Name ______________________________

Problem #1:	_______
2:	_______
3:	_______
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Total:	_______

ECE 580 FEEDBACK SYSTEMS (I)

Fall 2011

Final Exam

Tuesday, December 12, 2011

Consistent with the ECE Honor Code, you are asked to read the following voluntary statement carefully and sign it before beginning your work in each exam booklet:

I have not given or received unauthorized aid on this exam.

____________________________________ Signature ____________________ Date

Instructions

The exam is open book and notes. No electronic devices of any kind (including calculators) can be used during the exam. Questions are to be answered on the exam.

There are ten questions with the indicated values.

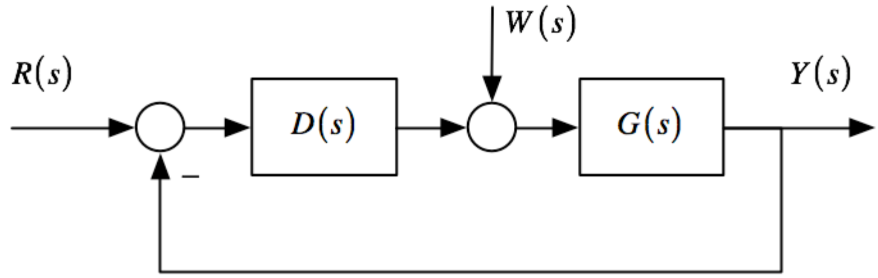


Figure 1

Questions 1 through 7 address the unity feedback system shown in Figure 1. The bode plot of the plant transfer function $G(s)$ is shown in Figure 2.

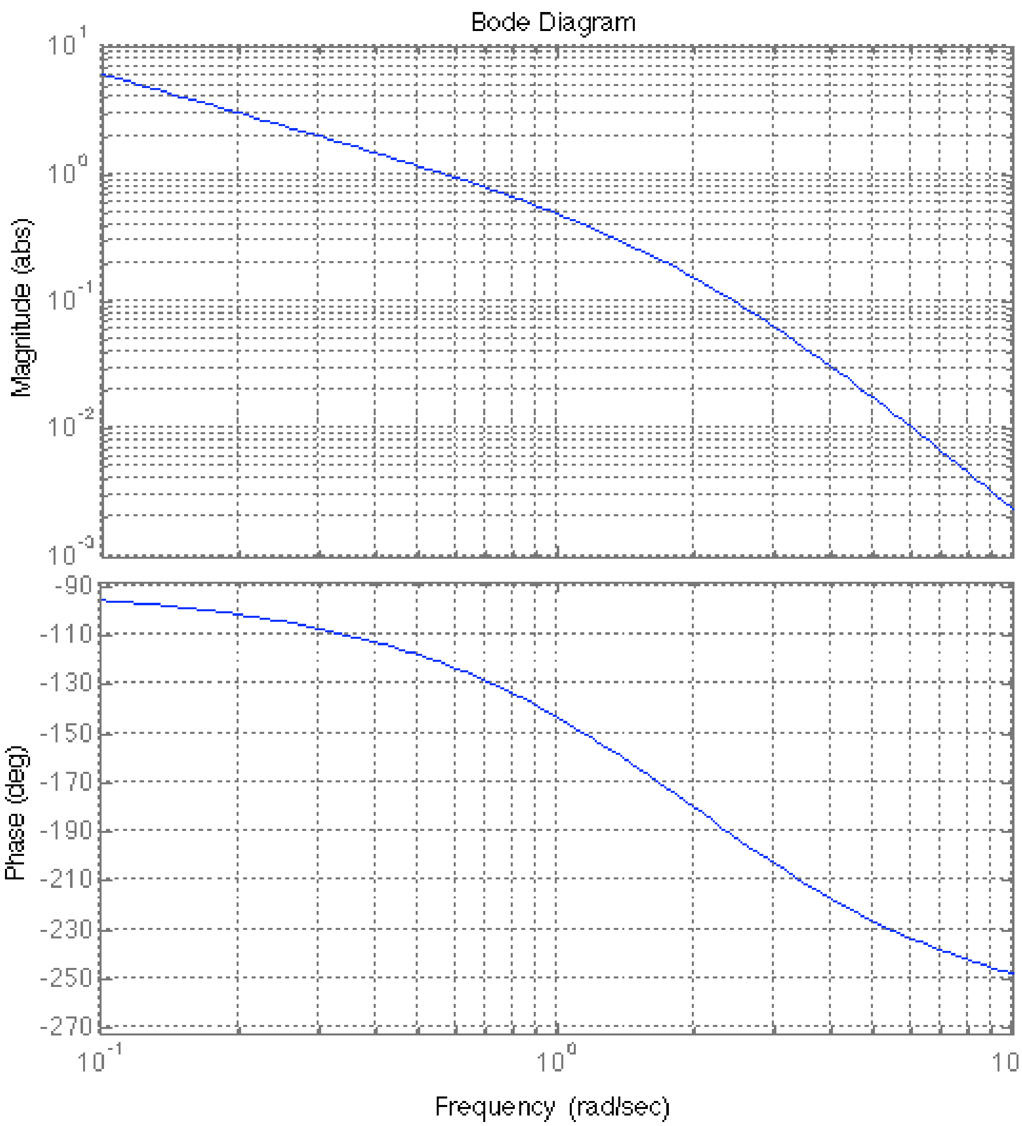


Figure 2

Problem 1: (15 points) Assume the compensator is $D(s)=1$. Find the position constant K_p, the velocity constant K_v, and the acceleration constant K_a of the closed-loop system.

$K_p = \infty$
$K_v = 2.5$
$K_a = 0$

The loop Bode magnitude plot Figure 2 has a low-frequency asymptotic slope of –20 db/dec which indicates 1 pole at the origin. Thus, the posotion constant is infinity and the acceleration constant is zero. The velocity constant is found from the low-frequency asymptote at 1 r/s: 0.6. Then

$$K_v = \frac{1}{0.6} = 2.5$$

Problem 2: (10 points) Assume the compensator consists of a single integrator element

$$D(s) = \frac{K_I}{s}$$

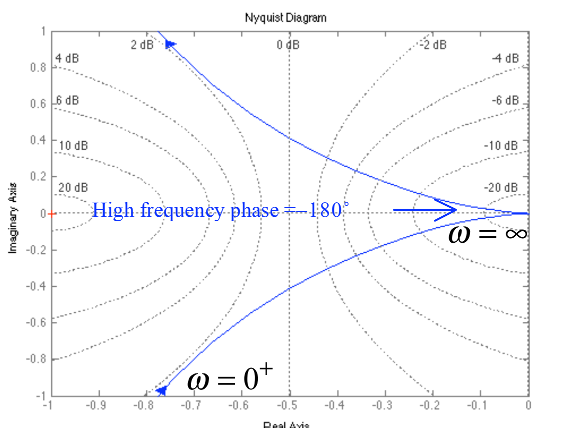
Can the system be Type 2 with respect to references? If not, why?

Answer (circle 1): Yes **No**

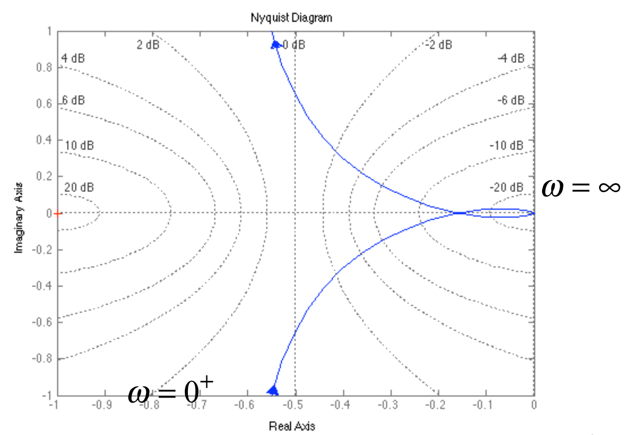
Reason if No: The phase of the plant transfer function Figure 2 is always less than –90˚. The phase of a compensator consisting of an integral element is –90˚. Thus, the phase of the loop transfer function is less than –180˚. The system closed-loop cannot be stable, and thus the error for a unit step input cannot go to zero.

Problem 3: (10 points) The following figures show the Nyquist diagrams of 4 systems. In each figure, if the diagram is not completely contained in the figure, the Nyquist locus grows to infinity in the direction it leaves the diagram. In these cases, the Nyquist curve for slightly negative frequencies $\omega = 0^{-}$ joins the Nyquist curve for slightly positive frequencies $\omega = 0^{+}$ with a large right semi-circle (the curve closes to the right). Which one corresponds the Bode plots in Figure 2? (i.e., which is the Nyquist diagram of $G(s)$?)

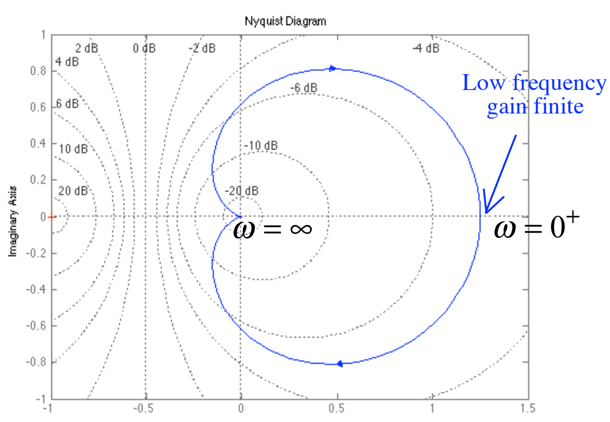
Nyquist diagram corresponding to Bode plot in Figure 2: (b)



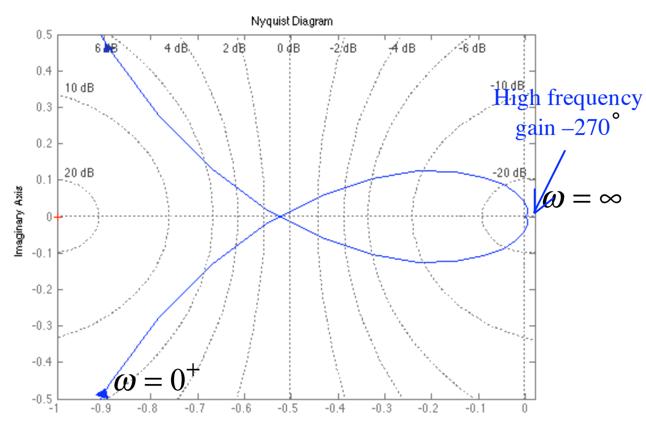
(a)



(b)



(c)



(d)

Problem 4: (10 points) The compensator consists of a proportional element: $D(s) = K$. What value for K results in a gain crossover $\omega_g = 1$ r/s?

$K = 2$

$$K = \frac{1}{\left|L\left(j\omega_g\right)\right|} = \frac{1}{0.5} = 2$$

Problem 5: (15 points) A proportional compensator $D(s) = K$ is selected so the gain crossover is $\omega_g = 1$ r/s. What are the gain increase margin γ_M, the gain decrease margin γ_m, the phase margin ϕ_M and the phase crossover ω_p? (Note: Approximate answers are acceptable.)

$\gamma_M = 6$
$\gamma_m = 0$
$\phi_M = 35^\circ$
$\omega_p = 2$ r/s

The phase of the loop transfer function is the same as the phase of the plant transfer function for a proportional compensator, and thus is the given by Figure 2. The magnitude plot of the loop transfer function is the same as Figure 2 shifted vertically to yield a magnitude of 1 at 1 r/s (gain crossover = 1 r/s). The phase crossover is the frequency at which the phase is -180°:

$$\omega_p = 2 \text{ r/s}$$

The gain at this frequency is approximately

$$\left|L(j2)\right| = 0.16$$

Thus, the gain increase margin is:

$$\gamma_M = \frac{1}{\left|L(j2)\right|} \approx 6$$

Since the phase is always greater than -180° below the gain crossover frequency, the gain reduction margin is 0. The phase margin is:

$$\phi_M = 180 + \measuredangle L(j1) \approx 180 - 145 = 35^\circ$$

Problem 6: (15 points) Using the symmetric optimum design technique, design a compensator for the system shown in Figure 1 and Figure 2 consisting of a single lead element

$$D(s) = D_{\text{lead}}(s) = K_l \frac{Ts+1}{\alpha Ts+1}$$

The gain crossover is to be $\omega_g = 1$ r/s, and the phase of the lead element is to be 30° at the gain crossover frequency ($\measuredangle D_{\text{lead}}(j\omega_g) = 30^\circ$).

$$K_l = \frac{2}{\sqrt{3}} \approx 1.2$$

$$T = \sqrt{3} \approx 1.7$$

$$\alpha = \frac{1}{3}$$

$$\alpha = \frac{1-\sin 30^\circ}{1+\sin 30^\circ} = \frac{1-0.5}{1+0.5} - \frac{1}{3}$$

$$T = \frac{1}{\omega_g \sqrt{\alpha}} = \sqrt{3} \approx 1.7$$

$$K_l = \frac{\sqrt{\alpha}}{\left|G_p\left(j\omega_g\right)\right|} = \frac{2}{\sqrt{3}} \approx \frac{2}{1.7} \approx 1.2$$

Problem 7: (5 points) The feedback system in Figure 1 is to have a step response with a peak overshoot no greater than 10%. The gain crossover is to be $\omega_g = 1$ r/s. Assume the closed-loop system can be approximated by a two-dominant pole model. What is the phase of the compensator at gain crossover?

$$\boxed{\measuredangle D\left(j\omega_g\right) = 25^\circ}$$

The damping ratio of the dominant pole pair should be $\zeta = 0.6$ to have a peak overshoot of less than 10%. The, the phase margin should be:

$$\phi_M = 100\zeta = 60^\circ$$

This means the phase of the compensator should be:

$$\measuredangle D\left(j\omega_g\right) = -180 + \phi_M - \measuredangle G_p\left(j\omega_g\right) = -180 + 60 - \left(-145\right) = 25^\circ$$

The following system will be used as the basis Problems 8 through 10. Consider the plant whose continuous-time transfer function is:

$$G(s) = \frac{1}{s+1} \qquad\qquad T = 0.2 \text{ sec} \qquad\qquad (1)$$

The output of this system is to be controlled using a sampled-data feedback architecture as shown in Figure 3.

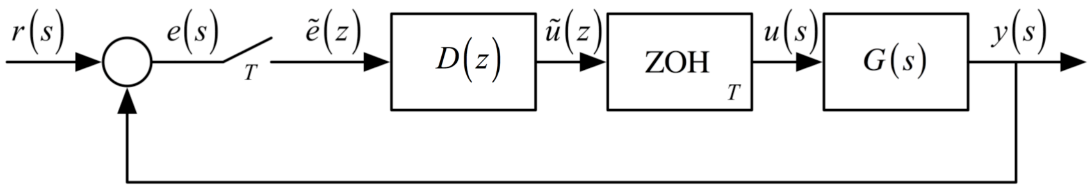


Figure 3

When the discrete-time controller design is to be based on the emulation of a continuous-time controller, the continuous-time feedback system shown in Figure 4 will be used. Design of the discrete-time controller can also be performed using the discrete-time feedback system shown in Figure 5.

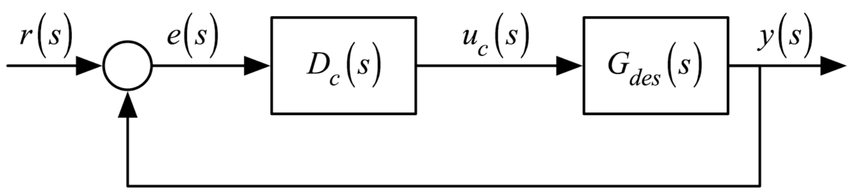


Figure 4

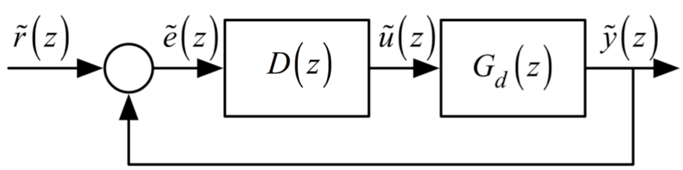


Figure 5

Problem 8: (5 points) The model of the plant in the discrete-time system Figure 5 is the step-invariant model of the plant $G(s)$ in (1). Find $G_d(z)$.

$$\boxed{G_d(z) = \frac{1-e^{-0.2}}{z-e^{-0.2}}}$$

The discrete-time plant model in Figure 5 is the step-invariant model:

$$G_d(z) = \left(1-z^{-1}\right)\mathbb{Z}\left\{\frac{G(s)}{s}\right\} = \left(1-z^{-1}\right)\mathbb{Z}\left\{\frac{1}{s(s+1)}\right\} = \left(1-z^{-1}\right)\mathbb{Z}\left\{\frac{1}{s} - \frac{1}{s+1}\right\}$$

$$= \left(1-z^{-1}\right)\mathbb{Z}\left\{\left(1-e^{-t}\right)_{t=kT}\right\} = \left(1-z^{-1}\right)\mathbb{Z}\left\{1-e^{-0.2k}\right\}$$

$$= \left(1-z^{-1}\right)\left(\frac{1}{1-z^{-1}} - \frac{1}{1-z^{-1}\left(e^{-0.2}\right)}\right) = 1 - \frac{z-1}{z-e^{-0.2}} = \frac{z-e^{-0.2}}{z-e^{-0.2}} - \frac{z-1}{z-e^{-0.2}}$$

$$= \frac{1-e^{-0.2}}{z-e^{-0.2}}$$

Problem 9: (5 points) The continuous-time design model $G_{des}(s)$ in Figure 4 includes a representation of the effects of the ZOH. If the ZOH is modeled by a first-order system, what is $G_{des}(s)$?

$$\boxed{G_{des}(s) = \frac{1}{(0.1s+1)(s+1)}}$$

$$G_{des}(s) = G_{ZOH}(s)G(s) \approx \frac{1}{{}^{T}\!/_{2}\,s+1}\frac{1}{s+1} = \frac{1}{(0.1s+1)(s+1)}$$

Problem 10: (5 points) The continuous-time controller in Figure 4 is a PI controller:

$$D(s) = \frac{s+1}{s}$$

Emulate this controller in discrete-time using the Tustin transformation.

$$\boxed{D(z) = \frac{1.1z - 0.9}{z-1}}$$

$$D(z) = \left.\frac{s+1}{s}\right|_{s=\frac{2}{T}\frac{z-1}{z+1}} = \frac{\frac{2}{T}\frac{z-1}{z+1}+1}{\frac{T}{2}\frac{z-1}{z+1}} = \frac{z-1+\frac{T}{2}z+\frac{T}{2}}{z-1} = \frac{\left(\frac{T}{2}+1\right)z-\left(1-\frac{T}{2}\right)}{z-1},$$

$$= \frac{1.1z - 0.9}{z-1}$$