
ECE 669

Parallel Computer Architecture

Lecture 25

Final Exam Review

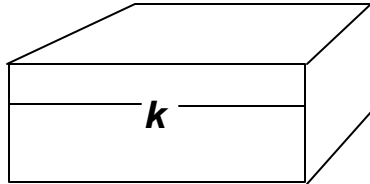


Bandwidth & Latency

Direct

Latency

- Average latency k_d :



nk – worst case

$$n \frac{(k-1)}{2} - \text{torus} - 1 \text{ dir channels}$$

$$\sim n \frac{k}{4} - \text{torus} - 2 \text{ dir channels}$$

$$\sim n \frac{k}{3} - \text{no end conns} - 2 \text{ dir channels}$$

- Bandwidth per node - more complex
 - $\frac{1}{B}$ if all near neighbor messages
 - If average travel dist then?
 - Let

$$\begin{aligned} \text{Avg DIST} &= \frac{nk}{3} \\ \text{Msg size} &= B \end{aligned}$$

Analogy

- If each student takes 8 years to graduate
- And if a Prof. can support 10 students max at any time

How many new students can Prof. take on in a year? $8x = 10$

Each year take on $\frac{10}{8}$

◦ **Similarly:**

- Network has Nn channels
- # of flits it can sustain = Nn
- # of msgs it can concurrently sustain = $\frac{Nn}{B}$
- Each msg flit uses $\frac{nk}{3}$ channels to dest
- So

$$N \times \frac{nk}{3} = \frac{Nn}{B}$$

or

$$BW \text{ per node} = x = \frac{3}{kB}$$

Another way of getting BW is:

- Max # msgs in net at any time = $\frac{Nn}{B}$
- These take = $\frac{kn}{3}$ cycles to get delivered, during which time no new mgs can get in
- I.e. we can inject $\frac{Nn}{B}$ msgs every $\frac{kn}{3}$ cycles

or # injected per node per cycle

$$\begin{aligned} &= \frac{Nn}{B} \cdot \frac{3}{kn} \cdot \frac{1}{N} \\ &= \frac{3}{Bk} \end{aligned}$$

- **Note: We have not considered contention thus far.**
- **In practice, latency shoots up much before we achieve the theoretical due to contention.**

Deriving U is not easy

◦ Notice

m = Probability of a message on a useful processor cycle

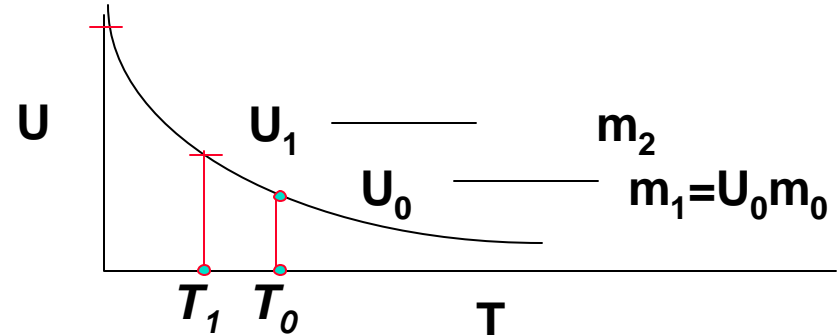
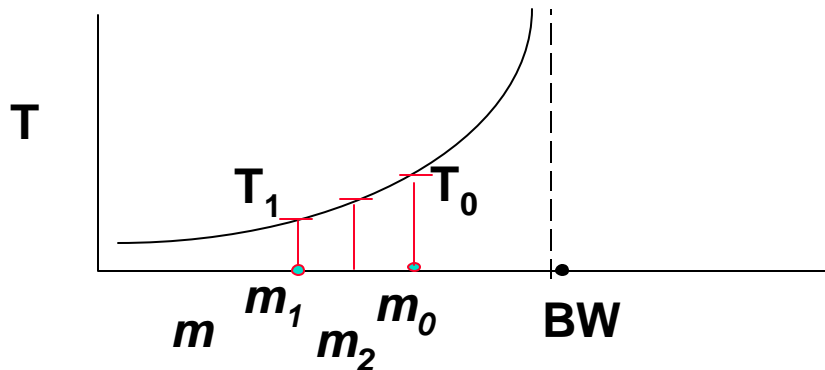
$m_{\text{eff}} = m \cdot U$ = probability of msg on any cycle

$T = f(m_{\text{eff}})$ = network delay as a function of m

$$U = \frac{1}{1 + mT}$$

or # of useful processor cycles depends on T and m_{eff}

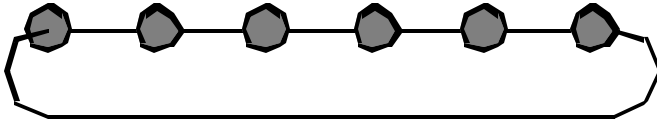
◦ Cyclic dependence!



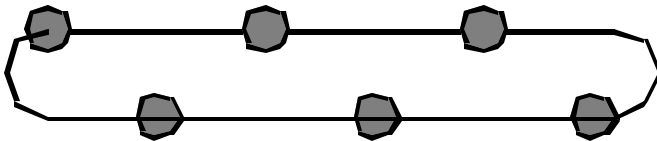
Linear Arrays and Rings



Linear Array



Torus



Torus arranged to use short wires

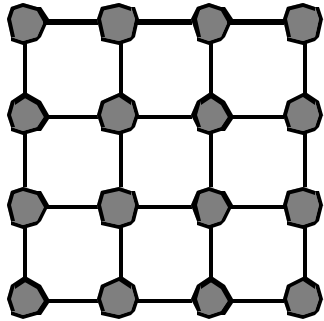
- **Linear Array**

- Diameter?
- Average Distance?
- Bisection bandwidth?
- Route A \rightarrow B given by relative address $R = B - A$

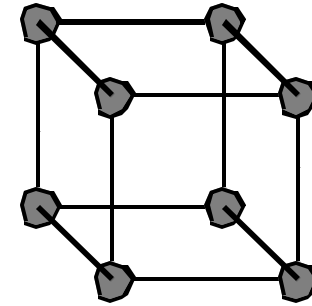
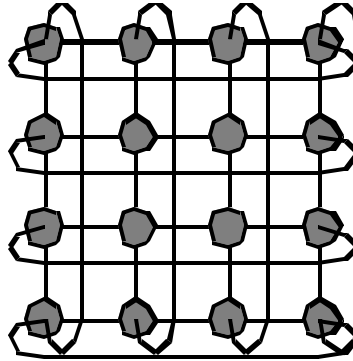
- **Torus?**

- **Examples: FDDI, SCI, KSR1**

Multidimensional Meshes and Tori



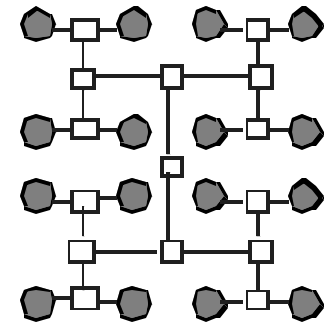
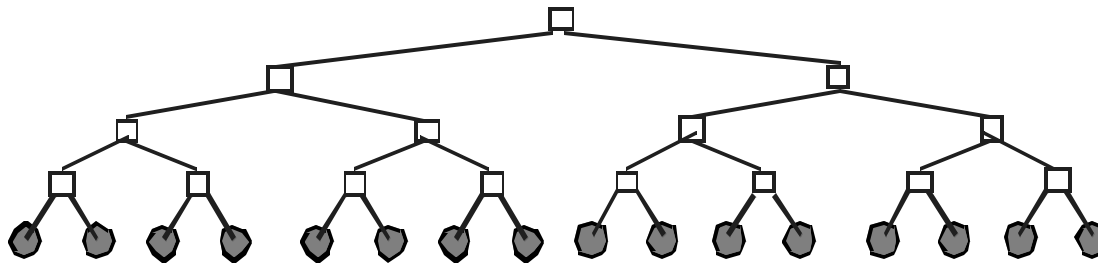
2D Grid



3D Cube

- **n -dimensional k -ary mesh: $N = k^n$**
 - $k = n\ddot{0}N$
 - described by n -vector of radix k coordinate
- **n -dimensional k -ary torus (or k -ary n -cube)?**

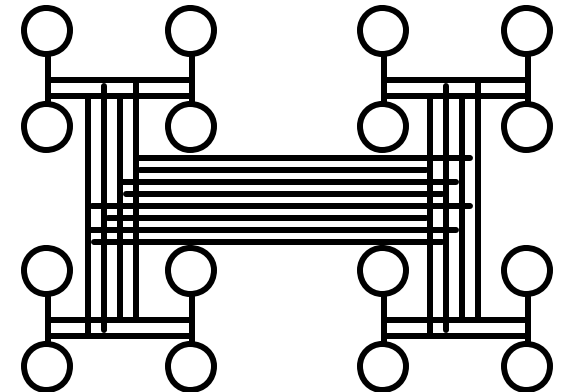
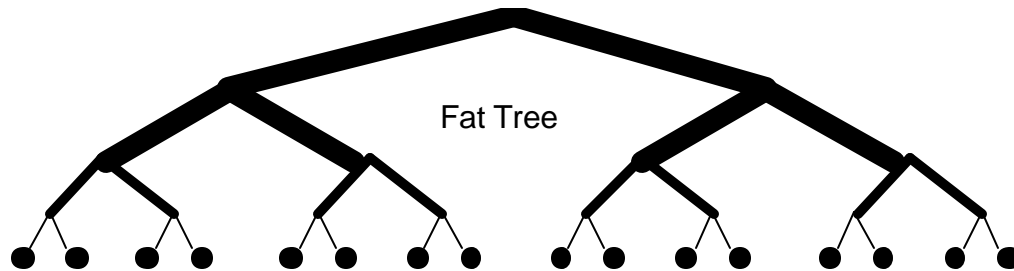
Trees



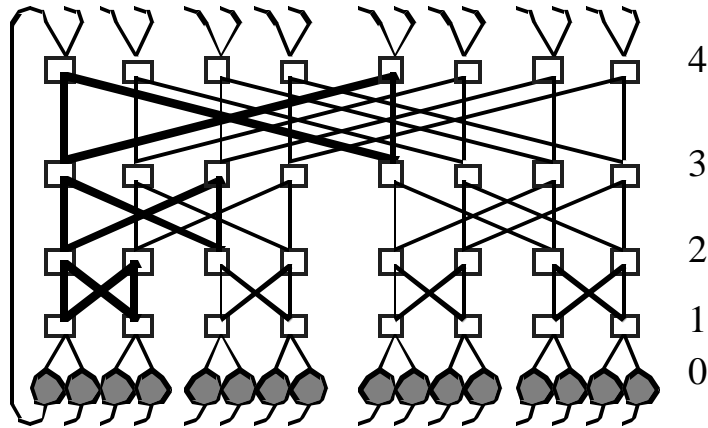
- **Diameter and ave distance logarithmic**
 - k-ary tree, height $d = \log_k N$
 - address specified d-vector of radix k coordinates describing path down from root
- **Fixed degree**
- **H-tree space is $O(N)$ with $O(\sqrt{N})$ long wires**
- **Bisection BW?**

Fat-Trees

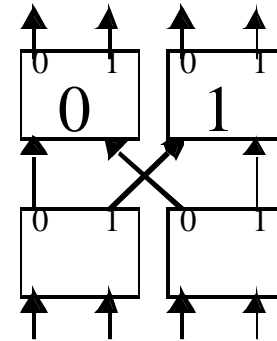
- **Fatter links (really more of them) as you go up, so bisection BW scales with N**



Butterflies



16 node butterfly



building block

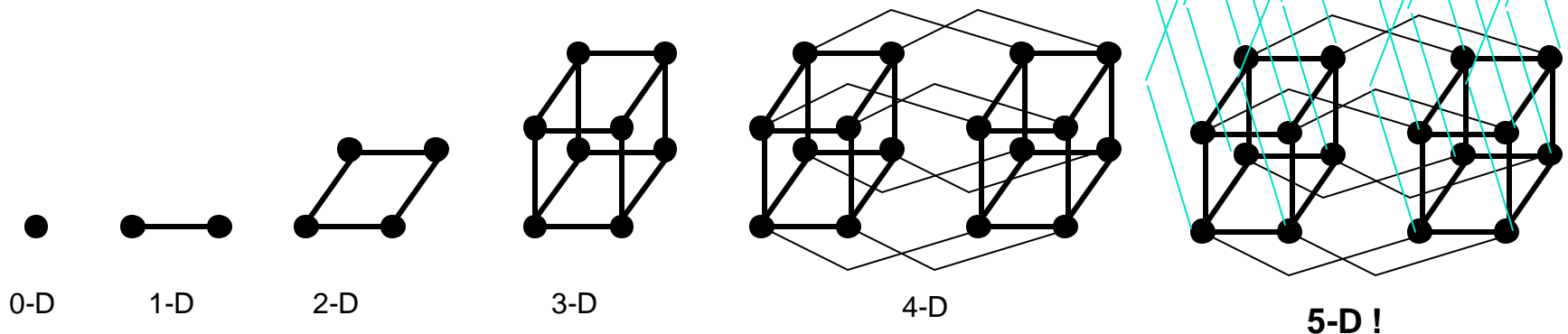
- **Tree with lots of roots!**
- **$N \log N$ (actually $N/2 \times \log N$)**
- **Exactly one route from any source to any dest**
- **Bisection $N/2$**

Hypercubes

- Also called binary n-cubes. # of nodes = $N = 2^n$.
- $O(\log N)$ Hops
- Good bisection BW
- Complexity
 - Out degree is $n = \log N$

correct dimensions in order

- with random comm. 2 ports per processor

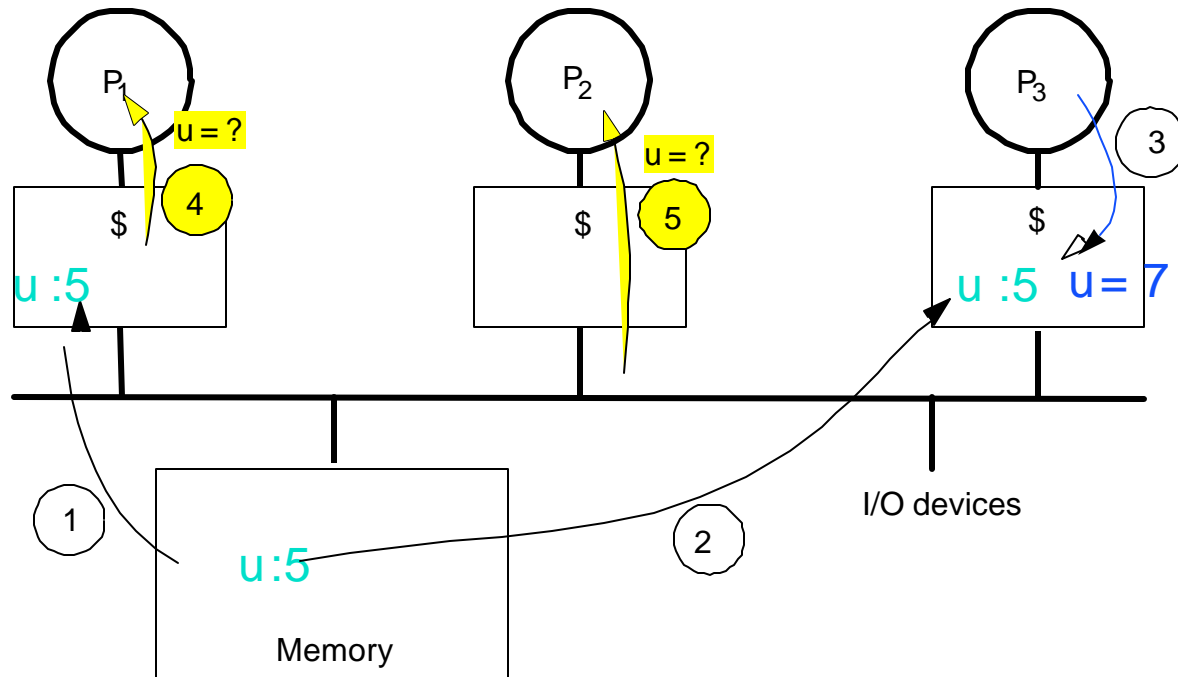


Topology Summary

Topology	Degree	Diameter	Ave Dist	Bisection	D (D ave) @ P=1024
1D Array	2	N-1	N / 3	1	huge
1D Ring	2	N/2	N/4	2	
2D Mesh	4	$2(N^{1/2} - 1)$	$2/3 N^{1/2}$	$N^{1/2}$	63 (21)
2D Torus	4	$N^{1/2}$	$1/2 N^{1/2}$	$2N^{1/2}$	32 (16)
k-ary n-cube	2n	nk/2	nk/4	nk/4	15 (7.5) @n=3
Hypercube	n =log N		n	n/2	N/2 10 (5)

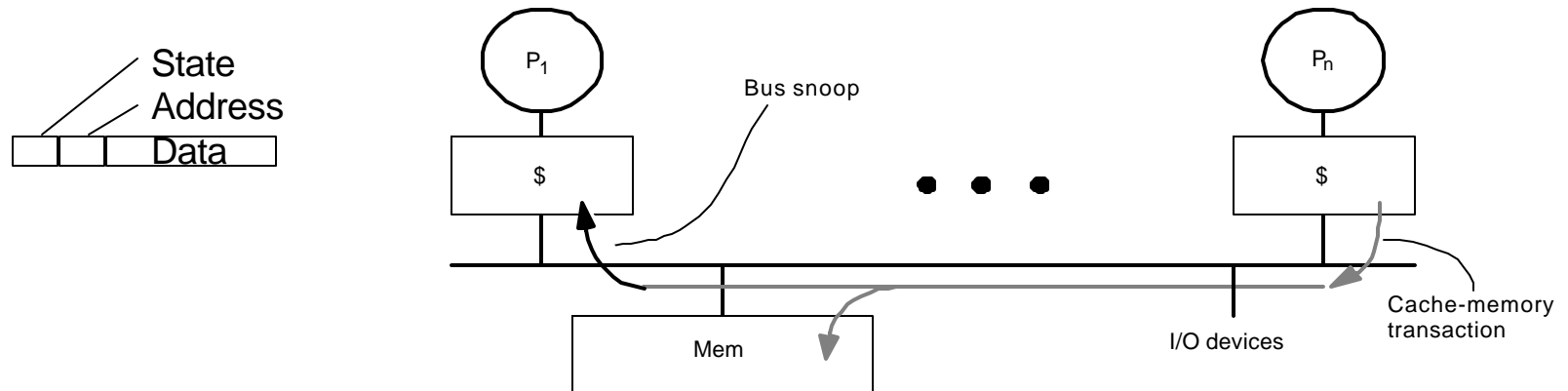
- **All have some “bad permutations”**
 - many popular permutations are very bad for meshes (transpose)
 - randomness in wiring or routing makes it hard to find a bad one!

Example Cache Coherence Problem



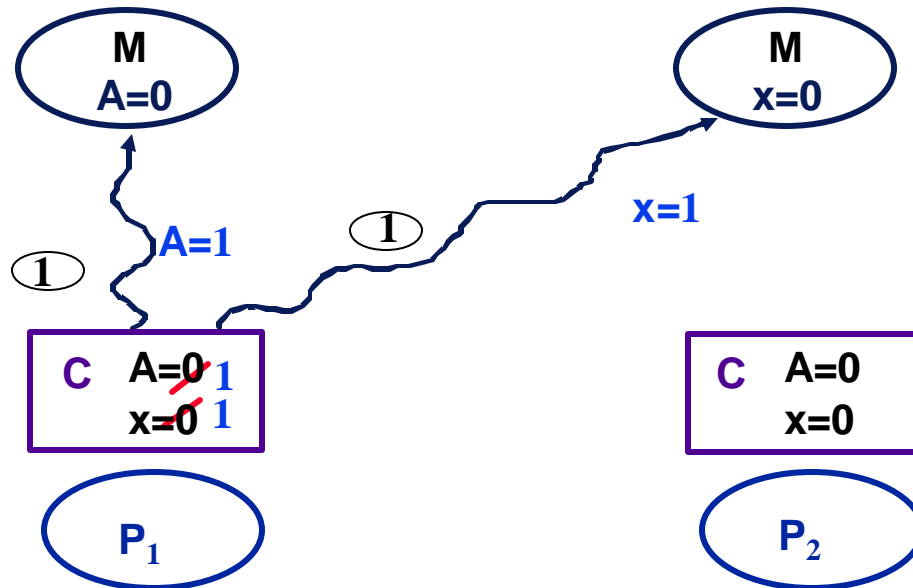
- **Processors see different values for u after event 3**
- **With write back caches, value written back to memory depends on happenstance of which cache flushes or writes back value when**
 - **Processes accessing main memory may see very stale value**
- **Unacceptable to programs, and frequent!**

Snoopy Cache-Coherence Protocols



- **Bus is a broadcast medium & Caches know what they have**
- **Cache Controller “snoops” all transactions on the shared bus**
 - relevant transaction if for a block it contains
 - take action to ensure coherence
 - invalidate, update, or supply value
 - depends on state of the block and the protocol

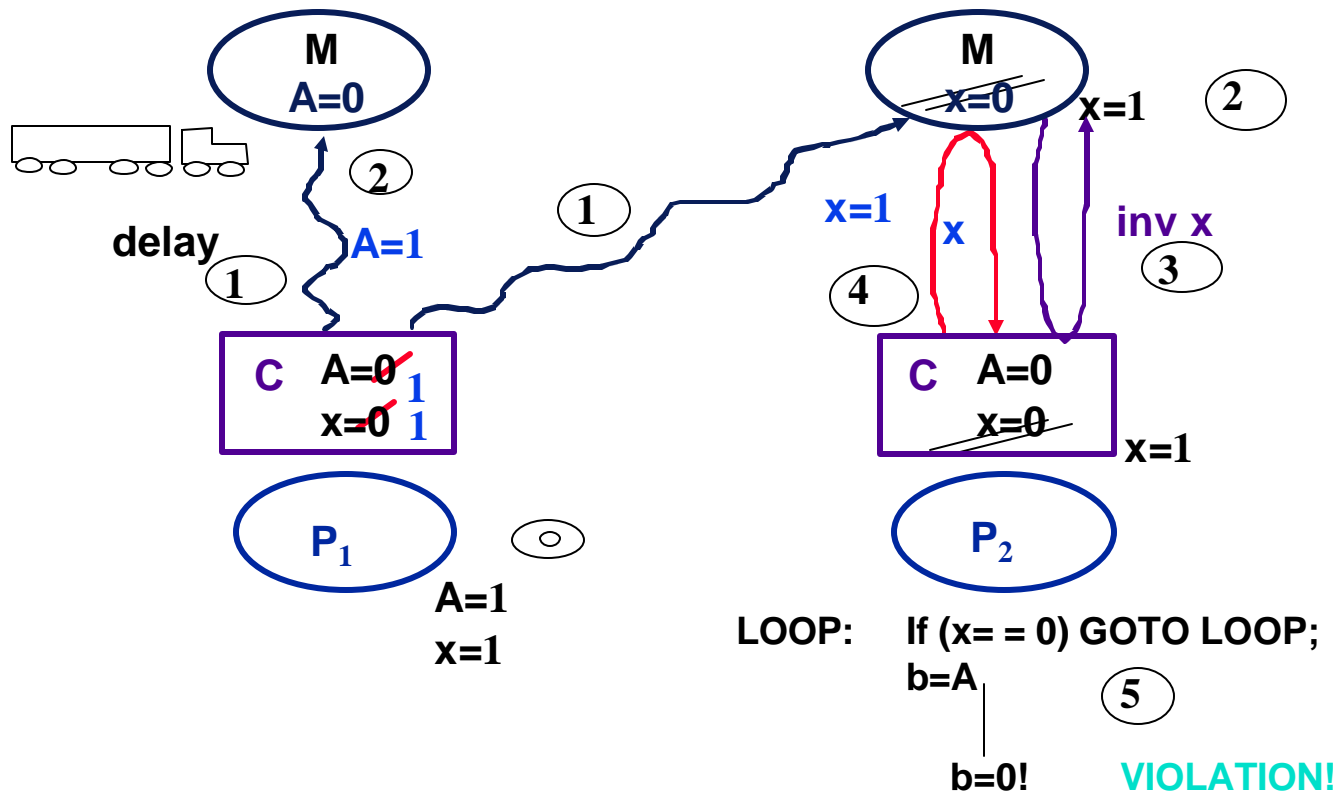
Does caching violate this model?



LOOP: If ($x = 0$) GOTO LOOP;
b=A

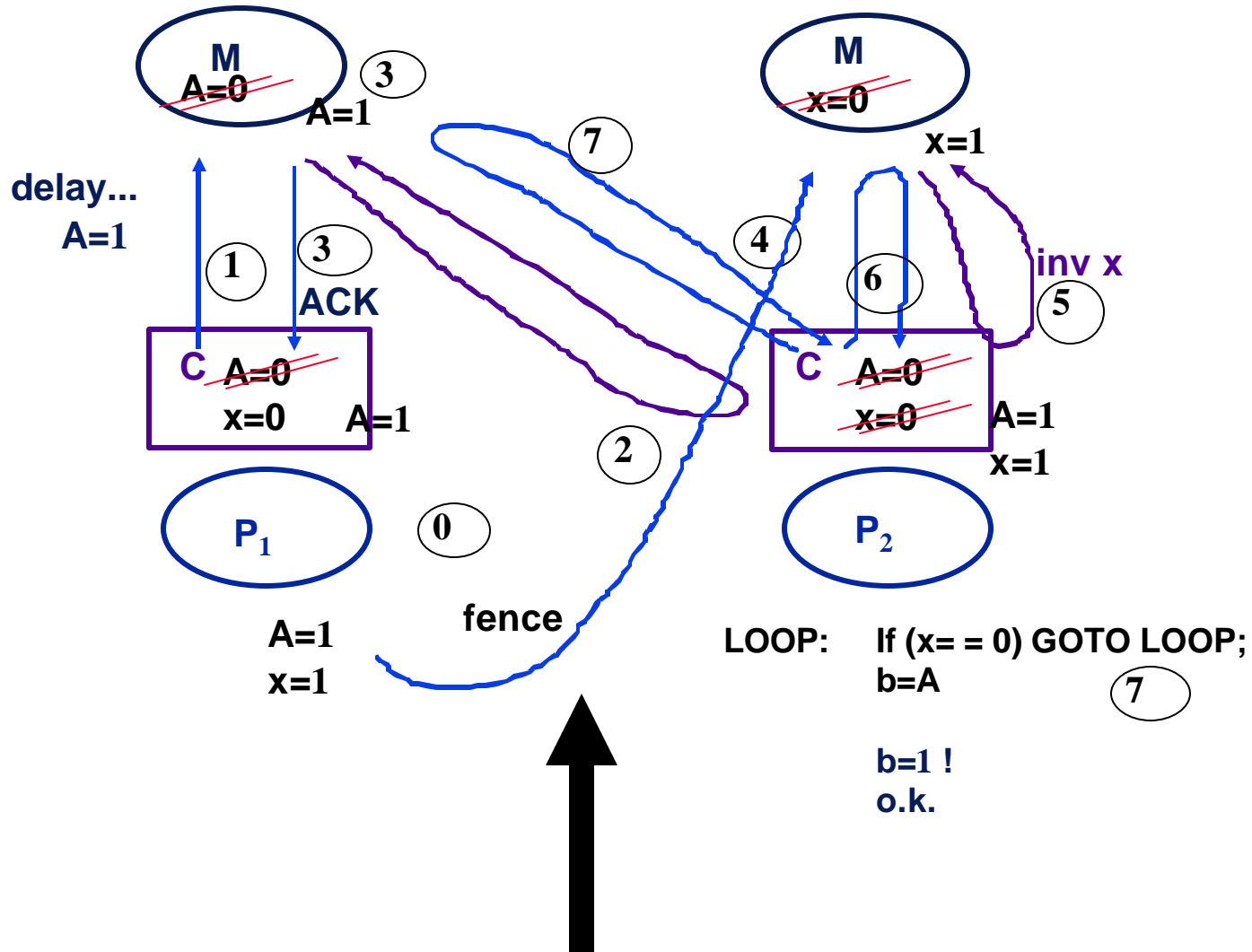
If $b = 0$ at the end, sequential consistency is violated

Does caching violate this model?

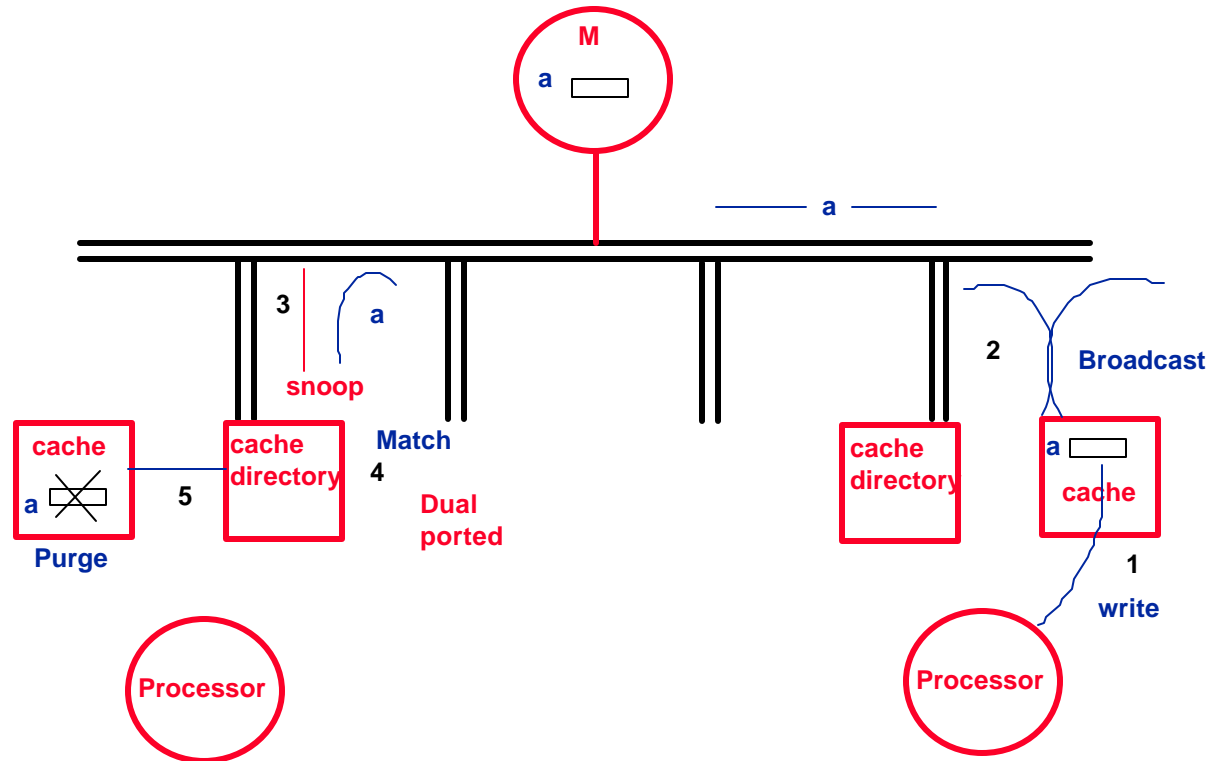


If $b == 0$ at the end, sequential consistency is violated

Does caching violate this model?



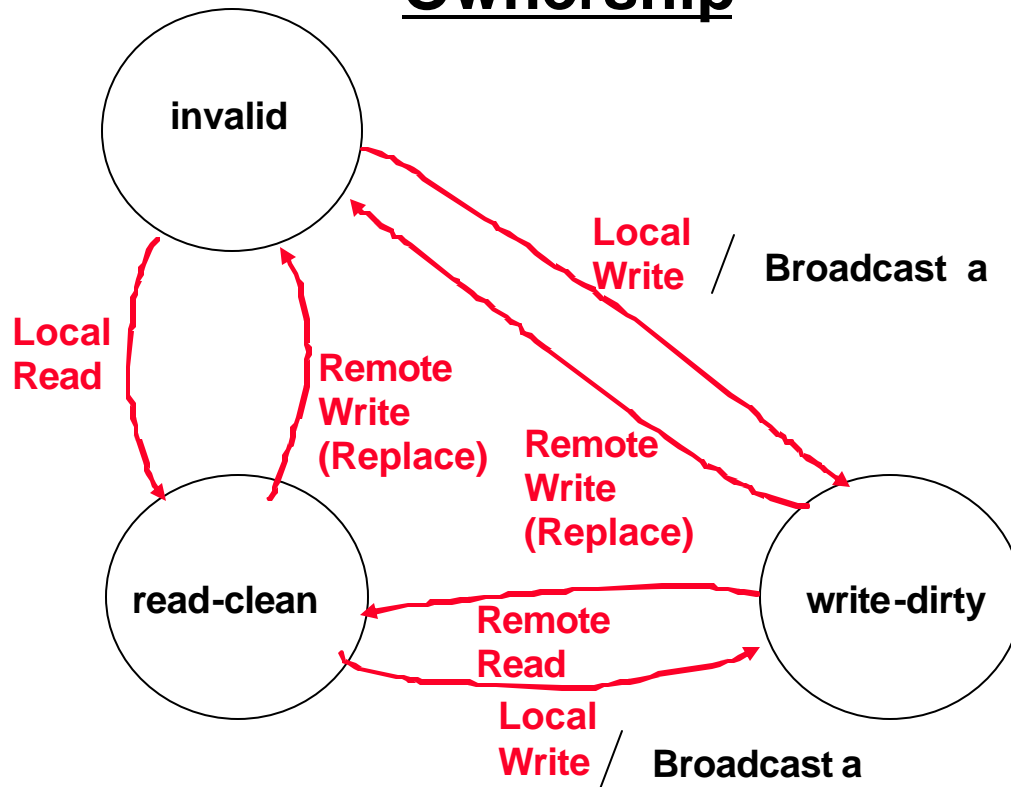
Coherence in small machines: Snooping Caches



- Broadcast address on shared write
- Everyone listens (snoops) on bus to see if any of their own addresses match
- How do you know when to broadcast, invalidate...
 - State associated with each cache line

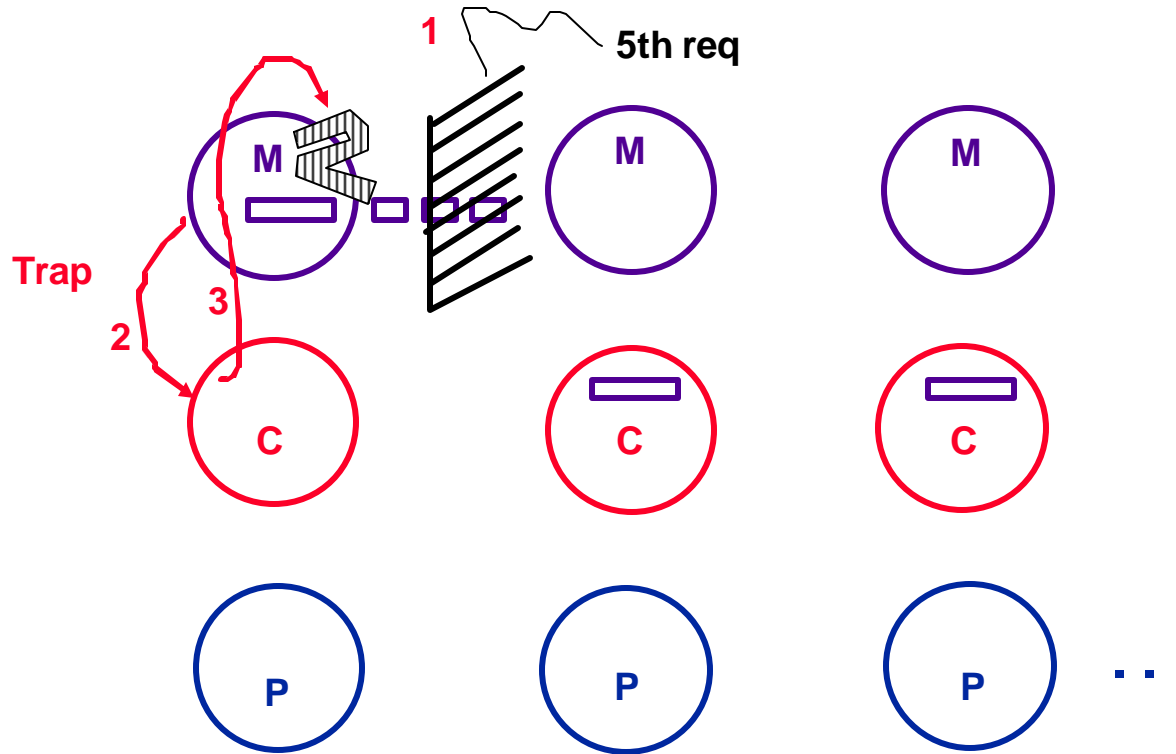
State diagram for ownership protocols

Ownership



- In ownership protocol: writer owns exclusive copy
- For each shared data cache block

Network

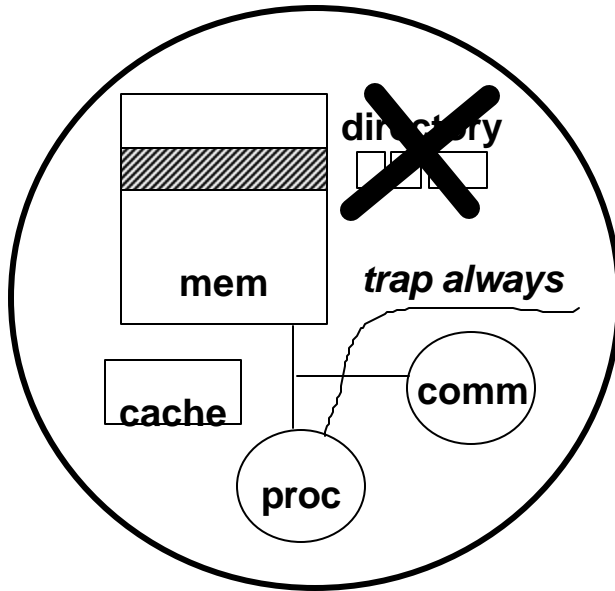


◦ LimitLESS directories:

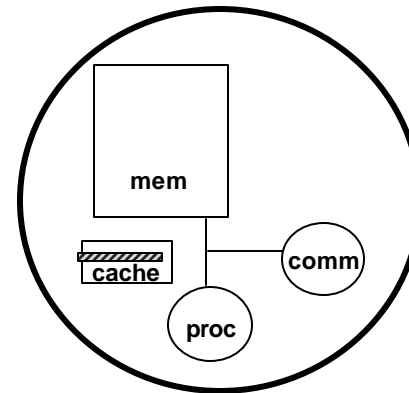
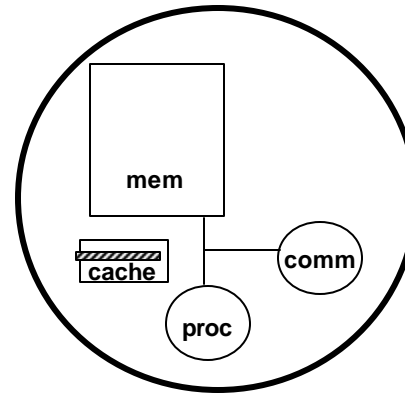
Limited directories Locally Extended through Software Support

- Trap processor when 5th request comes
- Processor extends directory into local memory

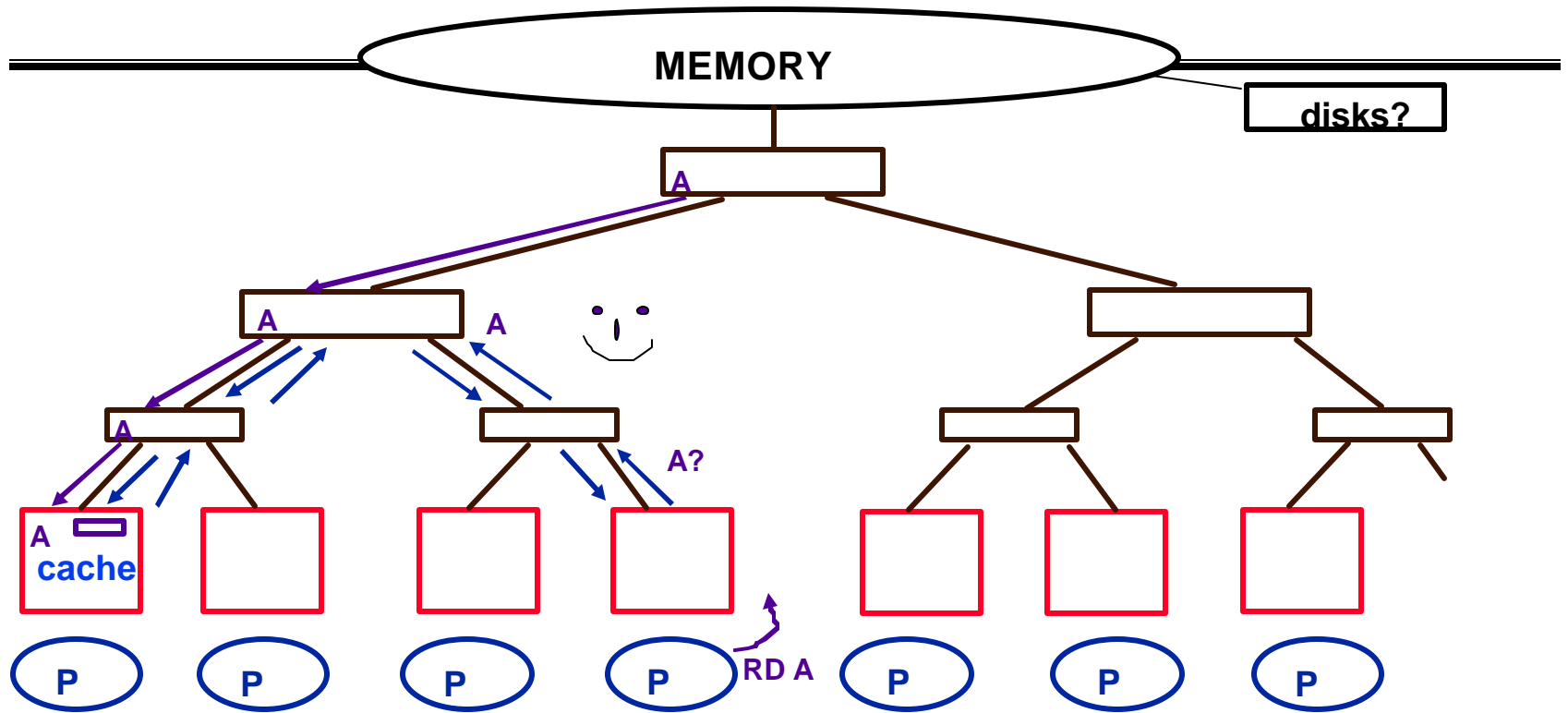
Zero pointer LimitLESS: All software coherence



remote



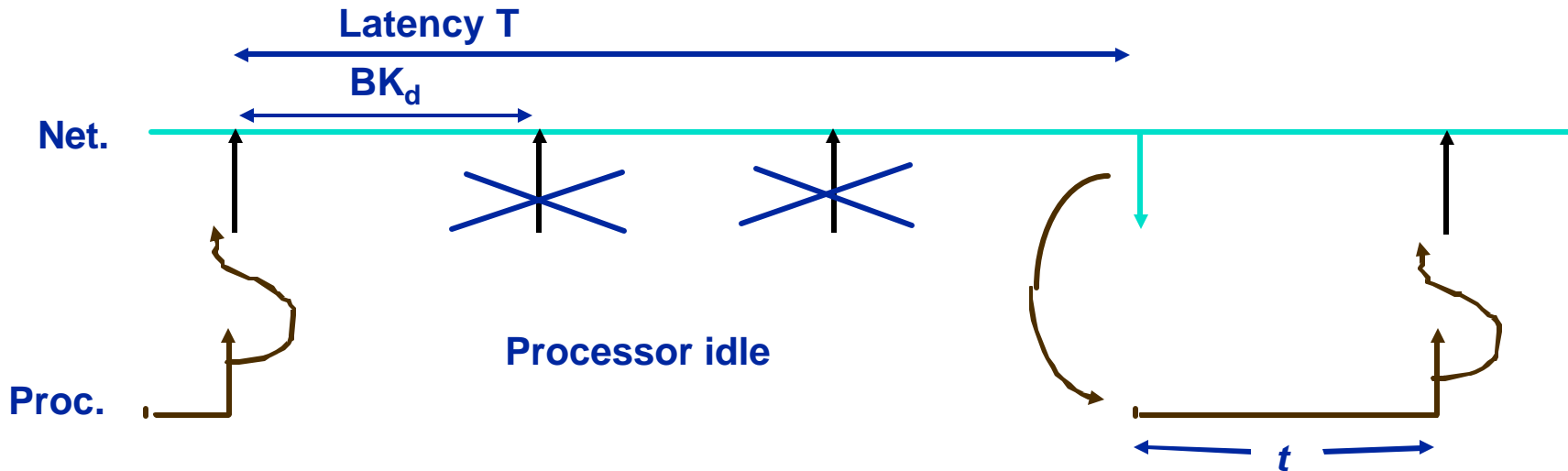
local



Hierarchical - E.g. KSR (actually has rings...)

Include network latency

- Each request suffers T cycles of latency



$$\frac{t}{t + T} = \frac{1}{1 + mT}$$

- Processor utilization =

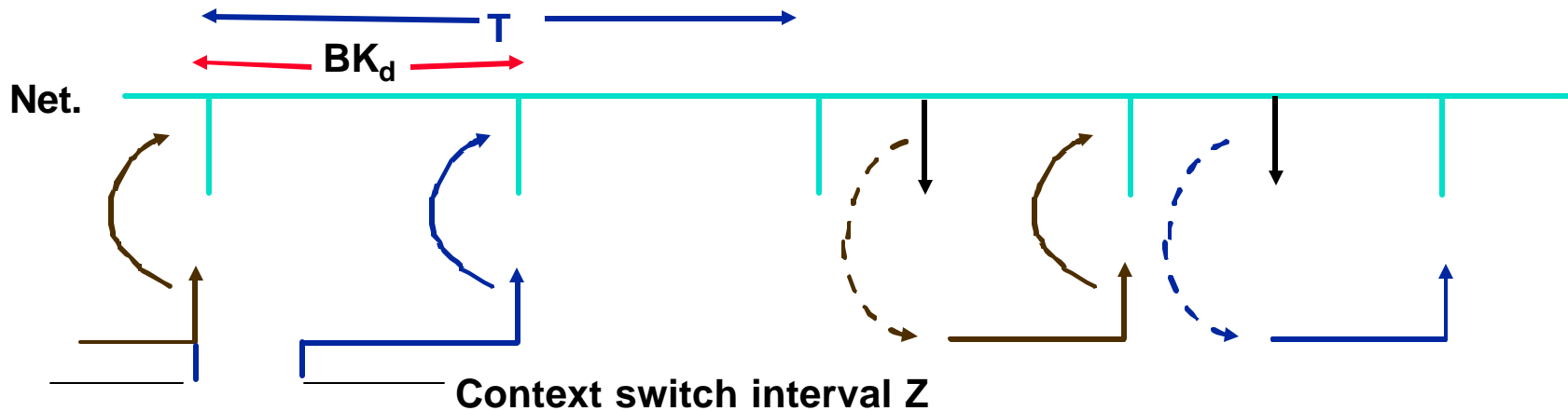
Processor utilization

Network bandwidth also wasted because of lost issue opportunities!

FIX?

One solution

Overlap communication with computation.



- Multithread" the processor

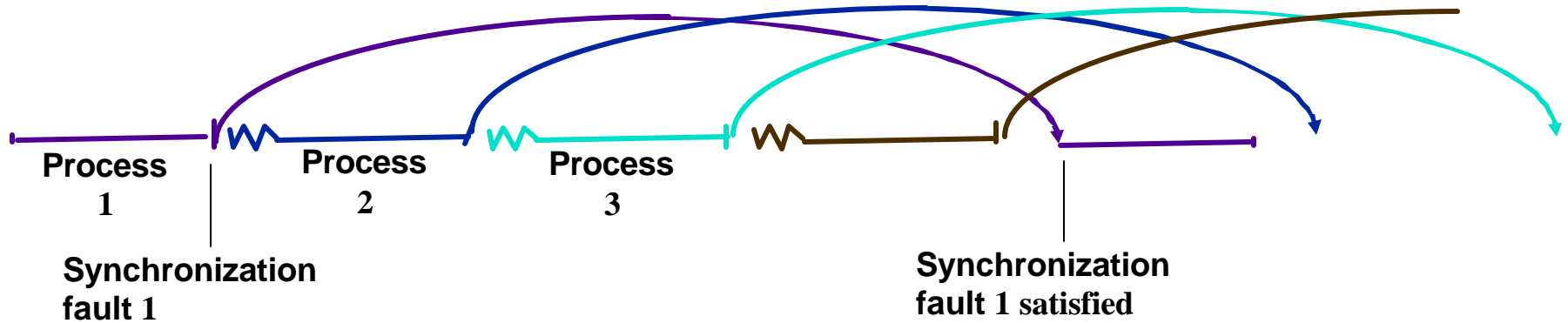
$$\text{Processor utilization} = \frac{pt}{t + T} \text{ if } pt < (t + T)$$

or

$$= \frac{t}{t + Z} \text{ otherwise}$$

- And/or allow multiple outstanding requests -- non-blocking memory

Synchronization delays

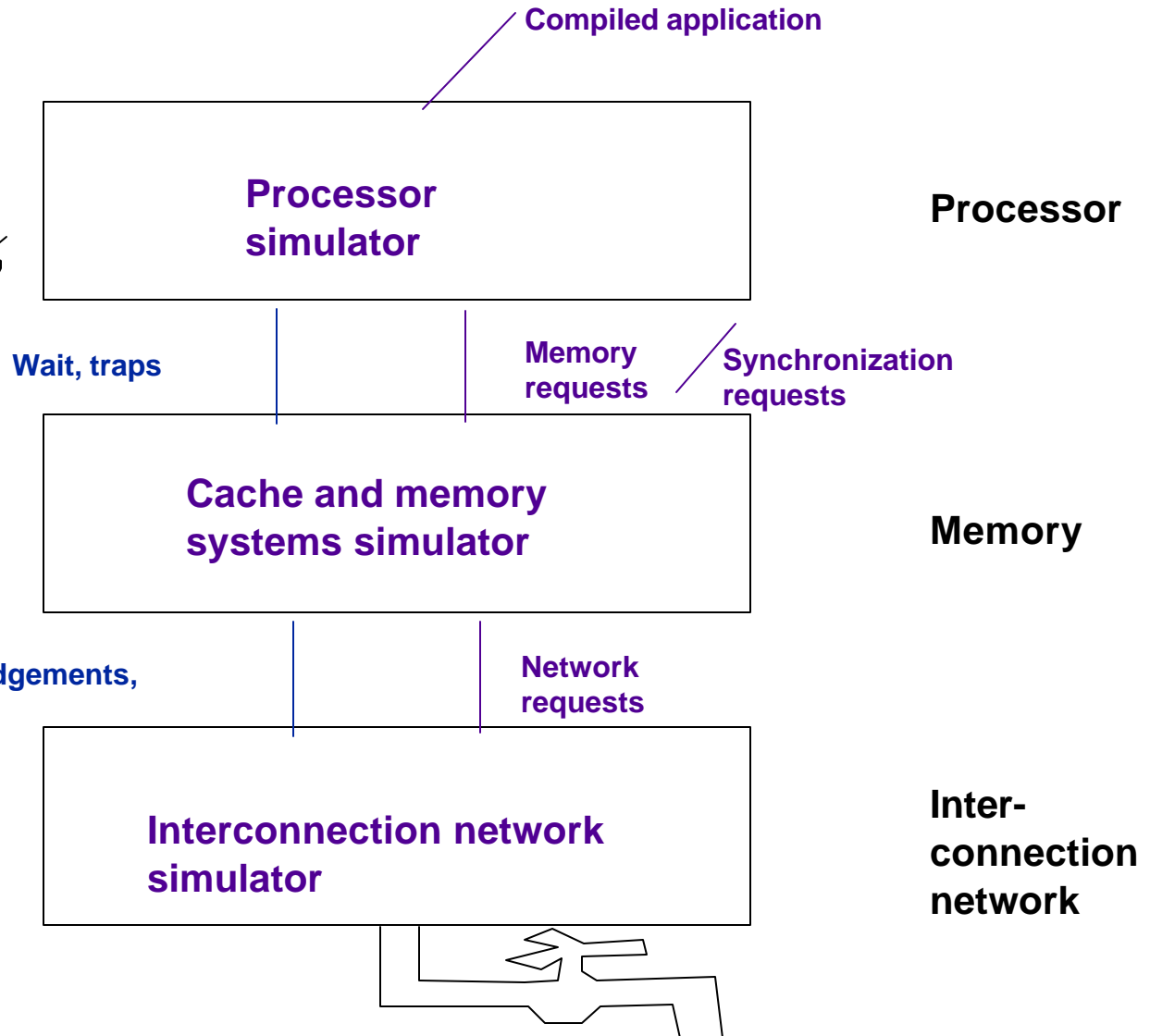
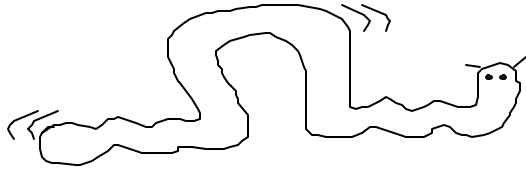


◦ If no multithreading



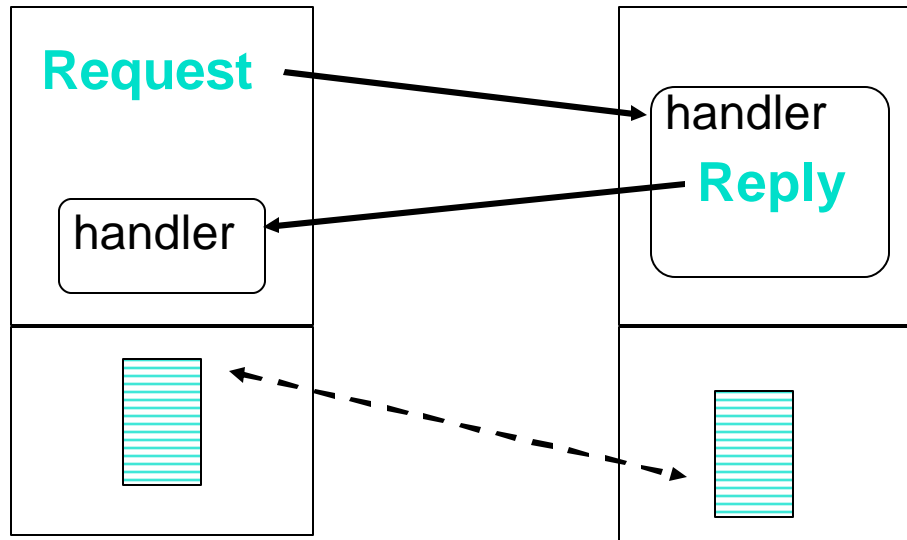
Full system simulation (coupled)

4000x slowdown per node
Very accurate



Measures:
Speedup, runtime
proc. util.
net latency
cache miss rate

Active Messages



- **User-level analog of network transaction**
 - transfer data packet and invoke handler to extract it from the network and integrate with on-going computation
- **Request/Reply**
- **Event notification: interrupts, polling, events?**
- **May also perform memory-to-memory transfer**

Deadlock Freedom

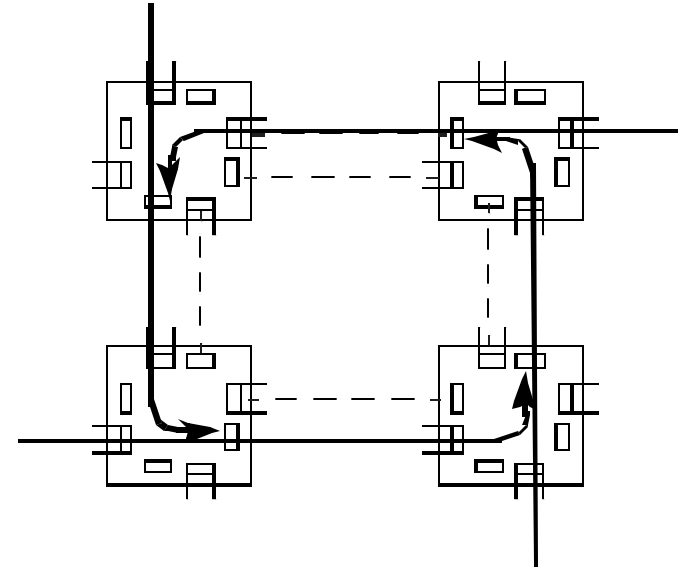
◦ How can it arise?

- necessary conditions:

- shared resource
- incrementally allocated
- non-preemptible

- think of a channel as a shared resource that is acquired incrementally

- source buffer then dest. buffer
- channels along a route



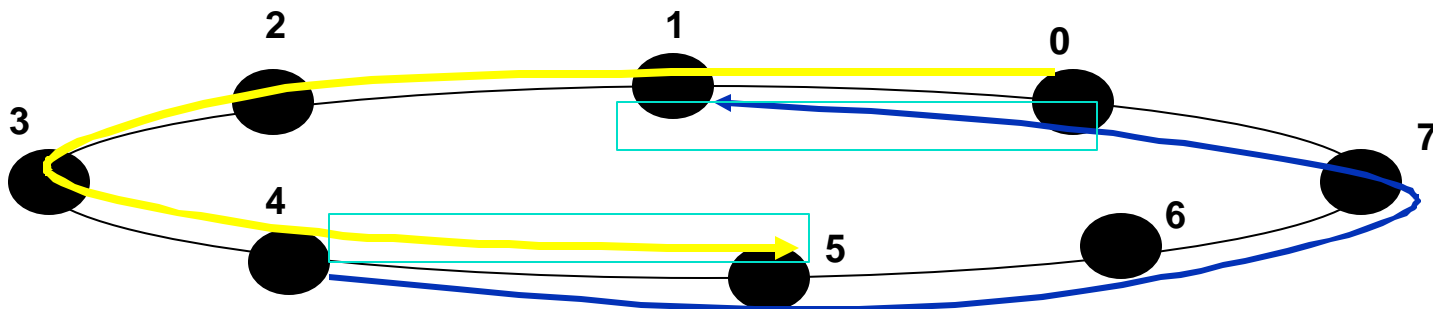
◦ How do you avoid it?

- constrain how channel resources are allocated
- ex: dimension order

◦ How do you prove that a routing algorithm is deadlock free

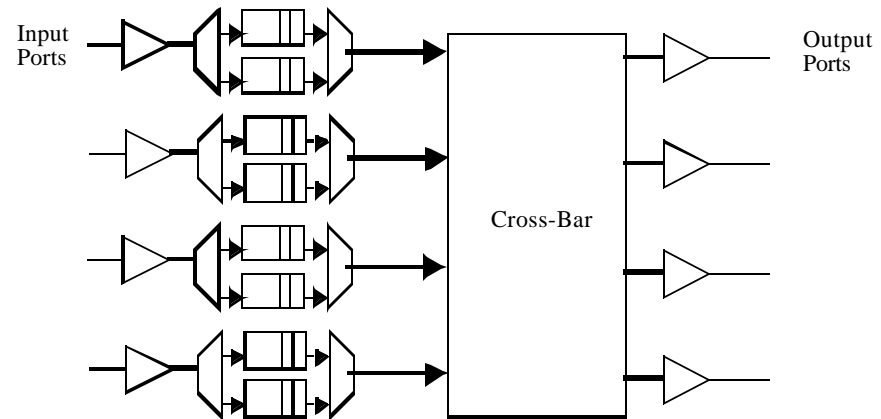
More examples:

- Consider other topologies
 - butterfly?
 - tree?
 - fat tree?
- Any assumptions about routing mechanism?
amount of buffering?
- What about wormhole routing on a ring?



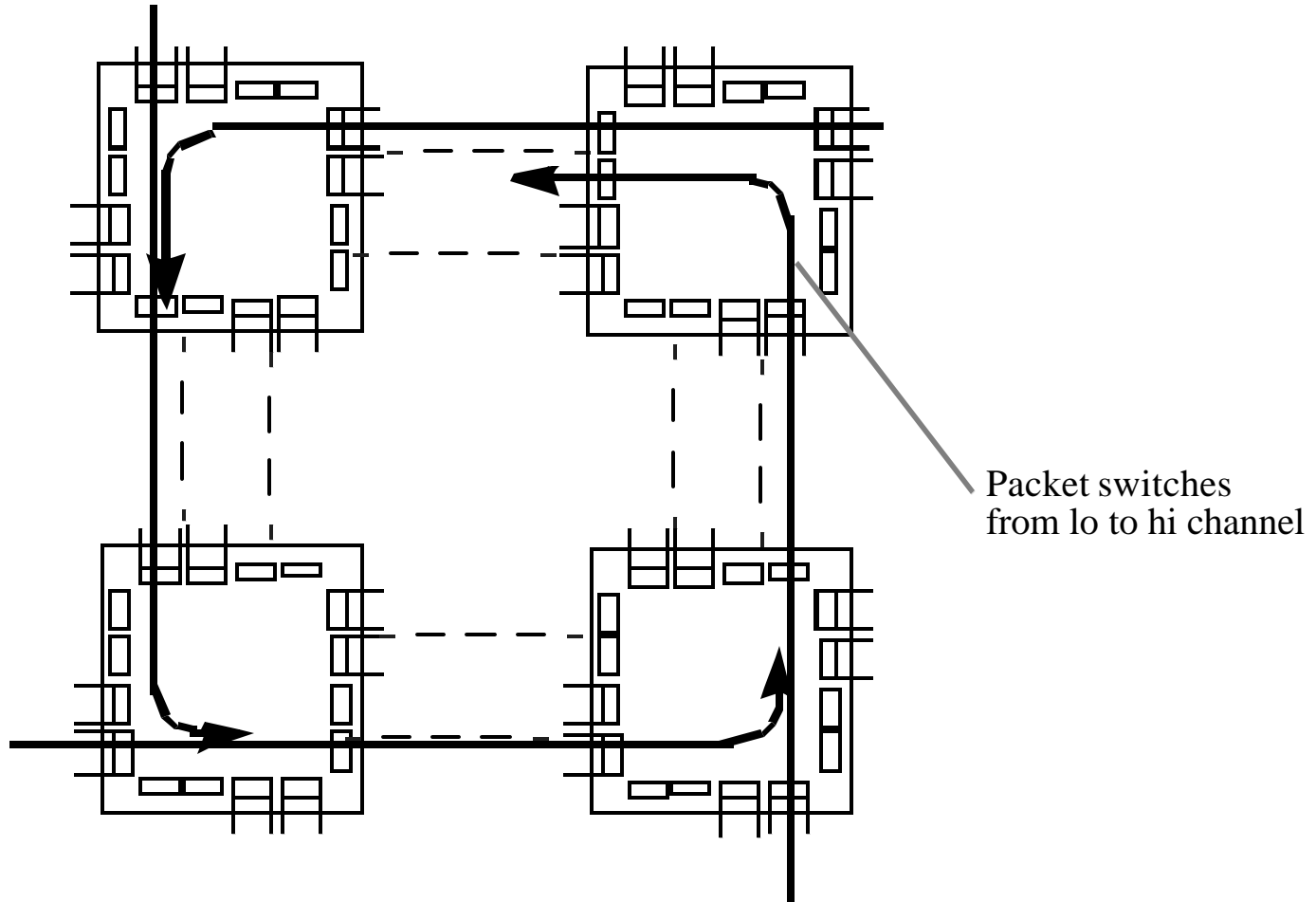
Deadlock free wormhole networks?

- **Basic dimension order routing techniques don't work for k-ary n-cubes**
 - only for k-ary n-arrays (bi-directional)
- **Idea: add channels!**
 - provide multiple “virtual channels” to break the dependence cycle
 - good for BW too!

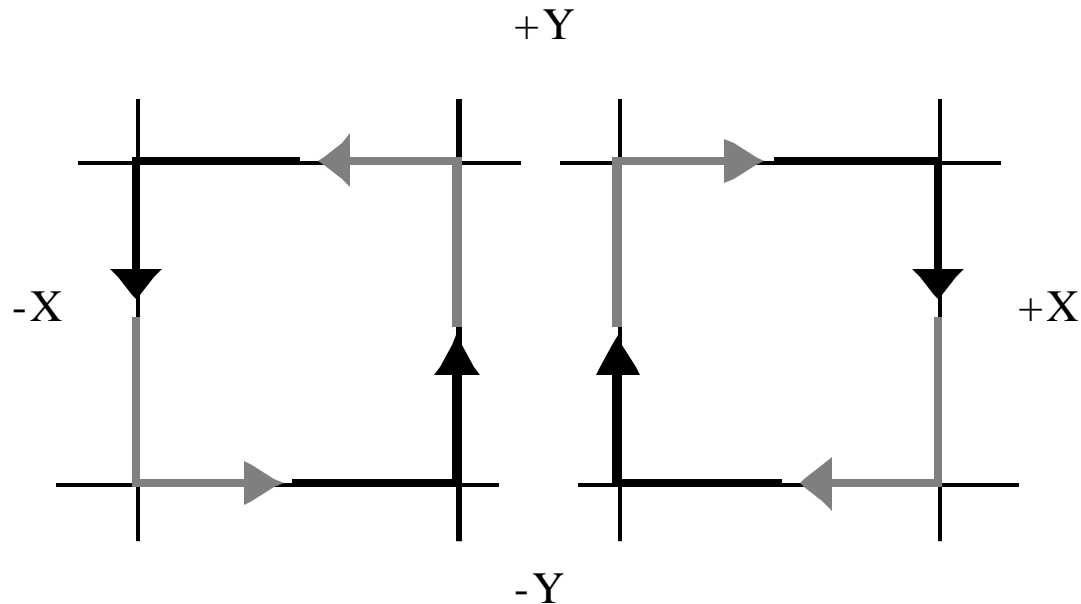


- Do not need to add links, or xbar, only buffer resources

Breaking deadlock with virtual channels

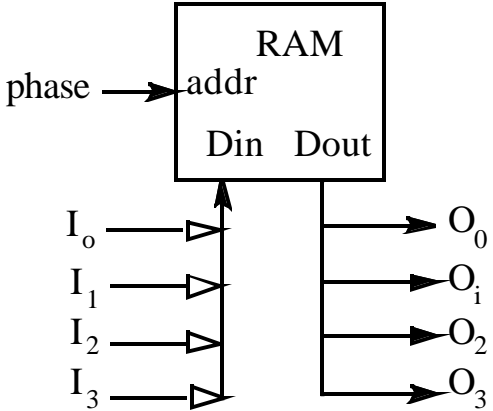
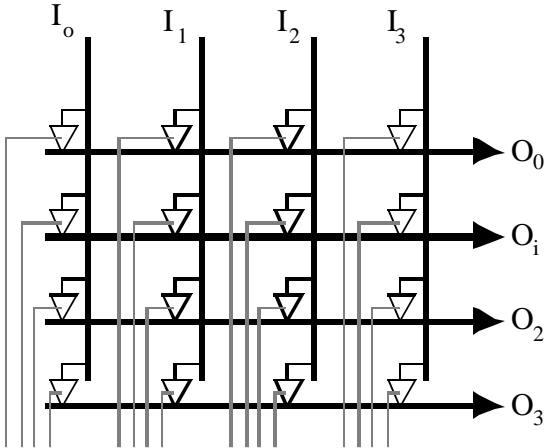
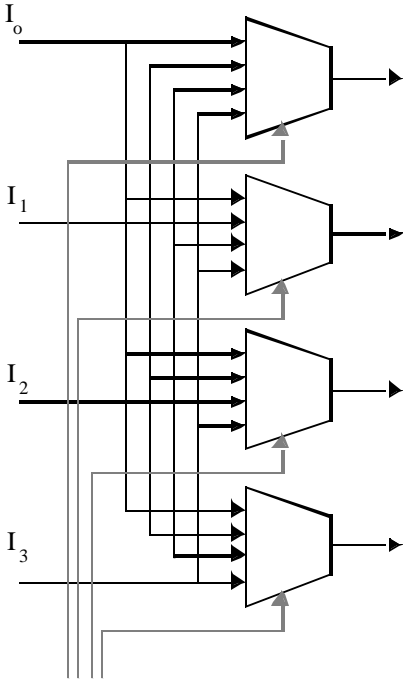


Turn Restrictions in X,Y



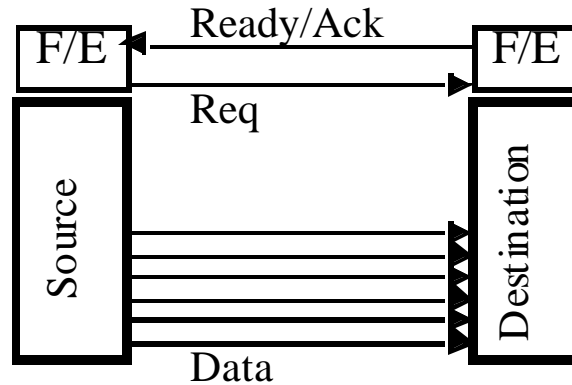
- **XY routing forbids 4 of 8 turns and leaves no room for adaptive routing**
- **Can you allow more turns and still be deadlock free**

How do you build a crossbar



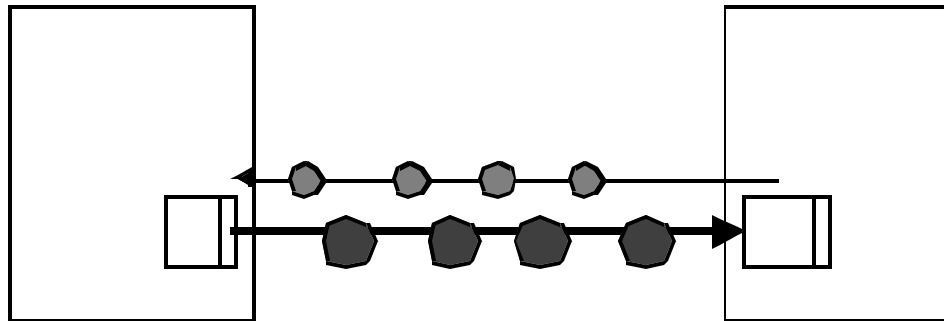
Examples

◦ Short Links



◦ long links

- several flits on the wire



Modulo Unrolling – Smart Memory

- Loop unrolling relies on dependencies
- Allow maximum parallelism
- Minimize communication



Code

```
...  
for(i=0;j<100;i++)  
  A[i]=A[i]*B[i+1]  
...
```

