# ECE 581 FEEDBACK CONTROL SYSTEMS (II)

## Final Exam

Thursday, May 20, 2005
4:00 P.M. – 6:00 P.M.

The exam is open book and open notes. You must show your work to receive partial credit. The problem values are:

| | |
|---|---|
| Problem #1 | 45 Points |
| Problem #2 | 30 Points |
| Problem #3 | 25 Points |
| Total | 100 Points |

**Problem 1:** (45 points)

A system consists of two tanks of water. The first tank is fed by a controllable source. The second tank is fed by the outflow of the first tank, which is proportional to the level of water in the first tank. The second tank also has an outflow proportional to the level of the water in the second tank. A dynamic model of the level of water in the second tank (using the controllable source to the first tank as the input) is given by:

$$G(s) = \frac{0.0005}{s^2 + 0.03s + 0.0002}$$

The step response of this model has the partial fraction expansion:

$$\frac{1}{s}G(s) = \frac{2.5}{s} - \frac{5}{s+0.01} + \frac{2.5}{s+0.02}$$

Assume the following specifications for controlling the level of water in the second tank:

1) The system is to be digitally controlled with a sample period of 10 seconds.
2) The steady-state error between the water level in the second tank and a commanded (reference) value following a step change in the reference value is zero.
3) The crossover frequency of the compensated system is 0.025 rad/sec.
4) The phase margin is 50°.

a) (5 points) Draw a block diagram of the unity feedback digital feedback system. The only exogenous signal that should be included in your diagram is the reference.

b) (10 points) Determine the step invariant model for $G(s)$, and draw a block diagram of the equivalent discrete-time system. (You do not need to perform any simplifications beyond the Z-transform of the step-invariant model).

c) (10 points) Define a continuous-time design model $G_{des}(s)$ that incorporates the zero-order hold.

d) (10 points) Design a continuous-time PI controller such that the specifications are satisfied.

e) (10 points) Transform the continuous-time PI controller to a discrete-time PI controller using the bilinear (Tustin's) method with pre-warping. Use the crossover frequency as the pre-warp frequency.

**Problem 2:** (30 points)

The system shown in Figure 1 is known to have a limit cycle with frequency 2 rad/sec and amplitude 0.5.

a) (10 points) Determine the ultimate frequency $\omega_u$ and ultimate gain $K_u$ (as defined by the Ziegler-Nichols closed-loop tuning method). **Hint**: Use describing functions to determine the limit cycle. The describing function of the ideal relay shown in Figure 1 is:

$$N(M) = \frac{4A}{\pi M} \qquad \text{where } A = 1$$

b) (15 points) Design a PID controller for $G(s)$ using the Ziegler-Nichols closed-loop tuning method. Assume $\omega_u = 2$ and $K_u = 2.5$.

c) (5 points) What is the expected damping ratio of the closed-loop poles?

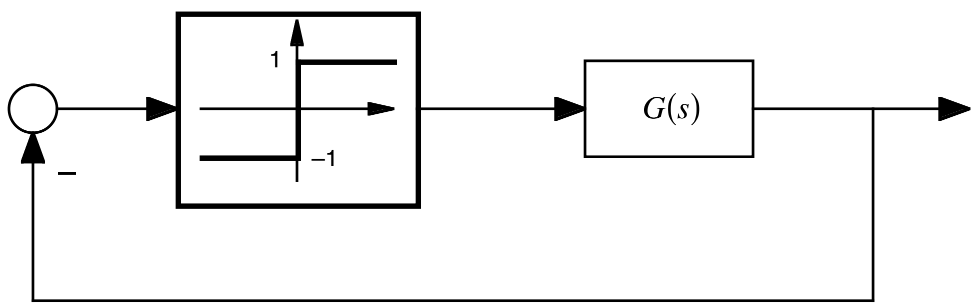


**Figure 1. Relay-based feedback system for Problem 2.**

**Problem 3:** (25 points)

Consider the system shown in Figure 2. The transfer function $G(s)$ is given by:

$$G(s) = K_p \frac{3(s+2)}{s(s-1)(0.1s+1)^2}$$

Figure 3 shows a Nyquist plot of $G(s)$ for $K_p = 1$, while Figure 4 shows the Bode plots of $G(s)$ for $K_p = 1$. Note that the open-loop transfer function is unstable, and that the Nyquist locus closes to the left of the critical point. Thus, if $G(s)$ were used in a unity feedback loop with $K_p = 1$, the system would be stable. The describing function for the dead-zone nonlinearity in Figure 2 is

$$N(M) = \begin{cases} 0 & M < A \\ K\left(1 - N_s\left(\frac{M}{A}\right)\right) & M \geq A \end{cases}$$

where $K = 2$ and $A = 0.5$. Note that $N_s(x)$ is a decreasing function of $x$ with $N_s(1) = 1$ and $\lim_{x \to \infty} N_s(x) = 0$

a) (15 points) Draw the gain locus of the describing function of the nonlinearity in the feedback loop..

b) (5 points) Find the frequency and magnitude of the potential limit cycle identified in part a).

c) (5 points) For each potential limit cycle, determine if it is stable.

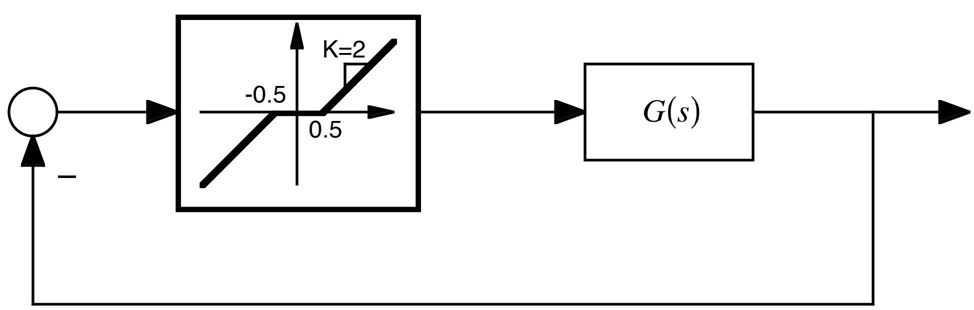


**Figure 2. Feedback system for problem 3.**

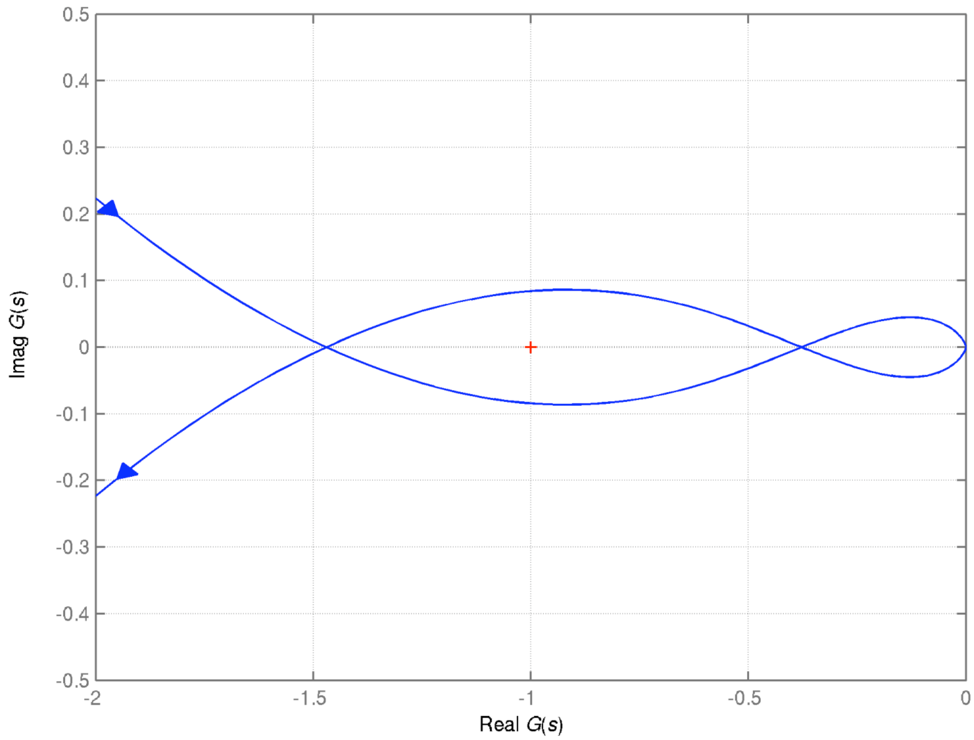


**Figure 3. Nyquist plot for Problem 3 with $K_P = 1$.**

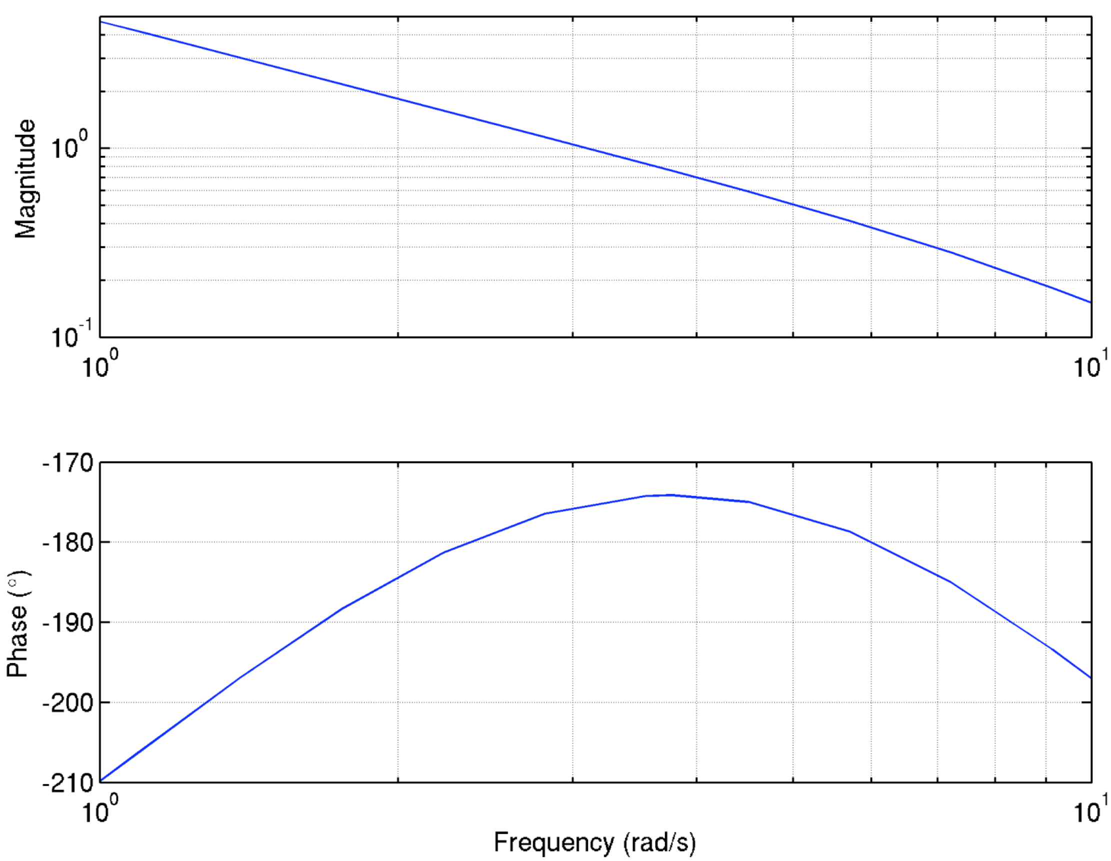


**Figure 4. Bode plot for Problem 3 with $K_P = 1$**