The remaining machine variable which can be adjusted in the field is the pool depth. Increasing the pool depth will, of course, increase the pool volume, and the effect of this has been discussed earlier. Changes in the pool depth are normally accomplished by overflow weirs located at the cylindrical end of the bowl. A decrease in pool depth will expose more area of the beach providing a greater opportunity for liquid drainage and producing a drier cake.

Figures 9 and 10 illustrate the results of increasing pool depth on the amount of solids recovered and cake dryness, and the incompatibility of these two parameters with pool depth (6).

If the solids are difficult to convey, increasing the pool depth would raise the point on the beach where the solids would leave the water. This will reduce the slippage force since the radius would decrease, permitting the solids conveying problem to be eased.

Screw Conveyor: The screw conveyors are designed to match the application and can be manufactured with a wide variety of pitches, with single or multiple

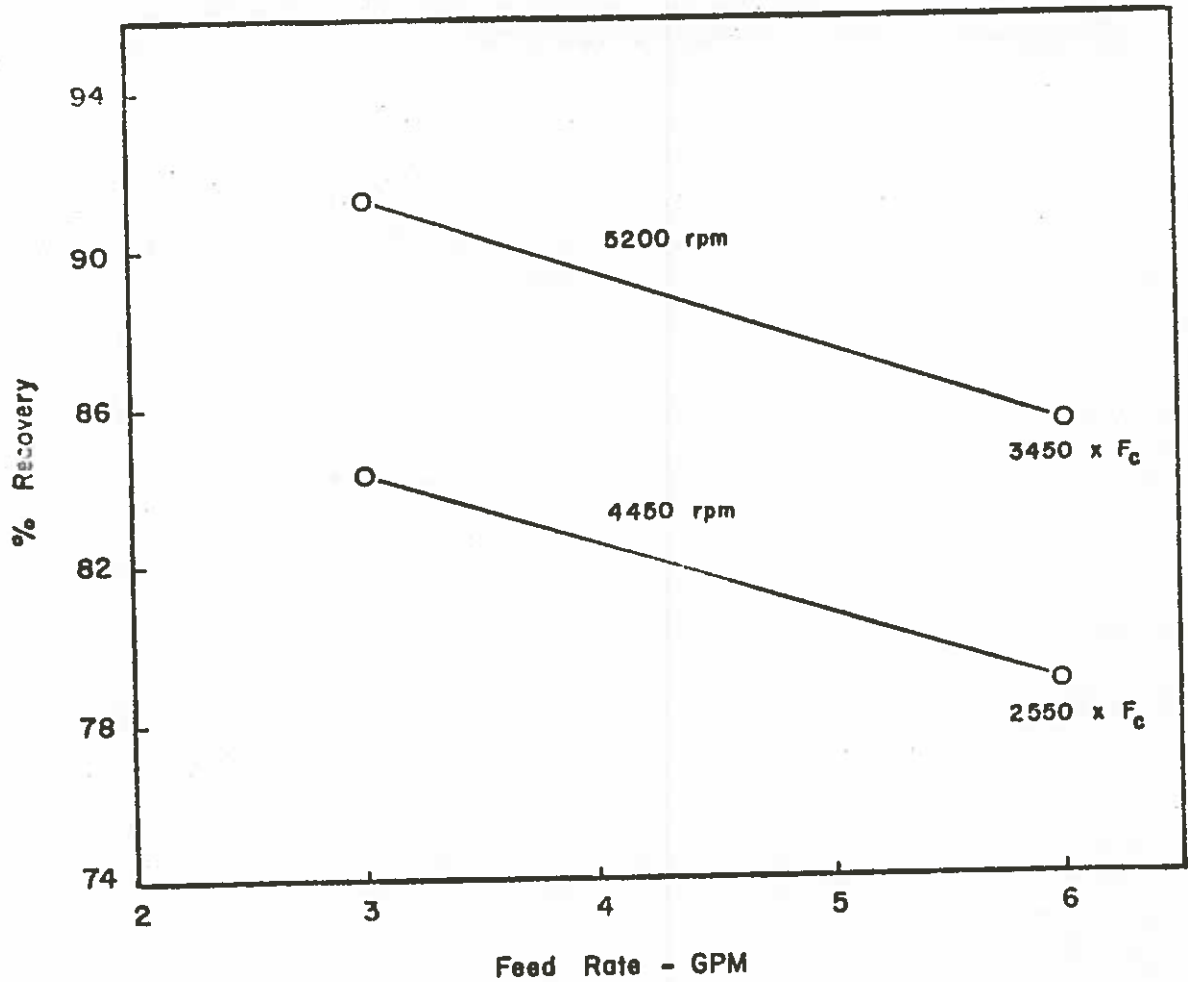


Figure 7: Relationship between bowl speed and recovery for mixed raw primary and waste activated sludge. (6)

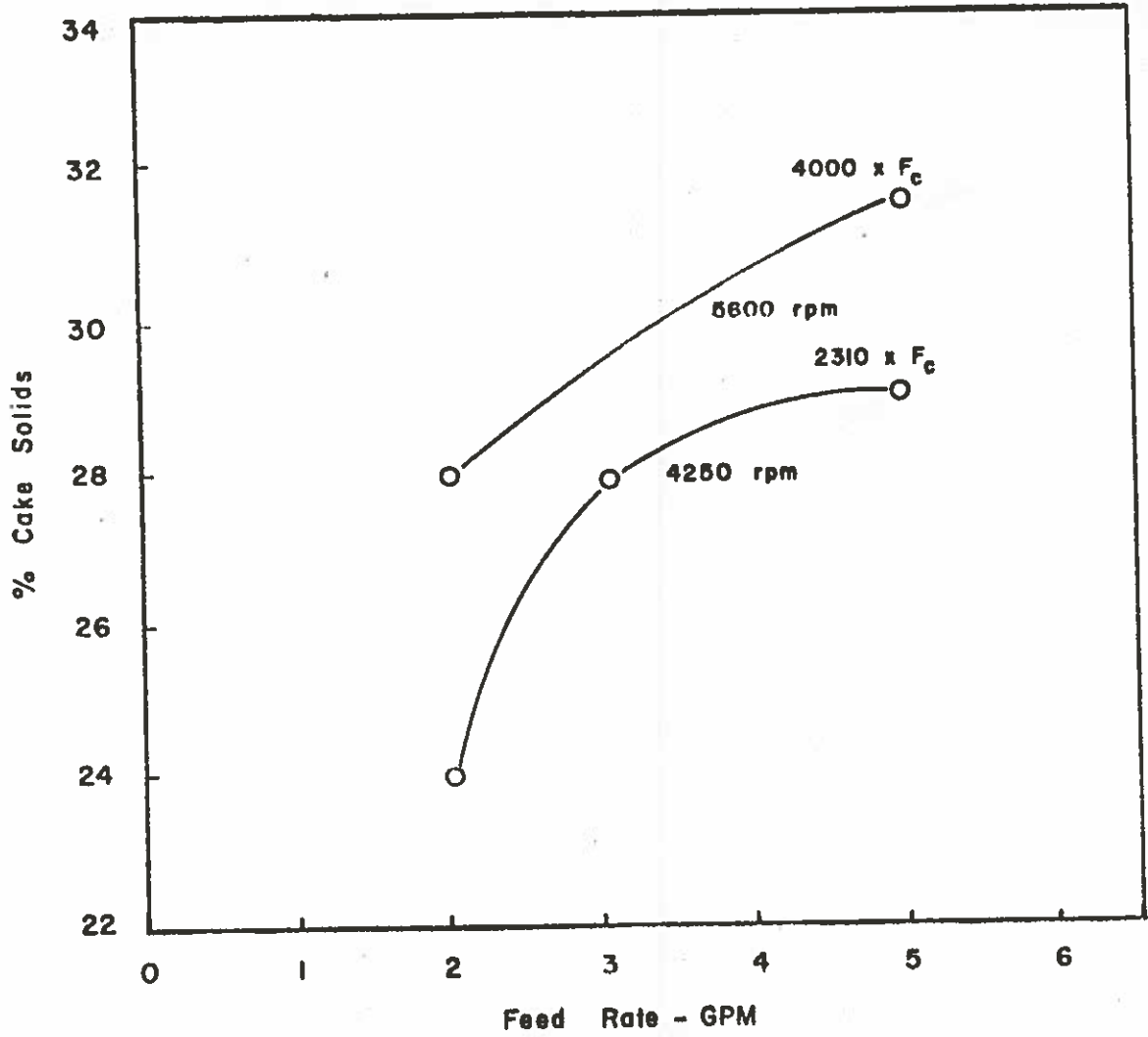


Figure 8: Relationship between bowl speed and cake dryness for digested primary sludge (6).

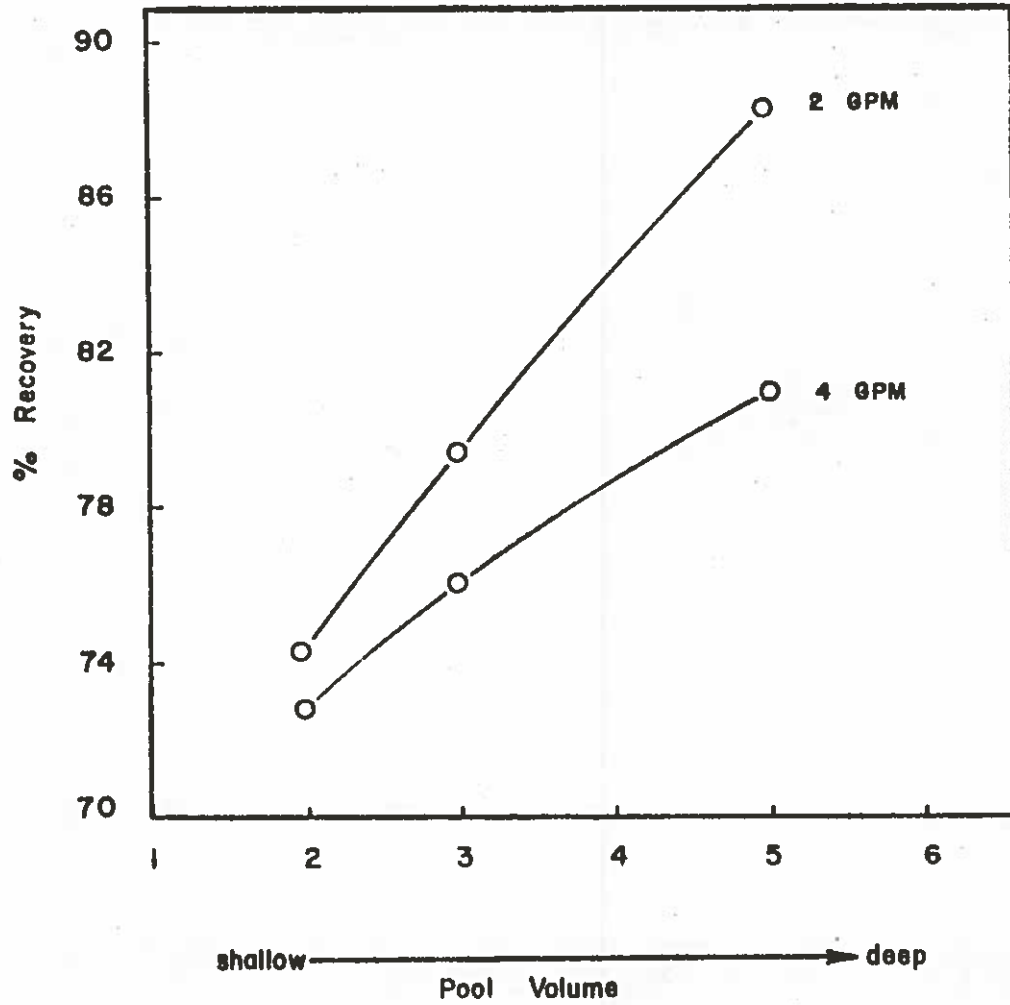


Figure 9: Relationship between recovery and pool volume for raw primary and secondary sludge (6).

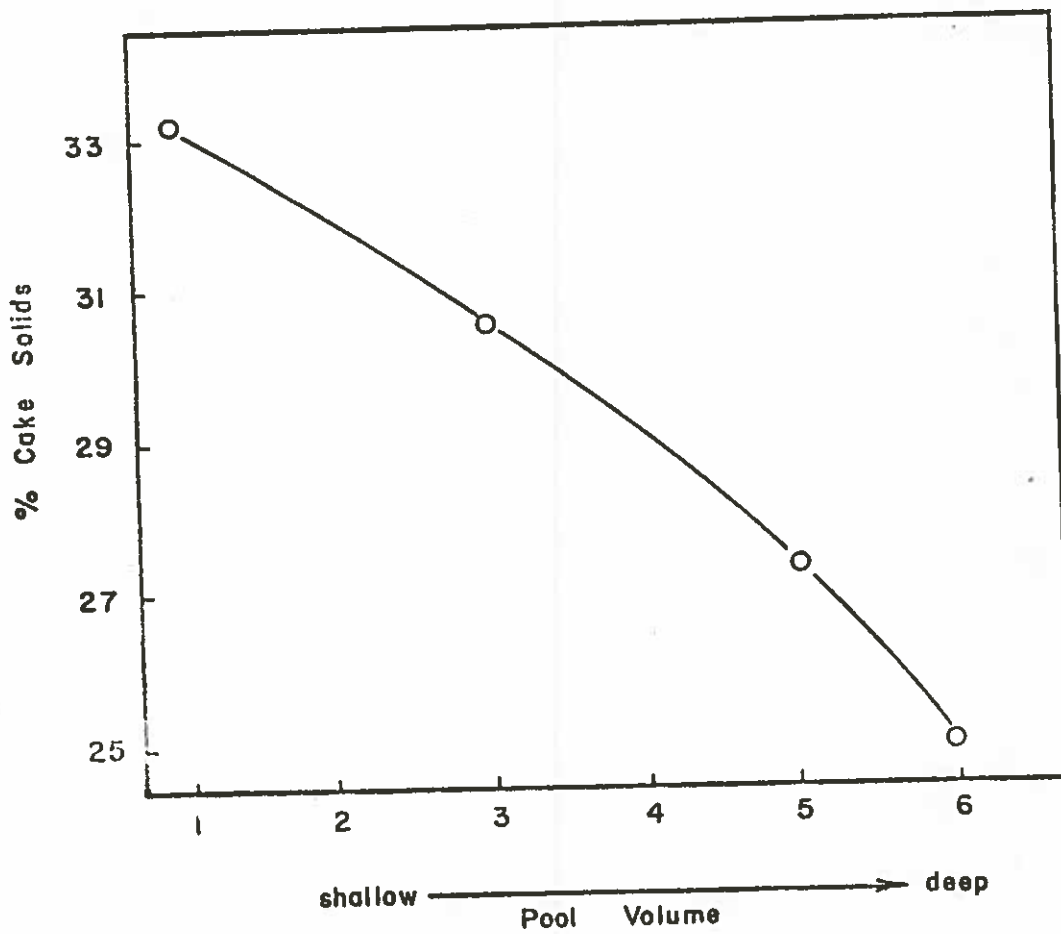


Figure 10: Relationship between cake dryness and pool volume for raw primary sludge (6).

leads. The conveyor removes solids from the bowl continuously due to the differential speed between the conveyor and bowl. This speed is determined by the solids load and its characteristics. Heavy easy-to-settle solids may be removed with a high differential speed, while solids with low structural strength are removed with a very low differential speed. Clarification requires that the conveyor operate at the minimum speed possible, depending on the torque developed by the buildup of solids in the bowl at very low speeds. High conveyor speeds may result in turbulence within the pool and loss of fines. This could be advantageous in classification processes such as elutriation of hydroxides from calcium carbonate sludges. Finally, the higher conveyor speeds will result in a wetter cake since the drainage time on the beach will be reduced.

In summary, the effect on solids recovery and cake dryness by modification of the centrifuge is shown in Fig. 11. In this figure a plus sign denotes increases in efficiency and minus signs are read as decreases. For example, to improve cake solids, one would increase bowl speed, decrease pool volume and decrease conveyor speed.

#### Process Variables (6, 13, 22)

Feed Rate: The feed rate of solids into the centrifuge is the prime process variable which may be modified to improve efficiency. This variable is the main determinant in the number and size of centrifuges necessary to handle the required sludge volumes and achieve the required effluent clarity and cake dryness. An increase in the feed rate will result in a decrease in the efficiency of solids removed but with an improved dryness of the cake (Fig. 12). The reason for this is that the increase in feed rate will decrease the detention time of a particle in the bowl. The larger particle will still settle

Machine Variables	Bowl Speed	Pool Volume	Conveyor Speed
To improve recovery	+	+	-
To improve cake solids	+	-	-

Figure 11: Effect of machine variables on the improvement of recovery and cake solids (6).



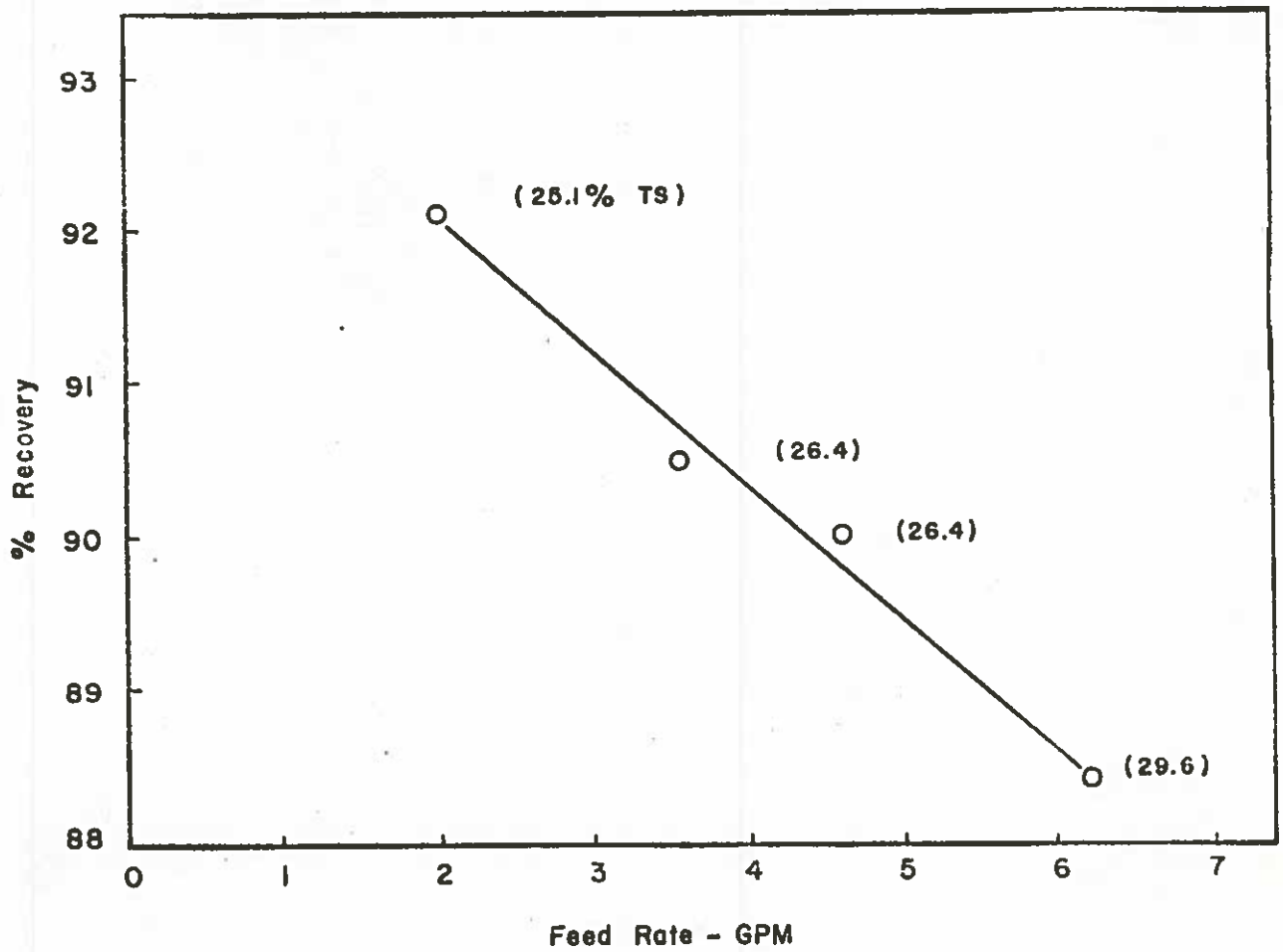


Figure 12: Relationship between recovery and feed rate for digested primary sludge without polymers. (6)

and the fine solids will be carried over the weirs in the centrate, leaving the cake produced much drier.

Feed Consistency: The limiting capacity of most centrifuges is the feed rate, or hydraulic capacity. The solids capacity is then a direct function of the feed consistency or the percent solids in the feed (6). An example of this effect is shown as follows:

Feed %TSS	Centrate % TSS	Cake %TSS	Recovery %	Relative Rate - %	
				Centrate	Cake
5	1	30	82.6	100	100
10	2	30	85.7	82	210

$$\begin{aligned} \% \text{ Recovery} &= \frac{\text{Cake}}{\text{Feed}} \left[ \frac{\text{Feed} - \text{Centrate}}{\text{Cake} - \text{Centrate}} \right] \times 100 \\ &= \frac{30}{5} \left[ \frac{5 - 1}{30 - 1} \right] 100 = 82.6\% \end{aligned}$$

It is noted that the overflow to feed solids ratio is the same (1/5) for both the 5% and 10% feed concentration. At constant solids capacity increasing the feed consistency will reduce the overflow rate of the machine and increase the solids recovery. The examples show that doubling the feed concentration increased the recovery (82.6% to 85.7%) while discharging over twice (100 to 210) as much cake. The overflow rate on the machine for the higher feed consistency is lower (100 : 82) since greater proportions of the feed solids appear as cake. For equivalent overflow rates a sludge with a 10% feed consistency will produce 250% more solids than a sludge with 5% feed consistency.

In most centrifuge applications, little direct effort is made to increase solids consistency in the feed. However, efforts should be made wherever possible to

obtain as high a feed solids concentration as possible in prior thickening processes, especially with waste organic sludge (13).

Because of the increase in solids recovered, an increase in feed consistency will result in a wetter cake.

Solids Characteristic: The characteristics of the solids which affect the operation of a centrifuge are the specific gravity of the particle, the size distribution of the particle, and the density of the solid. Discrete fibrous or granular particles which have good settling rates are generally easy to dewater and allow higher recoveries. Sludge with small particles and low density are hard to dewater and may require chemical treatment to cause coagulation resulting in flocs with better settling rates. Raw primary sludge is generally easier to dewater than biological sludge, such as activated sludge and trickling filter humus.

Temperature: The temperature has a direct relation to the viscosity and density of a liquid medium. As the temperature increases viscosity and density both decrease, but with viscosity decreasing at a more rapid rate than density, resulting in particles settling much faster (Eq.6). This will result in improved solids recovery and drier cake solids. With some types of sludge, the increase in temperature will decrease the structural strength of the solids making conveying of the sludge difficult. Adjustments in temperature of the feed sludge are not normally practiced for sewage sludge dewatering.

Chemical Aids: The most important adjustment which can be made with regards to the process, is to add chemicals. Use of these polyelectrolytes has greatly broadened the applications where centrifuges can be used successfully, and can increase the solids recovery to practically any level.

The addition of chemicals results in a much higher degree of clarification. Because of the higher number of fine particles removed, the cake is usually wetter. A second reason for the addition of chemicals is to improve the structural strength of the sludge so as to improve conveyability. This increase in structural stability is the result of removal of bound water from the hydrophilic fines.

The types of chemicals which have been added include alum and ferric chloride. Newer developments have resulted in cationic and anionic polymers which have proved very effective for use in flocculation and dewatering fine solids such as those present in digested and waste secondary sludge. Further developments in the field of polymer chemistry promise improved and cheaper conditioning agents.

A summary of the effects of changes in the process variables on improving cake solids and improving solid recovery is shown in Figure 13. It is illustrated that an increase in solids recovery can be achieved by decreasing feed rate, increasing feed consistency, increasing temperature and the addition of flocculants. An increase in cake solids can be accomplished by increasing the feed rate, decreasing feed consistency, and increasing the temperature. The cake is normally dryer when chemicals are not added. In general the changes made to improve solids recovery will result in wetter solids cakes, except where the dewatering of the solids is improved with an increase in temperature.

Figure 14 points out the fact that changing the operating variables to increase the solids recovery will result in a wetter cake, and compromises must be made to achieve the optimum levels desired.

Process Variables	Feed Rate	Feed Consistency	Temperature	Flocculants
To improve recovery	-	+	+	+
To improve cake solids	+	-	+	(+)

Figure 13: Effect of process variables on the improvement of recovery and cake solids (6).

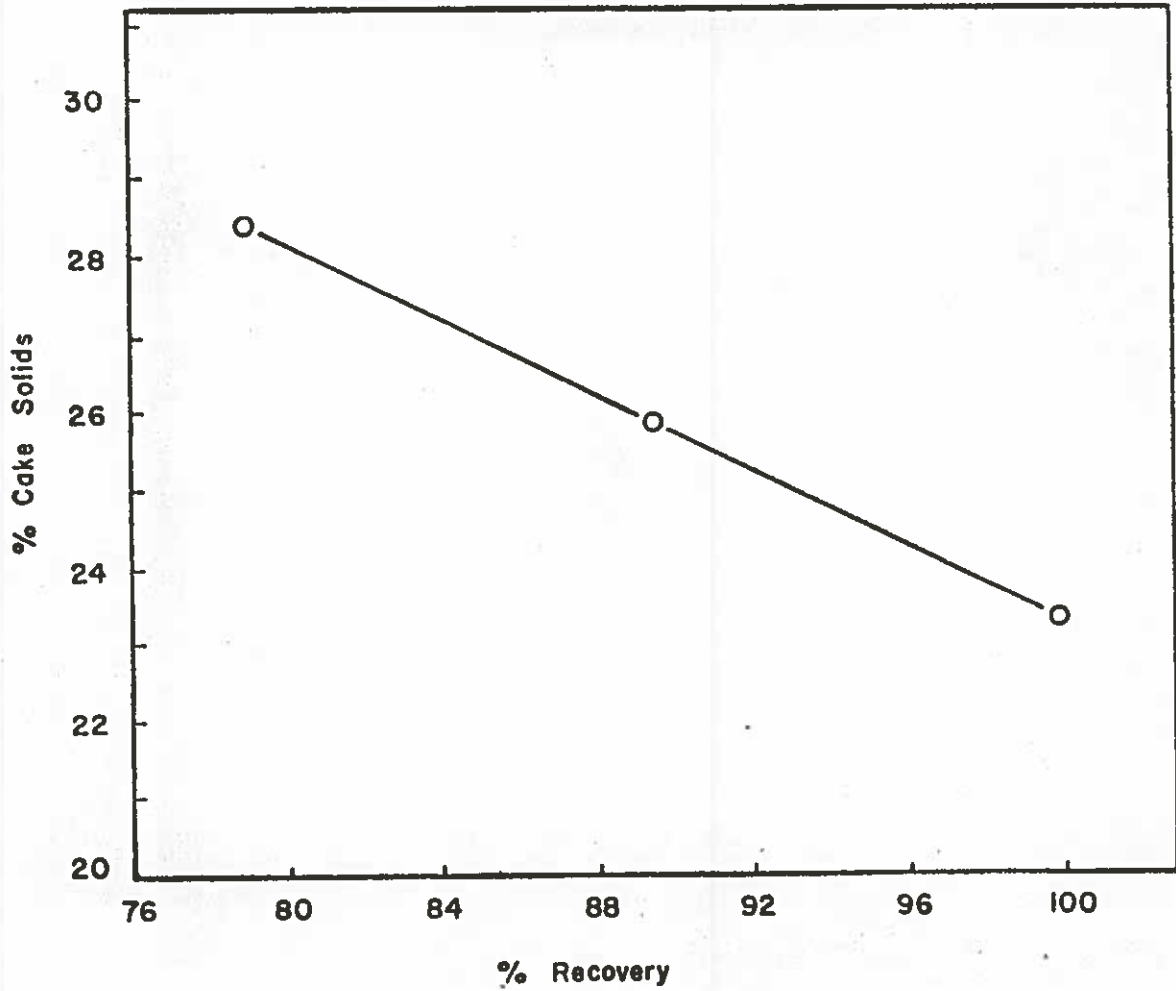


Figure 14: Relationship between cake dryness and recovery for digested primary and activated sludge with polymers added (6)

Adjustments in the process variables and the machine variables can usually be made in the field to produce the desired results. Because these adjustments can be made easily in the field the selection and evaluation of centrifuges has remained an art with little scientific background.

## C H A P T E R VII

## PERFORMANCE EVALUATION AND ENGINEERING SELECTION

In the selection of a centrifuge, the main consideration is to achieve the necessary degree of dewatering required for the particular disposal process in conjunction with a balance of the treatment processes at the least cost. It is desirable to obtain a solid cake at low moisture content with a centrate low in solids. This optimum balance for the entire treatment plant is the specific aim of the design engineer and plant operator.

At the present time there does not exist any basic design criteria from which to select a centrifuge. The three avenues of approach normally used are: to base the selection of a centrifuge on past performance in identical or similar conditions, the use of laboratory tests, and the utilization of pilot plants and manufacturer's scale-up techniques.

An analytical approach can be developed to determine the maximum pool depth which will permit a predetermined size particle to be removed after the centrifuge and the quantity of sludge to be dewatered has been selected.

#### Performance Characteristics

From past experiences gained at actual installations, and from test programs conducted by various manufacturers, it is claimed to be possible to predict, within reasonable ranges, the performance to be expected from a given centrifuge and a particular type of material to be dewatered. However, caution must be exercised since substantial differences exist between sludge produced by different processes, and also, in sludge produced by the same process but in different localities (9). In industrial waste, sludge produced from similar plants or mills may also differ widely.



The performance of a solid-bowl centrifuge on various types of sludge can be summarized as follows, and in Table III.

Raw Primary Sludge: Raw primary sludge is the easiest type of sludge to dewater because of the high fiber content of the feed solids (6). The cake can be dewatered to 30-40% solids with 70 - 90% solids capture without the use of chemicals.

Digested Primary Sludge: Digested primary sludge, like its raw counterpart, is easily dewatered (6). The cake can be dewatered to 30 - 40% solids with 70 - 90% solids capture. There is generally little need to use chemicals for dewatering digested or raw primary sludge. In the dewatering of digested sludge, the particles that escape to the centrifuge overflow have a volatile content that is often as high as the raw sludge. This overflow will also contain digestible grease solids which can be returned to the digestive system for further destruction. The majority of the retained solids can be disposed of as a low volatile, digested cake. Solid bowl centrifuges have been widely used to dewater this type of sludge.

Mixed Primary and Trickling Filter (Digested): It is reported (6) that bio-filter sludge may be dewatered to 25 - 30% solids. For solids capture greater than 85%, the addition of chemicals is required; the amounts of which would be approximately 6 lbs/ton of dry feed solids dewatered. These cakes are considerably wetter than primary sludge.

Mixed Primary and Activated Sludge (Digested): Activated sludge when mixed and digested with a primary sludge can be dewatered to 15 - 20% solids (6). Chemicals must be added to bring the recovery of solids to the 85% + level. This involves the addition of 10 to 35 lbs/ton of feed solid, thus it is the most difficult sludge to dewater.

TABLE III

## DEWATERING PERFORMANCE - VARIOUS SLUDGE TYPES (14)

TREATMENT	PROCESS	SLUDGE	CENTRIFUGE DATA		
			(% SOLIDS)	(%)	(LBS./TON)
			CAKE DRYNESS	CAPTURE	CHEMICAL
Primary	Conv.	Raw	30-40	70-90	None
Primary	Conv.	Digested	30-40	70-90	None
Secondary	Biofilter	Raw	25-35	85-99	5-10
Secondary	Biofilter	Digested	25-35	85-99	5-10
Secondary	Activated	Prim. & Sec.	15-20	65-99	10-15
Secondary	Activated	Dig. Prim. & Sec.	15-20	65-99	10-35
Secondary	Extended Actv.	Secondary	5-15	90-99	5-10
Secondary	Activated	Waste Act.	5-15	90-99	5-10

### Laboratory Tests

Simple clinical test-tube experiments will sometimes establish the practicality of centrifuge applications. These tests are valuable in the preliminary determination of the applicability of centrifuges rather than precise scale-up or sizing data.

A description of one such test is described as follows (23). A test tube or bottle which is filled with sludge is spun at about 2,000 times gravity for several minutes. The questions which can then be answered are - Do the solids settle or float? Do some settle and some float? Is the supernatant liquid clear? What is the nature of the settled cake? Is it compact, difficult to penetrate or easily reslurried? Is it soft, slimy?

If the solids do not settle or only partially settle, or if the supernatant liquid is not clear, then the use of sedimentation type centrifuges may be questionable. If the cake easily resuspends or is slimy, the solids will be difficult to convey up the beach of the centrifuge. Chemicals may be added to the sludge in the bottle or test tube, giving an indication of their efficacy.

### Sigma Concept

Charles Ambler developed in the early 1950's the sigma concept for relating the capacity and performance characteristics of one centrifuge to another (24). It may be observed that Equation 8, derived earlier can be rewritten and divided into two parts as follows:

$$Q_c = \frac{2(\rho_p - \rho_l)D_p^2 g}{18 \mu} \cdot \frac{\omega^2 r_m}{s g} \quad (\text{Eq.13})$$

Where  $r_m$  and  $s$  are the appropriate average values of the radius and layer thickness for a given condition. Since the first group on the right hand side of

the equation is concerned only with the parameters of the system that follows Stokes' Law, and the second group with parameters of the centrifuge unit, this may be written,

$$Q_c = 2u_t \cdot \Sigma \quad (\text{Eq.14})$$

where  $u_t$  is the terminal settling velocity in a gravity field, and is equal to,

$$u_t = \frac{(\rho_p - \rho_l) D_p^2 g}{18 \mu} \quad (\text{Eq.15})$$

and

$$\Sigma = \frac{V \omega^2 r_m}{s g} \quad (\text{Eq.16})$$

$\Sigma$  is an index of the centrifuge size, and has the dimension of length squared. It is equivalent to the area of a settling tank theoretically capable of doing the same amount of useful work.

This is shown as follows. The flow rate  $Q_c$  at which at least one-half of the solid particles of diameter,  $D_c$ , will be removed for a rectangular settling tank is,

$$Q_c = \frac{V}{t_c} \quad (\text{Eq.17})$$

and  $t_c$  is the time necessary for one-half of the particles with a diameter  $D_c$  to settle to the bottom,

$$t_c = 1/2 \cdot s/u_t \quad (\text{Eq.18})$$

and  $u_t$  as shown in Equation 6 is,

$$u_t = \frac{(\rho_s - \rho_l) D_c^2}{18 \mu} \cdot g \quad (\text{Eq.19})$$

so for the rectangular tank

$$Q = \frac{A \cdot 2(\rho_s - \rho_l) D_c^2 \cdot g}{18 \mu} \quad (\text{Eq.20})$$

Comparing this with Equation 14, we see the value of  $\Sigma$  is

$$\Sigma = A$$

Using this method of reasoning, the following values of sigma are developed by Lavanchy and Keith (25), for different centrifuges.

Bottle Centrifuge: Fig. 15 represents the model on which the analysis of the performance of a bottle centrifuge is based. A solid or liquid particle is considered in an initial position, P, at radius r from the axis of rotation. If equation 13 is applied to this specific particle with the additional consideration that;

$$u_t = dr/dt \quad (\text{Eq.21})$$

$$\int_r^{r_i} \frac{dr}{r} = \int_0^t u_t \frac{\omega^2}{g} \cdot dt \quad (\text{Eq.22})$$

and

$$\ln \frac{r_1}{r} = u_t \frac{\omega^2}{g} \cdot t \quad (\text{Eq.23})$$

The radius  $\bar{r}$ , which divides the volume of supernatant into two equal parts, can be defined in the following way;

$$\bar{r} = \frac{r_1 + r_2}{2} \quad (\text{Eq.24})$$

This point is referred to as the 50% cutoff point, since one-half of the particles which were in suspension at  $t=0$  are sedimented after the time,  $t$ ,

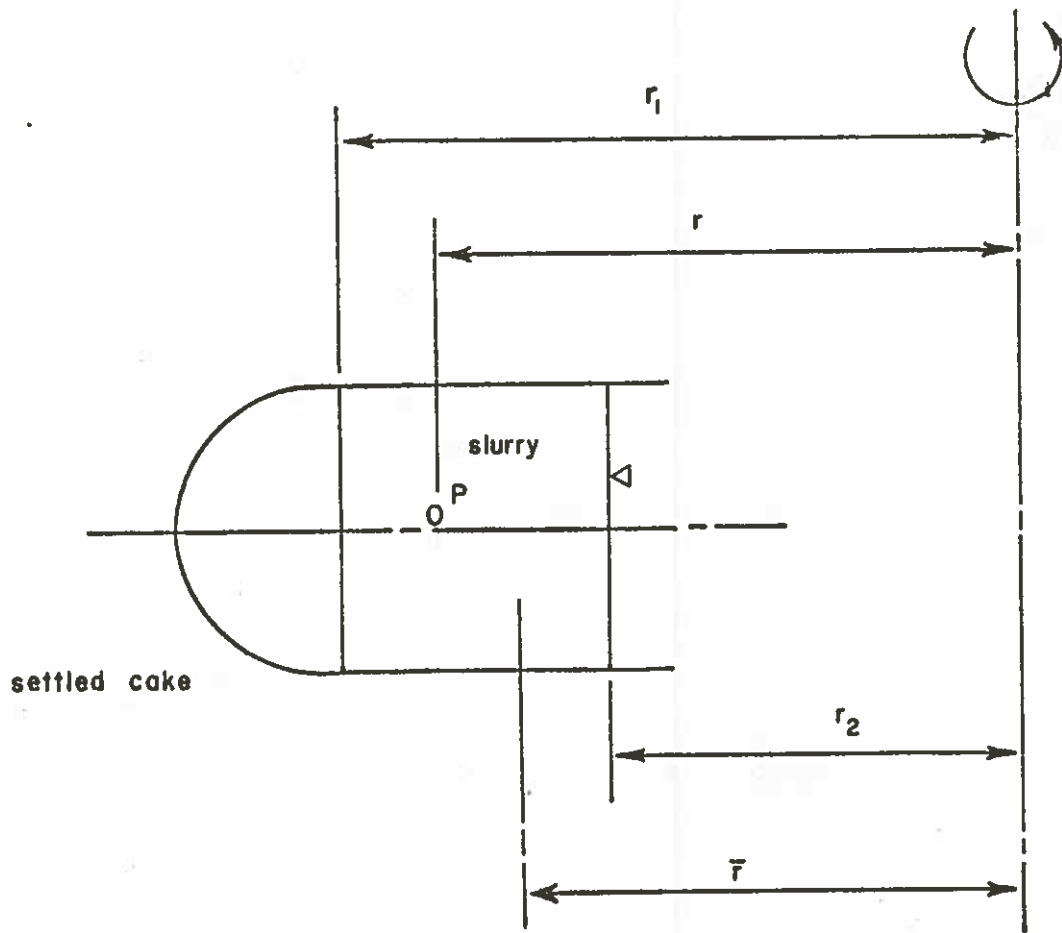


Figure 15: Model for analysis of the performance of a bottle centrifuge.

has elapsed. The effective capacity,  $Q_o$ , of the bottle centrifuge is determined by forming the ratio between the volume,  $\bar{V}$ , occupied by the sludge in the bottle in the spinning time,  $t$ , so that,

$$Q_o = \frac{\bar{V}}{t} = 2u_t \left( \frac{\omega^2}{g} \cdot \frac{\bar{V}}{2 \ln 2r_1/(r_1+r_2)} \right) \quad (\text{Eq.25})$$

This equation is then separated into two parts,  $u_g$ , characterizing the settling behavior of the solid particles, and the second factor, which refers to the physical parameters, which is called  $\Sigma_B$ . For a bottle centrifuge, it takes the form;

$$\Sigma_B = \frac{\omega^2 \bar{V}}{2g \ln 2r_1/(r_1+r_2)} \quad (\text{Eq.26})$$

By combining Equations 25 and 26 and eliminating the volume  $\bar{V}$ , the following relation is obtained;

$$\frac{Q_o}{\Sigma_B} = \frac{2g \ln 2r_1/(r_1+r_2)}{\omega^2 t} \quad (\text{Eq.27})$$

Tubular Bowl Centrifuge: The model chosen for this analysis is that of a cylinder rotating about its axis as shown in Fig. 16. The settling velocity determined from Equation 6 is;

$$u_t = \frac{dr}{dt} = \frac{\Delta \rho D_p^2 \omega^2 r}{18 \mu} \quad (\text{Eq.28})$$

The axial velocity of a particle is assumed identical to that of the continuous phase flowing through the cylinder, so that;

$$\frac{dZ}{dt} = \frac{Q_o}{\pi(r_2^2 - r_1^2)} = \frac{Q_o L}{\bar{V}} \quad (\text{Eq.29})$$

The ratio of Equation 28 to 29 when integrated between the limits  $r = r_2$  at

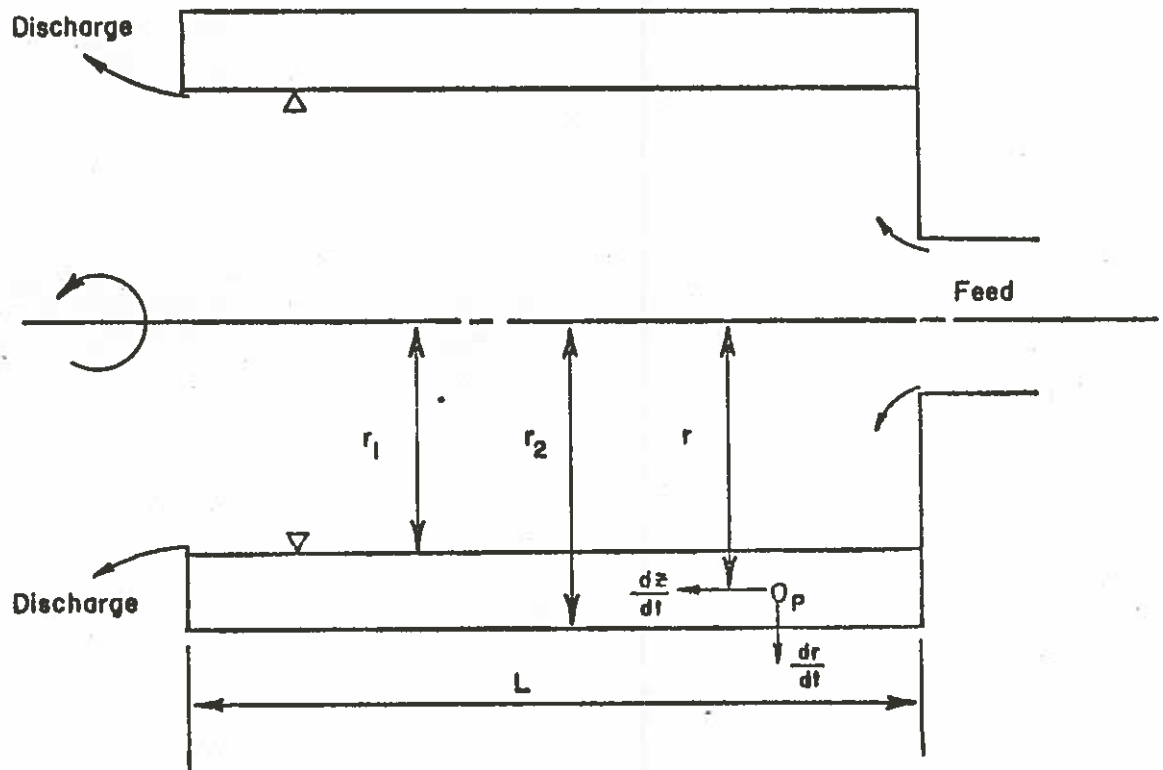


Figure 16: Model for analysis of the performance of a tubular bowl centrifuge.



$Z = L$ , and  $r = r$  at  $Z = 0$  determines the smallest radius,  $r$ , at which sedimentation of a particle of size  $D_p$ , may start in order that it reach the cylinder wall after covering the full length,  $L$ , of the separation zone. The radius,  $r$ , is expressed as;

$$\ln \frac{r_2}{r} = \frac{\bar{V} \Delta \rho D_p^2 \omega^2}{Q_0 18 \mu} \quad (\text{Eq.30})$$

if  $r$  is chosen in such a manner that;

$$\pi(r_2^2 - r^2) = \pi(r^2 - r_1^2) \quad (\text{Eq.31})$$

then the annular area confined between radius  $r_2$  and radius  $r$  equals the area between,  $r$ , and the free surface radius,  $r_1$ . This again denotes the 50% cutoff.

If  $r$ , from Equation 31 is substituted in Equation 30, and the logarithmic function approximated, Lavanchy and Keith,(25), show that the following relation is obtained;

$$Q_0 = 2u_t \left( \frac{\omega^2}{g} \frac{\bar{V}(3/2r_2^2 + 1/2r_1^2)}{(r_2^2 - r_1^2)} \right) \quad (\text{Eq.32})$$

$$Q_0 = 2u_t \left( \frac{\omega^2}{g} 2\pi L (3/4r_2^2 + 1/4r_1^2) \right) \quad (\text{Eq.33})$$

This again can be divided into two groups,  $2u_t$  and  $\Sigma_T$  where,

$$\Sigma_T = 2\pi L \frac{\omega^2}{g} (3/4r_2^2 + 1/4r_1^2) \quad (\text{Eq.34})$$

Disc-Type Centrifuge: A model showing the disc-stack is shown in Fig. 17. The flow of the continuous liquid phase containing solids is assumed to be

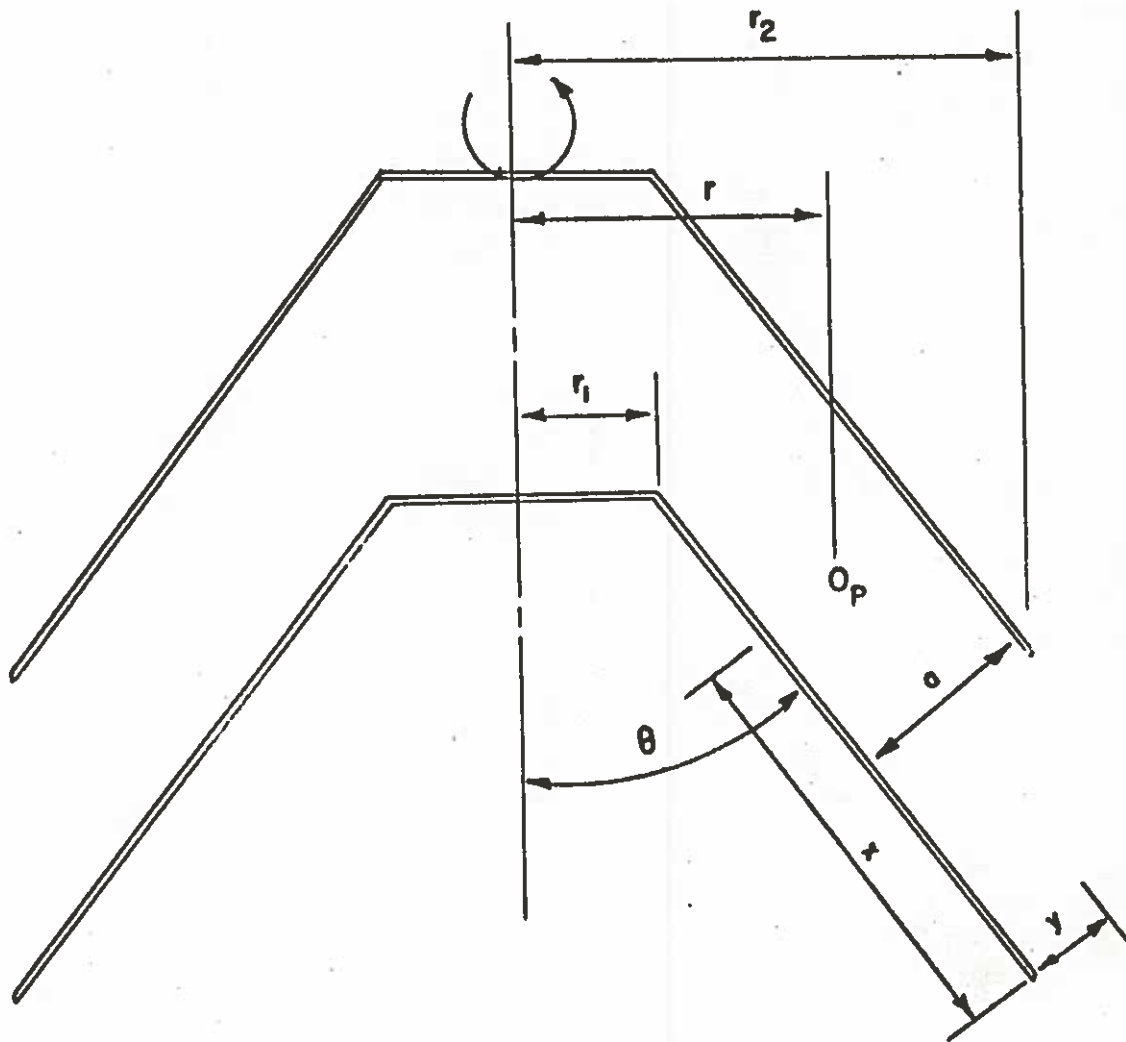


Figure 17: Model for analysis of the performance of a disc-type centrifuge.

evenly divided between the spaces formed by discs. The flow in each space is then  $Q_0/n$ . The velocity of the continuous liquid phase between the disc is;

$$\frac{dx}{dt} = - \frac{dr}{dt \sin \theta} = \frac{Q_0}{2\pi n r a} \quad (\text{Eq.35})$$

The settling motion of a particle of particular size  $d$  relative to the continuous phase is determined as previously by Stokes' Law. If the position,  $y$ , of the particle in the process of settling is measured perpendicularly from the upper surface of the lower of the two discs defining a disc space, the Equation 6 can be restated as;

$$\frac{dy}{dt} = \frac{dr}{dt} \cos \theta = u_t \frac{\omega^2}{g} \cdot r \cdot \cos \theta \quad (\text{Eq.36})$$

The ratio of Equation 36 to 35 is;

$$dy = - \frac{2\pi n a}{Q_0} \cot \theta u_t \frac{\omega^2}{g} r^2 dr \quad (\text{Eq.37})$$

Equation 37 integrated between the limits  $y = a/2$  at  $r = r_2$ ; and  $y = a$  at  $r = r_1$ , defines the throughput,  $Q_0$ , for which 50% separation of the entering particles is achieved, thus,

$$Q_0 = 2u_t \frac{2\pi n \omega^2}{3g} \cot \theta (r_2^3 - r_1^3) \quad (\text{Eq.38})$$

Again separation of this equation into two factors leads to the following relations.

$$Q_0 = 2u_t \Sigma_D \quad (\text{Eq.39})$$

where;

$$\Sigma_D = \frac{2\pi n}{3} \frac{\omega^2}{g} \cot \theta (r_2^3 - r_1^3) \quad (\text{Eq.40})$$

The  $\Sigma$  concept theoretically allows performance comparison between geometrically and hydrodynamically similar centrifuges operating on the same feed material. Equations 25, 33, and 38 shows that the sedimentation performance of any two similar centrifuges treating the same suspension will be the same, if the quantity  $Q_0/\Sigma$  has the same value for each. In practice, the introduction of an efficiency factor,  $e$ , is necessary to make possible the extension of the use of  $\Sigma$  to comparisons involving dissimilar centrifuges. This factor,  $e$ , takes into account the different levels of turbulence, remixing, etc. which exist in different centrifuges.

The overflow rate  $Q_{o2}$ , of a centrifuge #2 can now be compared with that rate,  $Q_{o1}$ , of a centrifuge #1 operating on the same feed. For equal performance, the following applies;

$$Q_{o2} = Q_{o1} \frac{e_2 \Sigma_2}{e_1 \Sigma_1} \quad (\text{Eq. 41})$$

If the centrifuges are geometrically and hydrodynamically similar, then,  $e_1 = e_2$ , and equation 41 is simplified to;

$$Q_{o2} = Q_{o1} \Sigma_2/\Sigma_1 \quad (\text{Eq. 42})$$

Tests have shown (25) that, if the efficiency of the bottle centrifuge is taken as 100%, the efficiency factors for tubular and disc centrifuges may be approximated as follows:  $e_T = 90\%$ ,  $e_D = 40\%$ . These figures do not apply to all practical separations, but do show the wide variation to be expected from calculated characteristics.

Ambler (26) has shown that the particle size distribution of the sludge should have a normal or Gaussian distribution, to provide accurate predictions of centrifuge performance. Collier (26) indicated that prediction from one centrifuge to

another should be attempted only when a plot of  $Q_0/\Sigma$  vs. % unsedimented solids is a straight line on log-probability paper.

The reason for requiring geometric and hydrodynamic similarity is to provide similar conditions of turbulence and conveyance of the sludge.

The sigma concept is restricted by the same assumptions as pertained to the development of Stokes' Law. Nevertheless, approximate predictions of performance can be made using this concept.

### Pilot Plants

It is easy to see that the previous methods of selecting a centrifuge are not ideal. They only provide a good starting point in approximating the type of centrifuge which would be applicable. Before a final selection is made tests on a centrifuge of equivalent proportions, but a smaller size, should be conducted.

These test machines are available on rental and/or trial basis from centrifuge manufacturers. Several of the manufacturers have also designed and built portable pilot plant units. These are truck, van, or skid mounted and can be moved from location to location with relative ease. Operation of the test unit through the entire range of process variables and with various chemical dosages usually provides accurate information necessary for selecting the most economical full scale installations.

Many times the solids handling properties of the centrifuge are more of a problem than clarification. The build-up of solids in the discharge housing, unusually high torque and/or power requirements, difficulty in conveying slick materials, discharging solid materials that mat, and other factors, may limit the applicability of certain centrifugal equipment (23).

When scaling up pilot plant data to larger units, it is necessary to consider the residence time of the feed in the bowl, the centrifugal force acting on the solids, the solids thickness, and residence time of the solids on the beach (6).

Calculation for the scale-up of continuous solid-bowl centrifuges can be performed as follows (6). The pool volume is determined from the geometry of a given centrifuge bowl (Fig. 18) using the location of the over-flow points at maximum level. The residence time in seconds of the feed in the bowl is determined by;

$$R_t = \frac{\text{Pool Volume}}{\text{Feed Rate}} \quad (\text{Eq. 43})$$

The average centrifugal force acting on the particles in the pool can be calculated from the bowl radius and the depth of liquid in the pool;

$$F_c = m\omega^2 \left( \frac{r_2^2 + r_1^2}{2} \right) \quad (\text{Eq.44})$$

The feed retention time plotted versus the solids content in the effluent liquor or centrate with  $F_c$  as a parameter will correlate the centrifuge performance for varying feed rates and pool depth.

The residence time of solids on the beach ( $R_s$ ) is an important consideration when the moisture content is critical.  $R_s$  can be approximated by the following;

$$R_s = \frac{L_2}{C_s} \quad (\text{Eq.45})$$

$$C_s = \frac{BP}{G} \quad (\text{Eq.46})$$

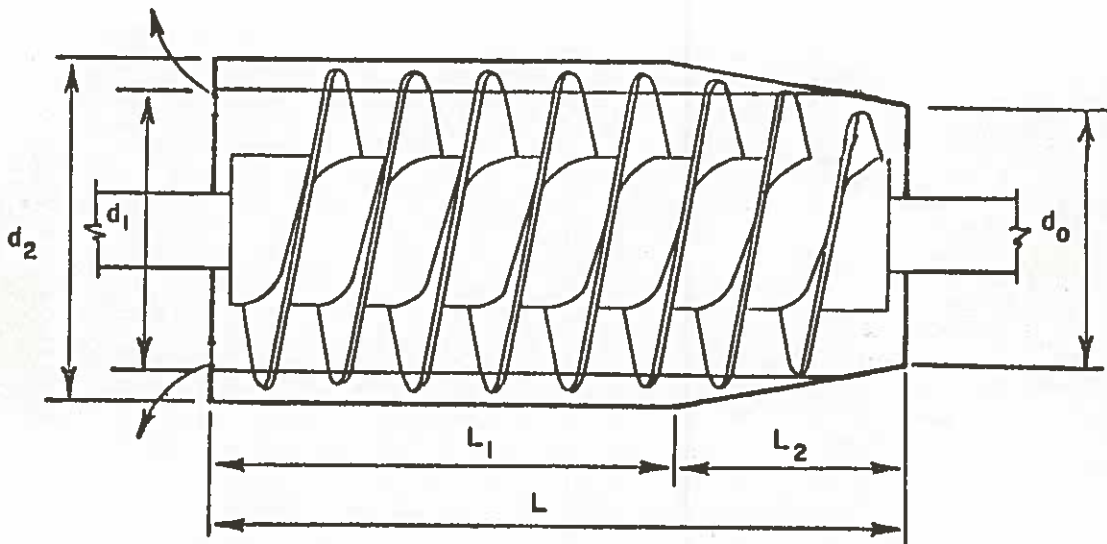


Figure 18: Model for the geometry of a centrifuge bowl.

where;

$L_2$  = length of beach L

$C_s$  = relative conveyor speed  $L T^{-1}$

B = bowl speed  $T^{-1}$

P = conveyor pitch L

G = gear ratio  $L^0 M^0 T^0$

The residence time of the solids on the dry beach plotted against solids moisture-content in the final cake (with average centrifugal force acting on the cake as a parameter) correlates most of the variables affecting final product dryness.

Cake thickness on the beach will affect final cake moisture. A correlation of the cake thickness versus cake moisture should be included in the data analysis and scale-up considerations. Variations of feed rate with other considerations kept constant will provide the data.

A cross-section of the cake as it is pushed in front of the conveyor is likely to be triangular. By assuming that the solids form a uniform annular layer on the periphery of the bowl at the point of discharge, the cake thickness can be calculated by;

$$T = 6(D_2 - D_4) \quad (\text{Eq.47})$$

where

$$D_4 = (D_2^2 - 4A/\pi)^{0.5} \quad (\text{Eq.48})$$

and;

$$A = P/C_s D_c \quad (\text{Eq.49})$$



where;

A = cake area at discharge

P = rate of wet cake discharge

$D_c$  = wet cake density.

Using this information with scale up factors provided by the manufacturer, correlation with larger units and performance characteristics can be predicted (6).

### Analytical Approach

After a centrifuge has been selected, the maximum pool depth to remove a particle size  $D_p$  can be determined mathematically. This approach is dependent on the geometry of the centrifuge bowl, the rotational speed of the bowl, the physical properties of the sludge and the flow rate through the centrifuge.

Considering a particle of diameter  $D_p$  the settling trajectory can be shown as in Figure 19. The horizontal velocity is:

$$u_H = \frac{Q}{A} = \frac{Q}{\pi(r_2^2 - r_1^2)} \quad (\text{Eq.50})$$

The terminal velocity in the radial direction is a function of the radius and has been determined previously to be:

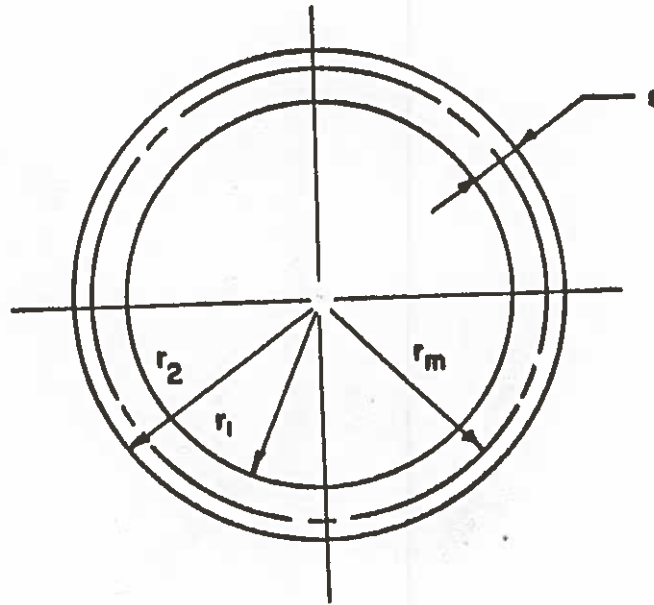
$$u_t = \frac{(\rho_p - \rho_l) D_p^2 \omega^2 r}{18 \mu} \quad (\text{Eq.6})$$

Also  $u_t$  can be expressed as:

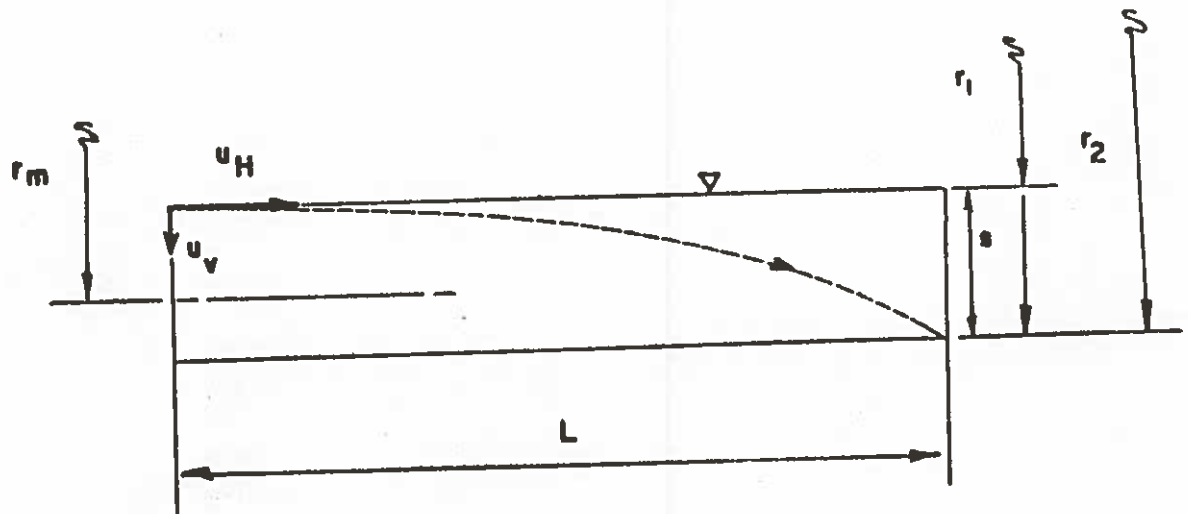
$$u_t = \frac{dr}{dt} = \frac{(\rho_p - \rho_l) D_p^2 \omega^2 r}{18 \mu}$$

rewriting and integrating,

$$\int_{r_1}^{r_2} \frac{dr}{r} = \frac{(\rho_p - \rho_l) D_p^2 \omega^2}{18 \mu} \int_0^t dt \quad (\text{Eq.51})$$



(a)



(b)

Figure 19: Settling trajectory of a solid particle.

$$\ln \frac{r_2}{r_1} = - \ln \frac{r_1}{r_2} = \frac{(\rho_p - \rho_1) D_p^2 \omega^2}{18 \mu} \cdot t \quad (\text{Eq.52})$$

letting

$$\frac{(\rho_p - \rho_1) D_p^2 \omega^2}{18 \mu} = \text{a constant, } C$$

we find

$$t = - \frac{1}{C} \ln \frac{r_1}{r_2} = \frac{1}{C} \ln \frac{r_2}{r_1} \quad (\text{Eq.53})$$

where  $t$  is the time to completely remove a particle of diameter  $D_p$ . Equation 53 is plotted in Figure 20. The abscissa of the graph is given as "pool depth" which is:

$$\text{pool depth} = r_2 - r_1 \quad (\text{Eq.54})$$

The detention time is expressed as:

$$\tau = \frac{L}{u_H} = \frac{L\pi(r_2^2 - r_1^2)}{Q_0} \quad (\text{Eq.55})$$

or

$$\tau = \frac{\pi L}{Q_0} (r_2^2) - \frac{\pi L}{Q_0} (r_1^2)$$

This equation is also plotted on Figure 20.

From this graph the value of  $r_1$  which would produce 100% removal of a particle with diameter  $D_p$  can be found. Increases in  $r_1$ , resulting in a shallower pool, would decrease the time required to remove the particle with a diameter  $D_p$ , and also allow smaller particles ( $D_p^1 < D_p$ ) to be 100% removed.

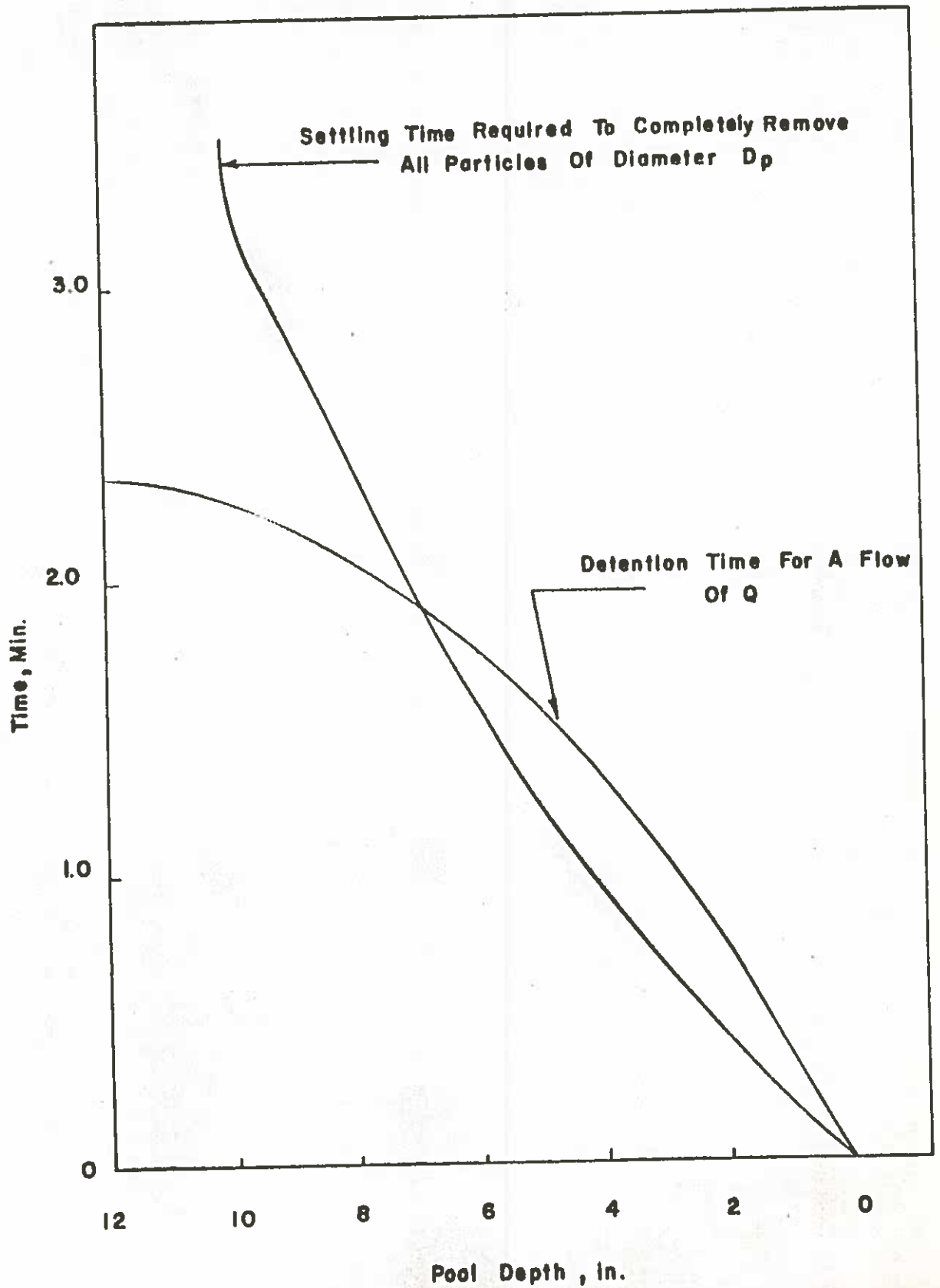


Figure 20: Detention time and settling time of a particle  $D_p$  as a function of pool depth.

## C H A P T E R VIII

## CONCLUSIONS

Centrifuges are becoming more and more popular as part of the answer to the problem of disposing of wastewater sludge. This increasing popularity is partially the result of some inherent advantages which it has over other types of mechanical dewatering equipment, and the unsuitability of sand beds and lagoons in densely populated areas. The fact that centrifuges have not monopolized the dewatering field indicates that they are not without their disadvantages.

The advantages are: (a) the low capital cost in comparison with other mechanical equipment; (b) moderate operating and maintenance cost; (c) minimization of odor problems due to the unit being completely enclosed; (d) small space required, and the fact that the unit does not have to be installed indoors; (e) chemical conditioning is not always required; (f) flexibility of the unit in handling a wide variety of sludge; (g) the ability to function as a thickener as well as a dewatering unit; (h) the unit requires a minimum of supervision, and (i) the ability to dewater some sludge which the vacuum filter cannot dewater (2).

The disadvantages of a centrifuge are: (a) the recovery of solids is very poor with some types of sludge, without the use of chemicals; (b) the use of chemicals eliminates one of the big advantages over vacuum filtration; (c) screening of influent is sometimes required; (d) the resulting cake is often wetter than the cake produced by vacuum filtration, and (e) maintenance costs are high (2).

The design of the sewage treatment plant plays an important role in determining the effectiveness of a centrifuge in dewatering sludge. It was shown that the lower the ratio of primary solids to secondary solids in the waste sludge to be dewatered, the more difficult the sludge was to dewater. By increasing the primary efficiency of a sewage treatment plant, the primary to secondary sludge ratio would be increased and dewatering would be facilitated. Besides the advantage of a drier cake and a lower polymer requirement, there is the added advantage of smaller secondary treatment system and increased capacity for a given dewatering station.

The problems of selection of a centrifuge are minimized by the many adjustments which can be made to centrifuges in the field to obtain satisfactory results. The advances which have been made in the development of polyelectrolytes, which permits the percent of solid recovery to be increased to any amount, has also aided in the selection of centrifuges.

Still, the lack of acceptable design criteria to properly determine the size of centrifuge required, would seem to be a serious drawback in their acceptance by consulting engineers. Development of this design criteria is difficult because of the flocculant nature of the waste sludge and the difficulty in developing bench-tests which adequately reflect the dewatering process of a centrifuge.

Many of the earlier problems in using centrifuges have been overcome by improved machine design and the application of improved chemical aid. Further developments in these areas will increase the acceptability of centrifuges for a wider range of the waste treatment field.

In conclusion, whenever it is desirable to employ mechanical processes for concentrations of waste sludge, consideration of a centrifugal unit could very well lead to the optimum answer for a dependable and economical process.

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