Lab 5: Building an AM Radio Receiver

Objectives
In this lab you will analyze and test a Tuned Radio Frequency (TRF) circuit for an AM radio receiver. You will determine the resonant frequency of the TRF circuit experimentally and compare this result to what you found analytically in the pre-lab (taking into account some of the parasitic components of the measuring devices). You will also revisit some filters from a previous lab experiment, and analyze the Bode plots of their transfer functions.

Pre-Lab Instructions

Calculations
Consider the TRF circuit shown in Figure 1. It consists of a variable capacitor and a coil, connected in parallel. Such a circuit exhibits a resonance at a certain frequency. The resonant frequency of a network is defined as the frequency at which the voltage and the current at the input of the network are in phase. This resonant frequency changes when C is changed, and is set to the AM carrier frequency to which the radio is “tuned.”

![Figure 1: TRF Circuit](image)

Now suppose we want to measure the resonant frequency, so we build the setup shown in Figure 2. We inductively couple the signal generator to deliver a current at frequency $\omega$ to the circuit. One can show that for large enough $R_2$ and small enough $L_2$, the mutual inductance is negligible and $L_2 = L_1$. While varying the frequency of the signal generator, we can experimentally find $\omega_0$, the frequency at which the measured voltage amplitude reaches its maximum value.

![Figure 2: Setup to Find the Resonant Frequency](image)
If we want to be more precise in our analysis, we must take into account the parasitic components introduced by the oscilloscope probe, as well as the resistance of the coil. Considering these elements, we have the equivalent circuit shown in Figure 3.

![Figure 3: Model for TRF Circuit and Oscilloscope Probe](image)

1. Derive an expression for the resonant frequency, $\omega_0$, in terms of the circuit components $R_{\text{COIL}}$, $L$, $C$, $C_{\text{PROBE}}$, and $R_{\text{PROBE}}$. The network will be in resonance when there is no phase shift between the node voltage $v$, and the current $i$ through the source. To put it another way, the complex number $I(j\omega)/V(j\omega)$ should have an angle of zero. Since $I(j\omega)/V(j\omega) = Y(j\omega)$ (the admittance of the network), we are solving for the value of $\omega$ that makes the admittance a purely real number.

2. Calculate $\omega_0$ for the values shown in Figure 3 (e.g., $C = 160\text{pF}$). Note that $\omega_0$ should be between $3 \times 10^6$ and $4 \times 10^6 \text{rad/s}$. Calculate the resonant frequency in Hz by dividing $\omega_0$ by $2\pi$.

3. The variable capacitor has a maximum value of $160\text{pF}$ and a minimum value of $20\text{pF}$. Calculate the resonant frequency (in Hz) for the minimum value ($C = 20\text{pF}$). Considering the AM radio band is 530kHz to 1710kHz, do the values you obtained for the resonant frequency at $C = 160\text{pF}$ and $C = 20\text{pF}$ make sense? Explain.

4. Derive an expression for the transfer function $H(j\omega)$, which in this case we will define as the voltage across the oscilloscope probe divided by the input current. That is to say: $H(j\omega) = V(j\omega)/I(j\omega) = Z(j\omega)$, the impedance of the circuit. For $C = 160\text{pF}$, evaluate $20\log_{10}|H(0)|$ and $20\log_{10}|H(j\omega_0)|$.

5. In PSpice, simulate the circuit shown in Figure 3 (n.b., you must type “Meg” and not simply “M” to denote $10^6$). Make a Bode plot for the transfer function by adding a voltage probe to the top node and conducting an AC sweep from 1Hz to 10,000,000Hz, with 1,000 points per
decade. On the plot of the output voltage, change the trace to “20*LOG10(V(i:+))” to obtain the gain in decibels (V(i:+) is the voltage of the top node; it may be labeled differently in your circuit). Change the x-axis from Hz to rad/s by selecting “Axis Setting” from the Plot menu, and clicking “Axis Variable” and changing “Frequency” to “2*pi*Frequency.” Compare the frequency value (in rad/s) of the voltage spike to the resonant frequency calculation done in question 2. Do they match?

6. Calculate $\omega_0$ for the case of no oscilloscope probe present (i.e., remove $C_{\text{PROBE}}$ and $R_{\text{PROBE}}$ from Figure 3) and $C = 160pF$. Generate the Bode plot to verify your answer, as was done in question 5.

7. Print out the following: the circuit schematic of Figure 3, with and without the probe components present, and the associated Bode plots of each. **NOTE: your name must appear in the filename of all circuit and waveform printouts!**

8. You will now compare the Bode plots generated in PSpice with ones generated in MATLAB. (Note: You may do this part of the assignment in Mathematica, MathCAD, or Excel if you prefer.) Create a new .m file in MATLAB and enter the following:

```matlab
Rprobe = 10e6;
Cprobe = 10e-12;
Rcoil = 10;
C = 160e-12;
L = 510e-6;

% derive the equation for omega0 in terms of the above variables
omega0 = omega0_hertz = omega0/(2*pi)
s = tf('s');
% derive the transfer function H(S) in terms of the given variables and s
H = ;
% the next two lines will evaluate the transfer function at s=0
% and s=omega0
Hzero = 20*log10(bode(H, 0))
Homega0 = 20*log10(bode(H, omega0))
% the next four lines create the bode plot of H(s)
figure (1)
clf
bodemag(H);
title('Bode Diagram with Probe: YOUR NAME HERE');
% derive the equation for omega0 without the probe present
omega0noprobe = omega0noprobe_hertz = omega0noprobe/(2*pi)
% derive the transfer function H(S) without the probe present
H2 = ;
% the next two lines will evaluate the transfer function at s=0
% and s=omega0
Hzero2 = 20*log10(bode(H2, 0))
Homega02 = 20*log10(bode(H2, omega0noprobe))
% the next four lines create the bode plot of H(s)
figure (2)
clf
bodemag(H2);
title('Bode Diagram without Probe: YOUR NAME HERE');
```
You will need to enter the equations for $\omega_0$ and the transfer function (NOTE: use the variable ‘s’ instead of the complex variable ‘j\omega’). To take a square root in MATLAB, simply use the sqrt() function, where the expression you want to find the square root of goes inside the parentheses. Run the .m file by typing the name of the .m file in the MATLAB Command Window. When you have done this, you should have two Bode plots identical to those generated in PSpice. Print out the two Bode plots, the .m file, and also the .m file output (the values of $\omega_0$, $20\log_{10}|H(0)|$, etc.). Make sure you change the title of each Bode plot to include your name.

9. Find the transfer functions for the three circuits shown in Figure 4. The transfer function for the low-pass filter should be of the form $H(s) = 1/(s/a + 1)$. The transfer function for the high-pass filter should be of the form $H(s) = Ks/(s/a + 1)$. The transfer function for the band-pass filter should be of the form $H(s) = Ks/((s/a + 1)(s/b + 1))$. $K$, $a$, and $b$ are constants (different for each filter).

Figure 4: Three Filtering Circuits

Transfer Functions:

LPF: \[
\frac{V_{out}(s)}{V_{in}(s)} = \]

HPF: \[
\frac{V_{out}(s)}{V_{in}(s)} = \]

BPF: \[
\frac{V_{out}(s)}{V_{in}(s)} = \]
In-Lab Instructions

Part One: Filters and Their Bode Plots

1. Sketch the Bode plots for all three circuits shown in Figure 4 and label all breakpoints.

The *passband* of a filter is the range of frequencies that are able to pass through unattenuated, and the *stopband* of a filter is the range of frequencies that are attenuated. Comment on the passband and the stopband of the three filters above, citing specific frequency ranges.

<table>
<thead>
<tr>
<th>Filter</th>
<th>Passband</th>
<th>Stopband</th>
</tr>
</thead>
<tbody>
<tr>
<td>LPF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>HPF</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BPF</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. To verify that your Bode plots are correct, build the three circuits and fill in the following table (set Vin to 20V peak-to-peak):

**Table 1: Frequency Response of the Three Filters from Figure 4**

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>Frequency (rad/s)</th>
<th>Vout/Vin (dB)</th>
<th>Vout/Vin (dB)</th>
<th>Vout/Vin (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.9Hz</td>
<td>$10^2$ rad/s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>159Hz</td>
<td>$10^3$ rad/s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,590Hz</td>
<td>$10^4$ rad/s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15,900Hz</td>
<td>$10^5$ rad/s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>159,000Hz</td>
<td>$10^6$ rad/s</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1,590,000Hz</td>
<td>$10^7$ rad/s</td>
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</tbody>
</table>

Do the results in Table 1 agree with the Bode plots you drew?

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**Part Two: Build the AM Radio Receiver**

1. **Set aside resistor R2 (100kΩ, brown-black-yellow), capacitor C1 (470nF, labeled 474) and integrated circuit IC1 (MK484).** Leaving these elements out allows for the isolation of the inductor coil and the variable capacitor. These three elements should be soldered in place only after the appropriate measurements have been made.

2. Assemble the kit according to the included instructions and the “white screen” printed on the PCB.

3. Wrap a wire three or four times around one end of the iron bar protruding from the inductor coil. Connect one end of this wire to the output terminal of the function generator (set to a sine wave of maximum amplitude), and the other end of the wire to a 100K resistor. Connect the ground terminal of the function generator to the other end of the 100K resistor. (This arrangement can be seen in Figure 6.)
4. Set the variable capacitor to its maximum value (160pF) by turning it all the way counter-clockwise. Consider the TRF circuit shown in Figure 5. It consists of a variable capacitor and a coil connected in parallel. As you have seen in the lectures, such a circuit exhibits a resonance peak at a certain frequency. This frequency changes when C is changed, and is set to resonate at the AM carrier frequency to which the radio is “tuned.”

![Figure 5: TRF Circuit](image)

![Figure 6: Setup Used in Finding the Resonant Frequency](image)

To measure the resonant frequency, build the setup shown in Figure 6. (The schematic is shown in Figure 2.) We inductively couple the signal generator to deliver a current at frequency $\omega$ to the circuit. One can show that for large enough $R_2$ and small enough $L_2$, the mutual inductance is negligible and $L_2 = L_1$. While varying the frequency of the signal generator, we can experimentally find $f_0$, the frequency at which the measured voltage amplitude reaches its maximum value. Measure the resonant frequency $f_0$ and record it below. Then convert the measured frequency to its corresponding angular frequency $\omega_0$.

<table>
<thead>
<tr>
<th>Maximum C (160pF):</th>
<th>$f_0$ (Hz)</th>
<th>$\omega_0$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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</tbody>
</table>

Now turn the variable capacitor all the way clockwise and find the resonant frequency.

<table>
<thead>
<tr>
<th>Minimum C (20pF):</th>
<th>$f_0$ (Hz)</th>
<th>$\omega_0$ (rad/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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</tbody>
</table>

How do these values compare with the values you found analytically?

5. Solder the remaining components on the PCB. Set the power supply you constructed previously to 9V. Unplug the power supply from the wall and connect its outputs to the AM radio power input terminals. Plug in the power supply and see if your radio works.

When you have completed the lab, sign and print your names below and have the TA initial next to each name.

TA

______________________________________________________

______________________________________________________
Appendix: Common Transfer Functions and Their Bode Plots

**H(s) = constant** (here the constant is 10)

![Bode Plot for H(s) = 10](image1)

*Figure A1: Bode Plot for H(s) = 10*

**H(s) = s**

![Bode Plot for H(s) = s](image2)

*Figure A2: Bode Plot for H(s) = s*
\( H(s) = s/a + 1 \) (here \( a = 1 \text{ rad/s} \))

**Figure A3: Bode Plot for \( H(s) = s/1 + 1 \)**

\( H(s) = 1/s \)

**Figure A4: Bode Plot for \( H(s) = 1/s \)**
H(s) = 1/(s/a + 1) (here a = 1 rad/s)

Figure A5: Bode Plot for H(s) = 1/(s/1+1)

Some Useful Properties of Logarithms:

\[ \log(A(s) \times B(s)) = \log A(s) + \log B(s) \]

\[ \log\left(\frac{A(s)}{B(s)}\right) = \log A(s) - \log B(s) \]

\[ \log(A^n(s)) = n \times \log A(s) \]