If the FOV is limited, can we use fewer control elements (switches, phase shifters, time delay units) to save cost?

Subarrays reduce control elements, but produce quantization lobes.
Suppose we reshape the subarray pattern,

Maximum scan range that suppresses quantization lobe is:

$|u_0| < \frac{\lambda}{2D}$
Therefore, the largest allowed subarray is
\[ D < \frac{\lambda}{2 \text{asin} \theta_{\text{max}}} \]

The number of elements in the subarray is
\[ n_{\text{elem}} = \frac{D}{d} \]

and the number of control units is reduced by \( 1/n_{\text{elem}} = d/D \).

How do we produce a subarray pattern?

Using Fourier Transform synthesis, the required aperture distribution is
Cannot produce the desired subarray pattern with a subarray of size $D$!

Solution: Use most, or all, of the array to produce the subarray pattern via overlapped subarrays.

Note that the formation of the array pattern from several subarrays is based on the phase centers of subarrays spaced $D$. 

We can use networks that form multiple beams from the array to simultaneously produce several $\sin(x)/x$ subarrays that are spaced $D$ apart (phase centers) and overlap to use the entire array.
The array of \( N \) elements can be used to form \( M \) narrow beams covering \( 1U \leq \sin \Theta_{\text{max}} \) as follows.

A signal \( S_m \) at one of the subarray ports is distributed to all \( N \) elements producing aperture illumination

\[
A_m(x) = \frac{\sin[\pi \frac{x-x_m}{D}]}{\pi \frac{x-x_m}{D}}
\]
To steer array beam, the phase of each subarray is adjusted.

\[ S_1 = 1 \text{e}^{j \theta_0} \] \[ S_2 = 1 \text{e}^{j \frac{1}{2} \pi \sin \theta_0} \]

The \( M \times M \) beamformer \( A \) produces these required \( S_m \) inputs to the subarray ports.
An input to one beam port, $B_m$, produces the $M$ signals $S_m$ with progressive phase to steer beam in direction $\theta_m$. For example, Butler matrix does this.
Discussion:

1. Where to put amplifiers?
   - At elements: Many, low-power
   - At subarrays: Few, high-power

2. As the operating frequency changes, beam orthogonality (if it exists) may be lost. In general, the multiple beam networks may have nonzero beam coupling, which affects performance.

3. We used ideal array theory, and the subarray patterns still differed because $\text{sinc}(x/k)$ is truncated differently for each subarray. Mutual coupling will affect performance, also.
4. Instead of using multiple beam forming network to produce overlapped subarrays, quantization lobes can be reduced by randomizing subarray phase centers.

Also, recall polyomino subarrays. See text for other randomizing schemes.

5. Angle filters in front of aperture can suppress quantization lobes, but the lobe power is still radiated from aperture and must be absorbed.
6. The MXN network to form M fully overlapped subarrays with \( \sin(x)/x \) amplitude is quite complicated. It is not practical to form more than 4 or possibly 8 subarrays.

7. A two-transform feed, for example a Butler matrix and a lens, can eliminate the MXN beam former.

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**Figure 8.33** Completely overlapped subarray (dual transform) lens perspective: (a) feed illumination and radiated subarray pattern; and (b) synthesized aperture taper and array radiation pattern. (From: [77]. © 1986 IEEE. Reprinted with permission.)
The scan range available for a two-transform system is approximately equal to the angle subtended by the feed array.

The text has analytical "derivation." In terms of $\sin(\theta) \times x$ subarrays:
\[ \theta_f \sim \frac{a \lambda}{W_f} \quad \text{(Null-to-Null)} \]

\[ 2D \sim F \theta_f = \frac{a F \lambda}{W_f} \]

\[ \theta_{ff} \sim \frac{\lambda}{D} = \frac{\lambda W_f}{F \lambda} = \frac{W_f}{F} \]

\[ \epsilon_{feed} = 2 \sin^{-1}(\frac{W_f}{2F}) \sim \frac{W_f}{F} = \theta_{ff} \]