Waveform Optimizations for Ultra-Wideband Radio Systems

David M. Pozar
Electrical and Computer Engineering
University of Massachusetts at Amherst
Amherst, MA 01003
USA

January 2002
Revised August 2003

Abstract
Solutions are presented for various optimizations of transient waveforms and signals used in ultra-wideband radio systems. These include the transmit antenna generator waveform required to maximize receive antenna voltage amplitude (with bounded input energy), the transmit antenna generator waveform that provides the "sharpest" received antenna voltage waveform, and the transmit antenna generator waveform that maximizes received energy with an inequality constraint on the radiated power spectral density. Using variational methods, general optimization results are derived for arbitrary antennas, including the effects of generator and load impedances, and numerical examples are provided for lossless dipoles and resistively loaded dipoles using moment method solutions. Closed-form results are provided for short dipole antennas for some special cases.

This work was supported by a MURI Project under Contract DAAD19-01-1-0477 from the US Army Research Laboratory.

1. Introduction

Ultra-wideband (UWB) radio is characterized by a wide system bandwidth and a low radiated power spectral density. Typical 3 dB bandwidths of UWB systems are 20% or greater, over frequency bands generally ranging from several hundred MHz to a few GHz (new FCC regulations restrict UWB operation primarily to the 3.1 – 10.6 GHz band). UWB radio does not employ a carrier, relying instead upon the radiation and propagation of bandlimited baseband transient pulses. Wide bandwidth provides potentially fine time resolution, while significant low-frequency components allow propagation through a wide variety of materials and structures. The following features of UWB radios are of interest in a number of future applications involving short-range communications, data networking, ranging, and location [1]-[2]:

- UWB radio maximizes the utility of under-used spectrum segments
- fine time resolution helps to mitigate indoor fading and multipath effects
- low radiated power levels provide an inherent measure of covertness
- wide bandwidth allows ranging accuracy of one foot or less
- low frequency content allows operation through walls, foliage, etc
- low power densities allow high levels of multi-user scaling

The UWB radio concept is not without potential drawbacks, however – notably the risk of interference with licensed spectrum users. Of particular concern is the potential for interference with GPS systems, cellular telephones, and public safety communications. Since power decreases with separation between an interfering transmitter and a legitimate receiver, the maximum allowable power spectral density radiated at a particular distance from a UWB transmitter should be specified to minimize the possibility of interference. The FCC is presently considering this issue [3].

The radiated power spectral density from a UWB transmitter should ideally be as flat as possible, but the frequency characteristics of practical transmit and receive antennas are seldom conducive to this goal. In fact, antenna performance is often the primary limiting factor on the overall performance of UWB systems. Since the transmit and receive antennas invariably distort the shape of the transmitted and received waveforms [4]-[5], it is worthwhile to consider the question of finding the best waveforms to optimize overall system performance in a particular sense.

This paper extends the variational methods developed in [6]-[7] for optimizing transient radiation to derive solutions for input and output waveforms in a UWB radio system that optimize either received signal amplitude, received signal energy, or received signal duration. As such, these solutions represent upper bounds on the performance that could be expected from practical UWB systems. The resulting fields satisfy Maxwell's equations, along with additional constraints on input energy and signal bandwidth. General solutions are derived for arbitrary antennas, and numerical examples are provided for specific cases of lossless and resistively loaded dipole antennas. Frequency domain moment method solutions are used for this purpose; the necessary transfer functions and input impedances can be obtained from the integral equation analysis. Finally, simple closed-form results for short dipole antennas are presented for some special cases

2. Analysis

The general configuration of a UWB radio is shown in Figure 1. The transmit antenna is driven with a voltage source $V_G(\omega)$ having an internal impedance $Z_G(\omega) = R_G(\omega) + jX_G(\omega)$, while the receive antenna is terminated with load impedance $Z_L(\omega) = R_L(\omega) + jX_L(\omega)$, and has a terminal voltage $V_L(\omega)$. The input impedance of the transmit and receive antennas are $Z_T(\omega) = R_T(\omega) + jX_T(\omega)$ and $Z_R(\omega) = R_R(\omega) + jX_R(\omega)$, respectively. The antennas are separated by a distance *r*, assumed to be large enough so that each antenna is in the far field region of the other over the operating bandwidth. The transmit antenna radiates an electric field $\bar{E}(\omega)$ incident at the position of the receive antenna.

The corresponding time domain quantities are given in terms of the inverse Fourier transforms:

$$v_G(t) = \frac{1}{2\pi} \int_{BW} V_G(\omega) e^{j\omega t} d\omega \tag{1a}$$

$$v_L(t) = \frac{1}{2\pi} \int_{BW} V_L(\omega) e^{j\omega t} d\omega \tag{1b}$$

$$\bar{e}(t) = \frac{1}{2\pi} \int_{BW} \bar{E}(\omega) e^{j\omega t} d\omega \tag{1c}$$

The integrations in (1a)-(1c) are over the bandlimited frequency range of $-B$ to B Hertz.

The current at the input to the transmit antenna is given by

$$I_T(\omega) = \frac{V_G(\omega)}{Z_G(\omega) + Z_T(\omega)}, \tag{2}$$

and the receiver load voltage in terms of $V_{oc}(\omega)$, the open-circuit voltage of the receive antenna, is given by

$$V_L(\omega) = \frac{V_{oc}(\omega) Z_L(\omega)}{Z_L(\omega) + Z_R(\omega)}. \tag{3}$$

We also define two transfer functions. Let $H_{LG}(\omega)$ be the voltage transfer function that relates the receive antenna load voltage to the generator voltage at the transmit antenna:

$$V_L(\omega) = H_{LG}(\omega) V_G(\omega) e^{-j\omega r/c}, \tag{4}$$

where c is the speed of light. This definition thus excludes the time delay between the transmit and receive antenna. Also, let $\bar{F}_{EG}(\omega)$ be a vector transfer function that relates the radiated electric field at the receive antenna to the transmit antenna generator voltage:

$$\bar{E}(\omega) = \bar{F}_{EG}(\omega) V_G(\omega) e^{-j\omega r/c}. \tag{5}$$

Although not explicitly shown, it should be understood that both of these transfer functions are functions of range as well as the elevation and azimuth angles at each antenna.

The open-circuit voltage at the receive antenna can be found from the vector effective height, $\bar{h}(\omega)$, of the receive antenna [8]:

$$V_{oc}(\omega) = \bar{h}(\omega) \cdot \bar{E}(\omega). \tag{6}$$

The time domain voltage waveform at the receive antenna can now be written in terms of the generator voltage using (1b) and (4):

$$v_L(t') = \frac{1}{2\pi} \int_{BW} H_{LG}(\omega) V_G(\omega) e^{j\omega t'} d\omega, \tag{7}$$

where $t' = t - r/c$ is the retarded time variable.

We also need to define various energy quantities. The energy available from the generator is given as [9],

$$W_{avail} = \frac{1}{8\pi} \int_{BW} \frac{\left|V_G(\omega)\right|^2}{R_G(\omega)} d\omega, \tag{8a}$$

where $R_G(\omega)$ is the real part of the generator impedance. The energy delivered to the transmit antenna is given by,

$$W_{in} = \frac{1}{2\pi} \int_{BW} \frac{\left|V_G(\omega)\right|^2 R_T(\omega)}{\left|Z_T(\omega) + Z_G(\omega)\right|^2} d\omega. \tag{8b}$$

The energy received by the load at the receive antenna is given by,

$$W_{rec} = \frac{1}{2\pi} \int_{BW} \frac{\left|V_L(\omega)\right|^2}{Z_L^*(\omega)} d\omega. \tag{8c}$$

Again, the integrations in (8a)-(8c) are over the bandwidth of $-B$ to B Hertz. Use of the fact that the imaginary part of a physically realizable input impedance is an odd function of frequency has been used to simplify (8b).

The following results are useful for extremizing linear and quadratic functionals of the function $V(\omega)$ [10]:

$$\nabla \int V(\omega) H(\omega) d\omega = H^*(\omega)$$
$$\nabla \int V(\omega) H(\omega) V^*(\omega) d\omega = \left[H(\omega) + H^*(\omega) \right] V(\omega)$$

In these results, $H(\omega)$ is a potentially non-self-adjoint operator, but the adjoint is conveniently given by the conjugate function for the problems considered here. ∇ is the gradient operator, defined in the context of variational calculus as in [10].

A. Maximization of Received Voltage Amplitude

We first consider the maximization of the received voltage amplitude at the receive antenna, for bandlimited signals, with a constraint on the energy delivered to the transmit antenna. Following the variational calculus procedures used in [6], [7], [10], we define the functional

$$J = -v_L(t' = 0) + \lambda W_{in}, \tag{9}$$

relative to the independent function $V_G(\omega)$, and where λ is a Lagrange multiplier. (The negative sign on v_L ensures maximization for the functionals being used in this work.) The constraint that $W_{in} = 1$ Joule must also be enforced by using (8b). We choose the maximization time as $t' = 0$ with no loss of generality.

Then the functional of (9) can be extremized with the following result:

$$\nabla J = 0 = \frac{-1}{2\pi} H_{LG}^*(\omega) + \frac{\lambda R_T(\omega) V_G(\omega)}{\pi \left| Z_T(\omega) + Z_G(\omega) \right|^2}. \tag{10}$$

Solving for $V_G(\omega)$ gives,

$$V_G(\omega) = \frac{H_{LG}^*(\omega) \left| Z_T(\omega) + Z_G(\omega) \right|^2}{2\lambda R_T(\omega)}. \tag{11}$$

The Lagrange multiplier is found by using (11) in (8b) and setting $W_{in} = 1$:

$$\lambda^2 = \frac{1}{2\pi} \int_{BW} \frac{\left| H_{LG}(\omega) \right|^2 \left| Z_T(\omega) + Z_G(\omega) \right|^2}{4 R_T(\omega)} \, d\omega. \tag{12}$$

The solution given in (11)-(12) is essentially the matched filter solution [10] for the linear system that consists of the transmit and receive antennas, along with their termination impedances.

A slightly different solution can be obtained by constraining the available energy from the generator, as opposed to the energy delivered to the transmit antenna. The required functional then becomes:

$$J = -v_L\left(t' = 0\right) + \lambda W_{avail}, \tag{13}$$

where the available energy is given by (8a). Then the optimum solution is,

$$V_G\left(\omega\right) = \frac{2R_G\left(\omega\right)H_{LG}^*\left(\omega\right)}{\lambda}, \tag{14}$$

with

$$\lambda^2 = \frac{1}{2\pi}\int_{BW} R_G\left(\omega\right)\left|H_{LG}\left(\omega\right)\right|^2 d\omega. \tag{15}$$

This solution accounts for power dissipated in the internal generator impedance, and so will generally result in a lower maximum receive voltage amplitude at $t' = 0$ than the solution of (11)-(12). But presumably less energy will be lost in the generator impedance.

B. Optimizing Received Waveform "Sharpness"

A technique proposed in [10] suggests that minimizing the received energy while constraining the received voltage amplitude to a fixed value at a specific point in time, as well as constraining the input energy, will enhance the sharpness of the output voltage waveform. The output voltage amplitude should be chosen less than the maximum that can be obtained from the matched filter solution given in (11)-(12) – this then introduces additional freedom that may allow the optimization process to reduce the effective duration of the output pulse.

The necessary function thus becomes,

$$J = W_{rec} + \lambda_1 v_L\left(t' = 0\right) + \lambda_2 W_{in}, \tag{16}$$

where we now have two Lagrange multipliers, λ_1 and λ_2. The constraints are applied to the receive voltage amplitude at $t' = 0$,

$$v_L\left(t' = 0\right) = v_0 \le v_{\max}, \tag{17}$$

and to the input energy, $W_{in} = 1$ Joule. We define $v_{\max}$ as the value of the maximum voltage amplitude as obtained from the matched filter solution of (11)-(12). Extremizing the functional of (16) gives,

$$\nabla J = 0 = \left|H_{LG}(\omega)\right|^2 \left[\frac{1}{Z_L(\omega)} + \frac{1}{Z_L^*(\omega)}\right] V_G(\omega) + \lambda_1 H_{LG}^*(\omega) + \frac{2\lambda_2 R_T(\omega)}{\left|Z_T(\omega)+Z_G(\omega)\right|^2} V_G(\omega) \tag{18}$$

Solving for $V_G(\omega)$ gives,

$$V_G(\omega) = \frac{-\lambda_1 H_{LG}^*(\omega)}{\dfrac{2R_L(\omega)\left|H_{LG}(\omega)\right|^2}{\left|Z_L(\omega)\right|^2} + \dfrac{2\lambda_2 R_T(\omega)}{\left|Z_T(\omega)+Z_G(\omega)\right|^2}}. \tag{19}$$

The normalizations are given by using (19) in (7) and (17), and in (8b):

$$v_0 = \frac{-\lambda_1}{2\pi} \int_{BW} \frac{\left|H_{LG}(\omega)\right|^2}{\dfrac{2R_L(\omega)\left|H_{LG}(\omega)\right|^2}{\left|Z_L(\omega)\right|^2} + \dfrac{2\lambda_2 R_T(\omega)}{\left|Z_T(\omega)+Z_G(\omega)\right|^2}} d\omega, \tag{20}$$

$$W_{in} = 1 \text{ Joule} = \frac{\lambda_1^2}{2\pi} \int_{BW} \frac{\left|H_{LG}(\omega)\right|^2 R_T(\omega)}{\left|Z_T(\omega)+Z_G(\omega)\right|^2 \left[\dfrac{2R_L(\omega)\left|H_{LG}(\omega)\right|^2}{\left|Z_L(\omega)\right|^2} + \dfrac{2\lambda_2 R_T(\omega)}{\left|Z_T(\omega)+Z_G(\omega)\right|^2}\right]} d\omega \tag{21}$$

Observe that for specified values of v_0 and W_{in}, (20) and (21) represent coupled equations for λ_1 and λ_2. Numerical root-finding techniques are generally required for solution.

C. Maximizing Received Energy with an Inequality Constraint on the Radiated Field

The expected FCC regulation that the radiated power spectral density at a specified distance from a UWB transmitter be less than a fixed value can be accommodated in optimization solutions through the use of an inequality constraint. Mathematically, we can attempt to maximize received voltage amplitude, or received energy, subject to the constraint that,

$$\left|\bar{E}(\omega)\right| \le E_0, \tag{22}$$

where E_0 is the maximum allowable (peak) electric field intensity at a specified distance from the receiver. In the far field of the transmit antenna, this can be translated to a maximum power density of $E_0^2/2\eta_0$, where $\eta_0 = 377$ ohms.

Inequality constraints of the form in (22) can be treated analytically in some cases by applying nonlinear programming techniques such as the Kuhn-Tucker theorem [11]. An alternative that can be applied more directly for the present case is the technique of slack functions, whereby the functional is modified with a non-negative auxiliary function [11]. Thus, we can rewrite (22) as

$$\left|\bar{E}(\omega)\right|^2 = E_0^2 - u^2(\omega), \tag{23}$$

where $u^2(\omega)$ is the slack function. Since $u^2(\omega)$ is real and non-negative, it is clear that (22) will always be satisfied. If we desire to maximize received energy, we can form the functional,

$$J = W_{rec} = \frac{1}{2\pi}\int_{BW} \frac{\left|\bar{h}(\omega)\cdot\bar{E}(\omega)\right|^2 R_L(\omega)}{\left|Z_R(\omega)+Z_L(\omega)\right|^2} d\omega = \frac{1}{2\pi}\int_{BW} \frac{\left|\bar{h}(\omega)\right|^2 R_L(\omega)\left[E_0^2 - u^2(\omega)\right]}{\left|Z_R(\omega)+Z_L(\omega)\right|^2} d\omega, \tag{24}$$

and extremize relative to $u(\omega)$, and thus indirectly relative to $\bar{E}(\omega)$. The result is,

$$\nabla J = 0 = \frac{-2\left|\bar{h}(\omega)\right|^2 R_L(\omega)}{\left|Z_R(\omega)+Z_L(\omega)\right|^2} u(\omega), \tag{25}$$

which implies that $u(\omega)=0$, and thus $\left|\bar{E}(\omega)\right| = E_0$. Once $\bar{E}(\omega)$ is found, we can work backward to find the necessary $V_G(\omega)$ that will produce this radiated electric field. Note that the phase of $\bar{E}(\omega)$ is not specified in this solution, as a result of the fact that the energy functional is independent of phase. Otherwise the result is not surprising, as it says that the available spectrum should be filled with the maximum allowable power density in order to maximize received energy. Maximizing the received voltage amplitude at a particular time would, in principle, define a phase distribution for the electric field, but maximizing voltage amplitude alone does not involve a quadratic functional, and so does not lead to an analytic solution using these techniques.

D. Maximizing Received Energy with a Constraint on the Generator Voltage Amplitude

In this case the required functional is given by,

$$J = -W_{rec} + \lambda v_G(t=0). \tag{26}$$

Note that the constraint only applies to the generator voltage at a specific instant of time, and so does not provide an overall limit on the generator amplitude. This might seem to limit the utility of this case, but there may be some situations where this is a useful approach. In addition, this result essentially completes the possible permutations of UWB antenna optimizations that can be carried out with a variational approach.

The optimum generator voltage is found to be

$$V_G(\omega) = \frac{\lambda}{\left|H_{LG}(\omega)\right|^2\left[\dfrac{1}{Z_R(\omega)} + \dfrac{1}{Z_R^*(\omega)}\right]}, \tag{27}$$

with the normalization

$$\lambda = \frac{2\pi v_0}{\int\limits_{BW} \frac{1}{\left|H_{LG}(\omega)\right|^2 \left[\frac{1}{Z_R(\omega)} + \frac{1}{Z_R^*(\omega)}\right]} d\omega}, \tag{28}$$

where v_0 is the constrained value of the generator voltage at $t = 0$.

3. Numerical Examples for Dipole Antennas

The above results have been derived for arbitrary transmit and receive antennas, and will be demonstrated here for several cases involving lossless and resistively loaded wire dipole antennas. The piecewise sinusoidal moment method is used to obtain the necessary quantities to implement the above optimization results. These include the input impedances of the antennas and the voltage transfer function, $H_{LG}(\omega)$, between the antennas. Since moment method solutions for dipole antennas are well-established, we refer the reader to the literature for details of the calculation of these quantities [12]-[13]. For simplicity, we assume both dipoles are identical, with length L, radius a, and conductivity σ. We also assume the dipoles are parallel, and radiating in the broadside directions. The range dependence of the received antenna voltage is removed.

A. Maximization of Received Voltage Amplitude

First consider the maximization of received voltage amplitude at $t' = 0$, with a constraint of 1 Joule for the available generator energy. The optimum generator voltage is given by (14)-(15). Applying these results to a lossless pair of dipoles with L = 15 cm, a = 0.02 cm, $\sigma = \infty$, Z_G = 50 Ω, and $Z_L = \infty$ leads to the generator and receiver voltages shown in Figures 2a-2b, where the peak receive voltage amplitude is 4.04E4 Volts, and the received energy is 1.53E-5 Joules (normalized by multiplying by r). The solutions are bandlimited to 2 GHz.

Next consider the same set of dipoles, but with a resistive loading modeled by setting the dipole conductivity to 1000 S/m. The resulting generator and receiver voltages are shown in Figures 3a-3b. The peak voltage has now dropped to about 2.3E4 Volts, and the received energy is 4.4E-6 Joules (normalized by multiplying by r). This represents a drop of about 4.9 dB in voltage, and -5.4 dB in energy. Also note that there is somewhat less overshoot and ringing in the response of the lossy dipoles, presumably due to the enhanced bandwidth introduced by the loading.

As a comparison with non-optimized pulse excitation, the same antenna geometries of Figures 2-3 were analyzed with a gaussian pulse excitation. For an available energy of 1 J, and a gaussian half-power width of about 5E-11 S, the resulting peak voltage amplitude for the lossless dipole was reduced by 6.3 dB from the optimum result of Figure 2, while the resulting peak voltage amplitude for the lossy dipole was reduced by 4.8 dB from the optimum result of Figure 3.

To further understand the effect of resistive loading and termination impedance on the optimum solution, Figure 4 shows the magnitude of the transfer function magnitude versus frequency, for the dipoles used in the cases of Figures 2 and 3, along with lossless dipoles with a receiver load impedance of $Z_L = 50\ \Omega$. Observe that the receiver load impedance has a far greater effect on the transfer function than does the dipole conductivity.

Effects of generator and load impedances, bandwidth, orientation angles, and other parameters can easily be studied with these solutions, but space limitations prevent us from presenting extensive data on these results. One important observation is that the use of complex termination impedances at either the generator or the receiver generally has the effect of greatly reducing the peak amplitude at the receiver, and greatly increasing the ringing of the response. This is caused by the resulting resonant circuit introduced by reactive terminating impedances in conjunction with the resonant dipole response. Conjugate matching and other reactive matching networks should therefore be avoided in UWB antenna systems, even at the expense of lower efficiency. Another observation is that increasing the signal bandwidth generally has minimal effect on the maximum amplitude, at least in the case of electrically large dipoles, since these antennas radiate effectively only over a relatively narrow band of frequencies near the first resonance.

B. Optimizing Received Waveform "Sharpness"

Next consider the optimization of the receive voltage waveform "sharpness", for a pair of lossy dipoles with $L = 15$ cm, $a = 0.02$ cm, $\sigma = 1.0\text{E}4$ S/m, $Z_G = 50\ \Omega$, and $Z_L = \infty$, over a bandwidth of 4 GHz. The optimum solution is given by (19)-(21). A numerical root-finding method is used to solve (20)-(21) for λ_1 and λ_2, the Lagrange multipliers. This can be facilitated by first using (20) to eliminate λ_1 from (21). Then, for a specified value of v_0 (the constrained receive voltage amplitude at $t' = 0$), (21) can be solved for λ_2. The required root-finding procedure is very sensitive, generally requiring double precision computation.

Figure 5 shows the relation between these two quantities. Note that for a specified value of v_0 there are two possible roots for λ_2. We have found that the negative root leads to a maximization of received pulse width, while the positive root leads to a minimum value. The maximum value of v_0 is v_{max} – the matched filter solution of Section 2A for constrained input energy. In the present example $v_{max} = 5381$ Volts. Clearly the solution for optimized "sharpness" cannot produce a larger receive voltage amplitude than the matched filter case – this can be demonstrated mathematically by manipulating the results of (19)-(20).

Setting v_0 to values progressively less than v_{max} leads to output waveforms that show increasing compression, as evidenced by lower amplitudes away from the main pulse at $t' = 0$. The effect is demonstrated in Figures 6a,b,c. (The results in these figures are normalized to a maximum value of unity in order to more easily compare the waveform shapes.) Observe that the response for $v_0 = v_{max} = 5381$ Volts in Figure 6a has a sinx/x form that exhibits a considerable amount of energy outside the region of the central pulse, continuing out to (normalized) time values of ±8 or more. Lowering the constrained output voltage amplitude to 3000 Volts (Figure 6b) causes considerable sharpening of the response – the first overshoots are about half the values for the case of Figure 6a, and the ringing of the pulse is negligible for (normalized) time values of ±4 or more. Further reduction of v_0 to 500 Volts (Figure 6c) continues this trend, although with diminishing rewards. The first overshoots are reduced to about a third of the values in the response of Figure 6a, and the ringing is effectively stopped for (normalized) time values of ±3.

A quantitative measure of the improvement in receive pulse "sharpness" can be defined as the following "compression ratio":

$$CR = 10\log\frac{W_{rec-\max}v_0^2}{W_{rec}v_{\max}^2}. \tag{29}$$

Where $W_{rec-\max}$ is the receiver energy associated with the maximum received voltage amplitude (which occurs when $v_0 = v_{\max}$). Thus, as v_0 approaches v_{max}, W_{rec} approaches $W_{rec-max}$, and the compression ratio approaches 0 dB (no compression, or improvement in "sharpness"). But as v_0 is decreased, the received energy may decrease faster than the square of the receiver voltage (which is roughly proportional to the energy of the main pulse), resulting in an overall ratio greater than one. Figure 7 shows the compression ratio for the dipoles of Figures 5-6, and for solutions for the same dipoles but with two other values of bandwidth and conductivity. We see than compression ratios as high as 4 – 5 dB can be obtained. Such waveforms have very little ringing, and so can be advantageous for reducing intersymbol interference in UWB systems.

4. Analytical Results for Short Dipoles

Essentially closed-form results for several of the optimizations of Section 2 can be obtained for electrically small antennas such as short dipoles and small loops, as long as the bandwidth is such that closed-form expressions can be found for the necessary input impedances and transfer functions. This section presents such results for short perfectly conducting transmit and receive dipole antennas. In this case, the input impedance of each antenna (assumed identical) can be approximated as,

$$Z_T(\omega) = Z_R(\omega) = R(\omega) - \frac{j}{\omega C_0}, \tag{30}$$

where the radiation resistance is given by,

$$R(\omega) = \frac{5L^2\omega^2}{c^2} = \alpha\omega^2, \tag{31}$$

where $\alpha = 5L^2/c^2$ is a constant. The dipole capacitance is given by,

$$C_0 = \frac{L}{240c\left(\ln\frac{L}{2a} - 1\right)}. \tag{32}$$

The capacitance expression in (32) has been derived from the exact induced EMF result after using small argument approximations for the sine and cosine integrals – it has been compared to numerical moment method results and found to be more accurate than the usual expressions found in the literature.

The voltage transfer function between two short dipoles can be derived as,

$$H_{LG}(\omega) = \frac{-j\omega\mu_0 h^2 Z_L(\omega)}{4\pi r\left[Z_G(\omega)+Z_T(\omega)\right]\left[Z_L(\omega)+Z_R(\omega)\right]}, \tag{33}$$

where $h = L/2$ is the dipole half-length. These expressions assume a piecewise sinusoidal current distribution on each dipole, and generally give good results for frequencies such that $L < \lambda/20$

A. Maximize Received Voltage Amplitude, Constrained Input Energy, $Z_G = 0$, $Z_L = \infty$

The solution for this problem is given by the general expressions in (11) and (12). For the special case of short dipoles with $Z_G = 0$ and $Z_L = \infty$, the transfer function of (33) further reduces to,

$$H_{LG}(\omega) = \frac{-j\omega\mu_0 h^2}{4\pi r Z_T(\omega)}, \tag{34}$$

and then (11) can be evaluated as,

$$V_G(\omega) = \frac{j\omega\mu_0 h^2 Z_T(\omega)}{8\pi r\lambda R_T(\omega)}. \tag{35}$$

The normalization of (12) can also be evaluated in closed-form:

$$\lambda^2 = \frac{\mu_0^2 h^4 B}{32\pi^2 r^2 \alpha}, \tag{36}$$

where B is the bandwidth in Hertz, and α is a constant defined in (31). Combining these results and using the inverse transform of (7) provides the optimized time domain receiver voltage:

$$v_L(t') = \frac{3h}{r}\sqrt{10B}\,\frac{\sin 2\pi Bt'}{2\pi Bt'}, \tag{37}$$

showing that the response has a sinx/x form, with a peak value that increases as the square root of bandwidth. The expected $1/r$ range dependence is also apparent. The peak value of the receiver voltage response is $3h\sqrt{10B}/r$. The optimized amplitude moment method results of Section 3A were compared with these results for electrically short dipoles, with good agreement.

It is probably logical to next derive the required generator voltage for this solution, but this is not possible because $V_G(\omega)$ in (35) has a non-removable singularity at $\omega = 0$ (due to the fact that $R_T(\omega)=0$ at DC). The physical meaning of this result seems to be that the solution is trying to capture low frequency energy at the source, even though low frequency components will not propagate to the receive antenna, and also do not contribute to the input energy (due to the very high reactive impedance of the transmit antenna). If we restrict the operating band to some minimum frequency (above DC), the inverse transform of (35) can then be obtained – the resulting receiver response is virtually unaffected by this lower limit, but it has a substantial effect on the transmit antenna waveform.

B. Maximize Received Voltage Amplitude, Constrained Input Energy, $Z_G(\omega)=Z_T^*(\omega)$, $Z_L=\infty$.

In this case we conjugate match the generator to the transmit antenna – in principal this maximizes power transfer to the transmit antenna. Of course, this requires a generator reactance that is positive with a slope of $1/\omega$ - conditions that are not possible for a physically realizable passive element. Nevertheless, the solution gives an upper bound on what can be achieved, and it may be possible to approximate the required frequency dependence with active circuit matching.

Setting $Z_G(\omega)=Z_T^*(\omega)$ simplifies the transfer function of (33) to the following:

$$H_{LG}(\omega)=\frac{-j\omega\mu_0 h^2}{8\pi r R(\omega)}=\frac{-j\mu_0 h^2}{8\pi r\alpha\omega}. \tag{38}$$

Then applying the general solution of (11)-(12) gives the following optimization results:

$$V_G(\omega)=\frac{j\omega\mu_0 h^2}{4\pi\lambda r}, \tag{39}$$

$$\lambda^2=\frac{\mu_0^2 h^4 B}{64\pi^3\alpha r^2}, \tag{40}$$

Then the time domain receiver voltage is found as,

$$v_L(t')=\frac{12\pi h}{r}\sqrt{5\pi B}\,\frac{\sin 2\pi B t'}{2\pi B t'}, \tag{41}$$

which has a peak value of $\frac{12\pi h}{r}\sqrt{5\pi B}$. This value is larger than the previous case where $Z_G = 0$ by a factor of $4\pi\sqrt{\pi/2}$, or about 24 dB. Again, the generator voltage has a singularity at DC.

C. Maximize Received Voltage Amplitude, Constrained Available Energy, $Z_G(\omega) = Z_T^*(\omega)$, $Z_L = \infty$

In this case we maximize receiver voltage with a constraint on the available generator energy. The general solutions are given by (14)-(15). The transfer function is the same as in (38). The optimum generator voltage is then,

$$V_G(\omega) = \frac{j\omega\mu_0 h^2}{4\pi\lambda r}, \tag{42}$$

and the normalization condition is,

$$\lambda^2 = \frac{\mu_0^2 h^4 B}{64\pi^3 \alpha r^2}. \tag{43}$$

Finally, the time domain receiver voltage is found as,

$$v_L(t') = \frac{30h}{r}\sqrt{\frac{\pi B}{5}}\frac{\sin 2\pi B t'}{2\pi B t'}. \tag{44}$$

The peak value of this response is $\frac{30h}{r}\sqrt{\frac{\pi B}{5}}$, which is about 4 dB larger than the case where the input energy was constrained and $Z_G = 0$.

Note that similar closed-form optimizations can be obtained for other small antennas such as electrically small loops, slots, and monopoles, although the required integrals can become very complicated. In fact, similar optimizations can be derived for any pair of antennas that can be represented with simple RC or RL equivalent circuits that are valid over the frequency band of interest.

5. Conclusions

Several possible optimization solutions for bandlimited radiated waveforms for ultra-wideband radio systems have been presented for general radiating elements, with arbitrary generator and load impedances. Constraints include input energy, available energy, received voltage amplitude, generator voltage amplitude, and radiated power spectral density. These solutions represent upper bounds on the performance of actual UWB systems. Examples of optimized results have been presented for lossless and resistively loaded wire dipole antennas. Closed-form results have been presented for optimization solutions for short dipoles. These results can be applied to arbitrary UWB antenna elements, and to any set of antennas that can be represented with lumped element equivalent circuits over the frequency band of interest.

6. Acknowledgement

The author would like to acknowledge helpful discussions with Professors D. Goeckel and L. Franks of the University of Massachusetts at Amherst.

7. References

[1] R. A. Scholtz, R. Weaver, E. Homier, J. Lee, P. Hilmes, A. Taha, and R. Wilson, "UWB deployment challenges", *Proc. PIMRC 2000*, September 2000.

[2] M. Z. Win and R. A. Scholtz, "Impulse radio: how it works", *IEEE Communications Letters*, vol. 2, pp. 10-12, February 1998.

[3] Federal Communications Commission, "Revision of Part 15 of the Commission's Rules Regarding Ultra-Wideband Transmission Systems", NPRM 00-163, May 10, 2000.

[4] D. Lamensdorf and L. Susman, "Baseband-pulse-antenna techniques", *IEEE Antennas and Propagation Magazine*, vol. 36, pp. 20-30, February 1994.

[5] A. Shlivinski, E. Heyman, and R. Kastner, "Antenna characterization in the time domain", *IEEE Trans. Antennas and Propagation*, Vol. 45, pp. 1140-1149, July 1997.

[6] D. M. Pozar, R. E. McIntosh, and S. G. Walker, "The optimum feed voltage for a dipole antenna for pulse radiation," *IEEE Trans. Antennas and Propagation*, Vol. AP-31, pp. 563-569, July 1983.

[7] D. M. Pozar, Y.-W. Kang, D. H. Schaubert, and R. E. McIntosh, "Optimization of the transient radiation from a dipole array," *IEEE Trans. Antennas and Propagation*, Vol. AP-33, pp. 69-75, January 1985.

[8] C. A. Balanis, **Antenna Theory: Analysis and Design**, 2nd ed. John Wiley & Sons, New York, NY, 1997.

[9] D. M. Pozar, **Microwave Engineering**, 2nd ed., John Wiley & Sons, New York, NY, 1998.

[10] L. E. Franks, **Signal Theory**, revised edition, Dowden & Culver, Stroudsberg, PA, 1981.

[11] D. R. Smith, **Variational Methods in Optimization**, Prentice-Hall, New Jersey, 1974.

[12] J. H. Richmond, "Radiation and scattering by thin-wire structures in the complex frequency domain", Technical Report 2902-10, Ohio State University ElectroScience Lab, Columbus, OH, 1973.

[13] N. N. Wang, J. H. Richmond, and M. C. Gilreath, “Sinusoidal reaction formulation for radiation and scattering from conducting surfaces”, *IEEE Trans. Antennas Propagation*, vol. 23, pp. 376 - 382, May 1975.

Figure captions:

Figure 1. Frequency domain model of transmit and receive antennas for a UWB radio system.

Figure 2a. Generator voltage waveform to maximize receive voltage amplitude for lossless dipoles with L = 15 cm, a = 0.02 cm, $\sigma = \infty$, Z_G = 50 Ω, $Z_L = \infty$. Available generator energy constrained to 1 Joule; bandwidth is 2 GHz.

Figure 2b. Resulting optimized receive antenna voltage waveform for the antenna parameters of Figure 2a.

Figure 3a. Generator voltage waveform to maximize receive voltage amplitude for dipoles with L= 15 cm, a = 0.02 cm, σ = 1000 S/m, Z_G = 50 Ω, $Z_L = \infty$. Available generator energy constrained to 1 Joule; bandwidth is 2 GHz.

Figure 3b. Resulting optimized receive antenna voltage waveform for the antenna parameters of Figure 3a.

Figure 4. Voltage transfer function magnitude versus frequency for a pair of dipoles with L = 15 cm, a = 0.02 cm, Z_G = 50 Ω, for various conductivities and receiver load impedance.

Figure 5. Constrained receive voltage amplitude at $t' = 0$ versus the Lagrange multiplier λ_2, for lossy dipoles having L = 15 cm, a = 0.02 cm, σ = 1.0E4 S/m, Z_G = 50 Ω, and $Z_L = \infty$. The signal bandwidth is 2 GHz, and the constrained input energy is W_{in} = 1 Joule.

Figure 6a. Normalized receive voltage versus time for optimized waveform "sharpness" with v_0 = 5381 Volts for the lossy dipoles defined in Figure 5.

Figure 6b. Normalized receive voltage versus time for optimized waveform "sharpness" with v_0 = 3000 Volts for the lossy dipoles defined in Figure 5.

Figure 6c. Normalized receive voltage versus time for optimized waveform "sharpness" with v_0 = 500 Volts for the lossy dipoles defined in Figure 5.

Figure 7. Compression ratio versus normalized constrained receiver voltage for dipoles defined in Figure 5.

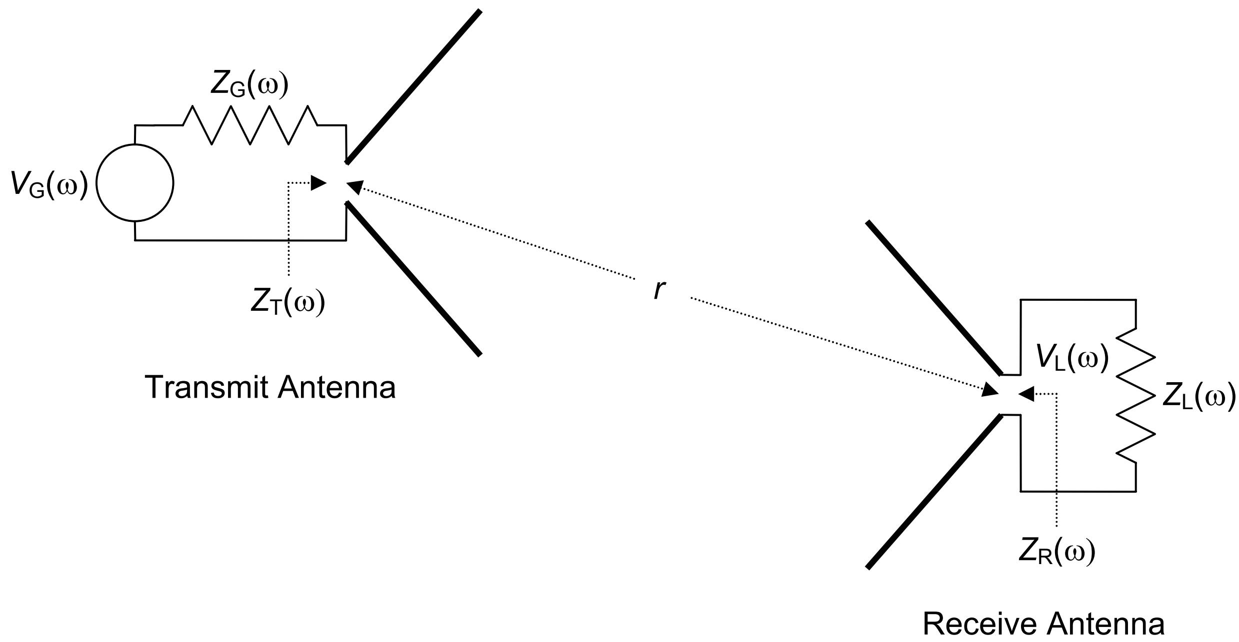

$Z_G(\omega)$
$V_G(\omega)$
$Z_T(\omega)$
Transmit Antenna
r
$V_L(\omega)$
$Z_L(\omega)$
$Z_R(\omega)$
Receive Antenna

Figure 1.

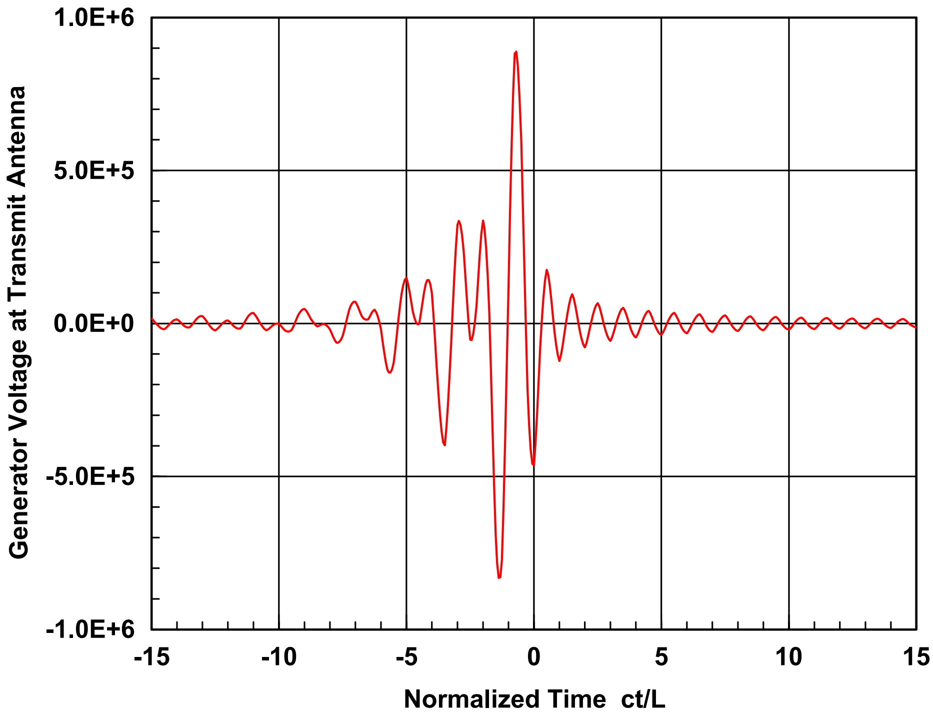

1.0E+6
5.0E+5
0.0E+0
-5.0E+5
-1.0E+6
-15
-10
-5
0
5
10
15
Normalized Time ct/L
Generator Voltage at Transmit Antenna

Figure 2a.

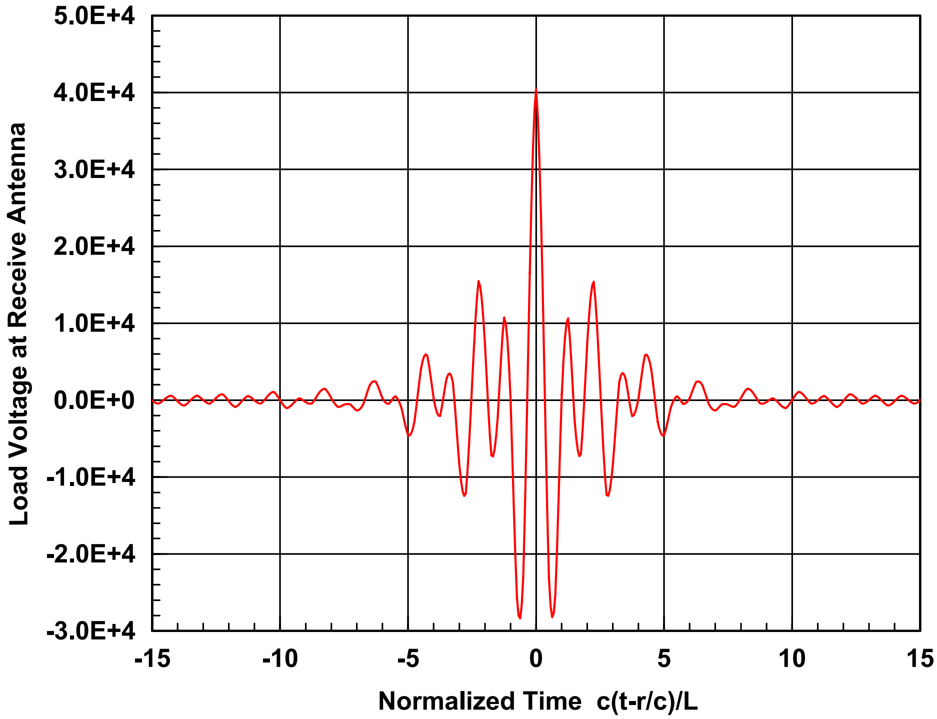

5.0E+4
4.0E+4
3.0E+4
2.0E+4
1.0E+4
0.0E+0
-1.0E+4
-2.0E+4
-3.0E+4
Load Voltage at Receive Antenna
-15
-10
-5
0
5
10
15
Normalized Time c(t-r/c)/L

Figure 2b.

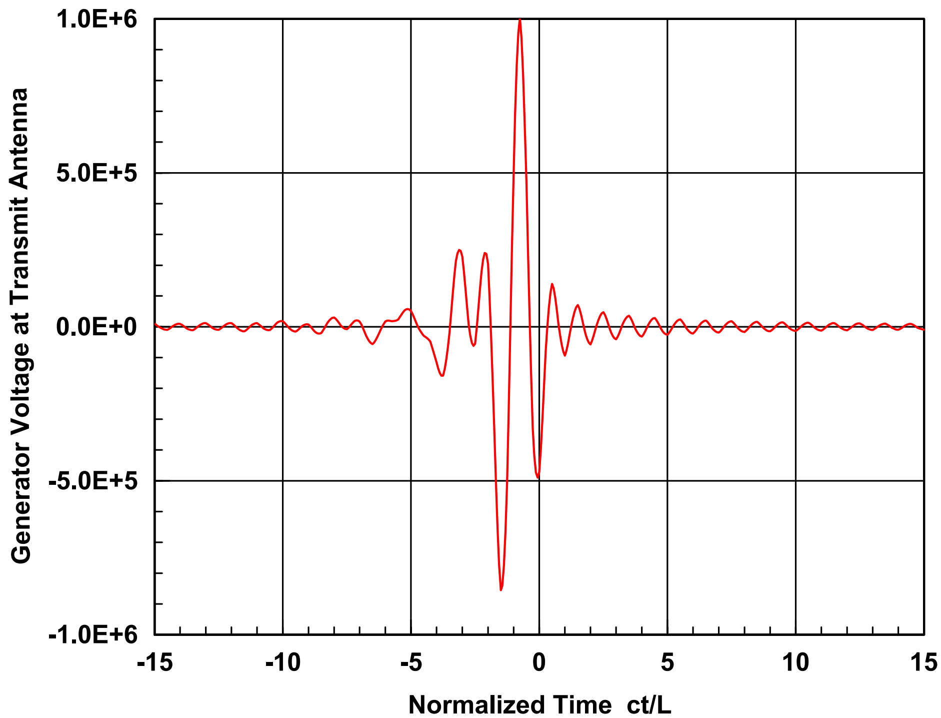

1.0E+6
5.0E+5
0.0E+0
-5.0E+5
-1.0E+6
Generator Voltage at Transmit Antenna
-15
-10
-5
0
5
10
15
Normalized Time ct/L

Figure 3a.

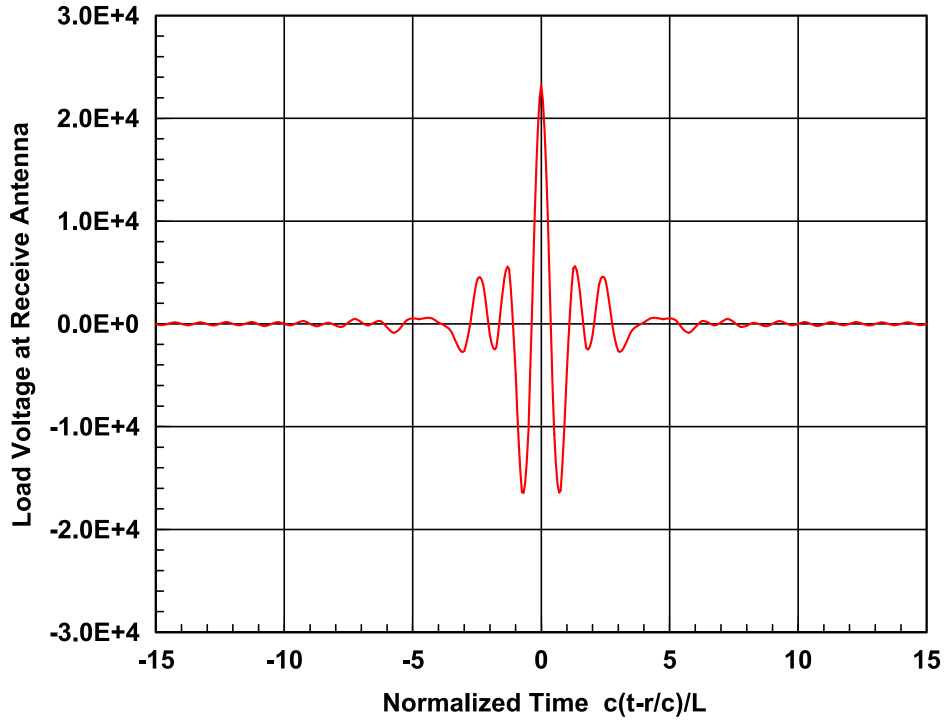
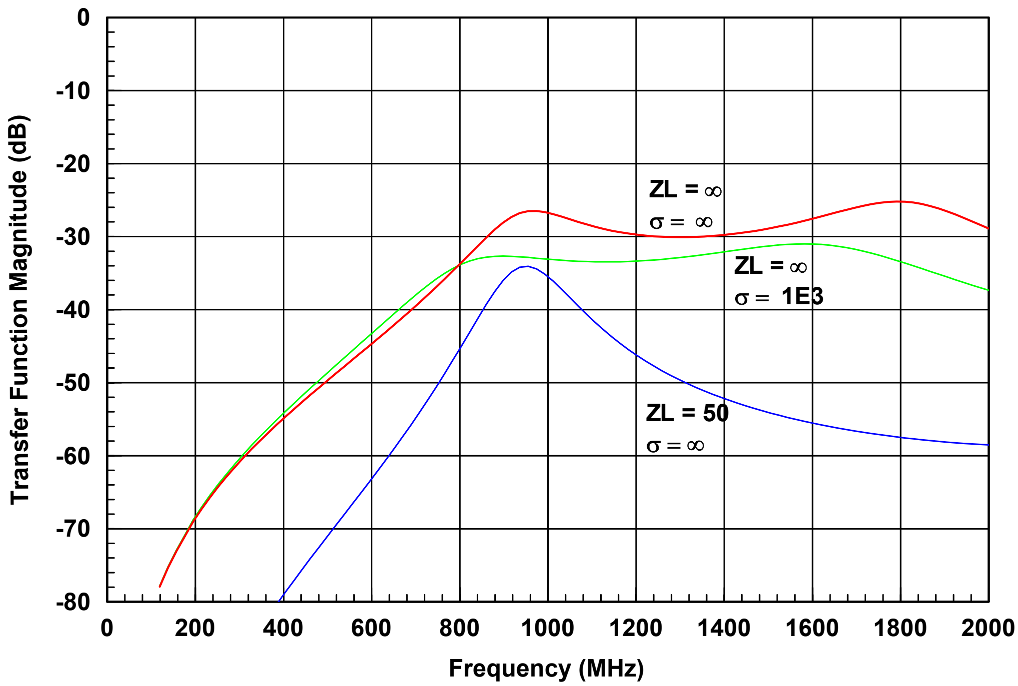

3.0E+4
2.0E+4
1.0E+4
0.0E+0
-1.0E+4
-2.0E+4
-3.0E+4
Load Voltage at Receive Antenna
-15
-10
-5
0
5
10
15
Normalized Time c(t-r/c)/L

Figure 3b.



Transfer Function Magnitude (dB)
0
-10
-20
-30
-40
-50
-60
-70
-80
0
200
400
600
800
1000
1200
1400
1600
1800
2000
Frequency (MHz)
ZL = ∞
σ = ∞
ZL = ∞
σ = 1E3
ZL = 50
σ = ∞

Figure 4.

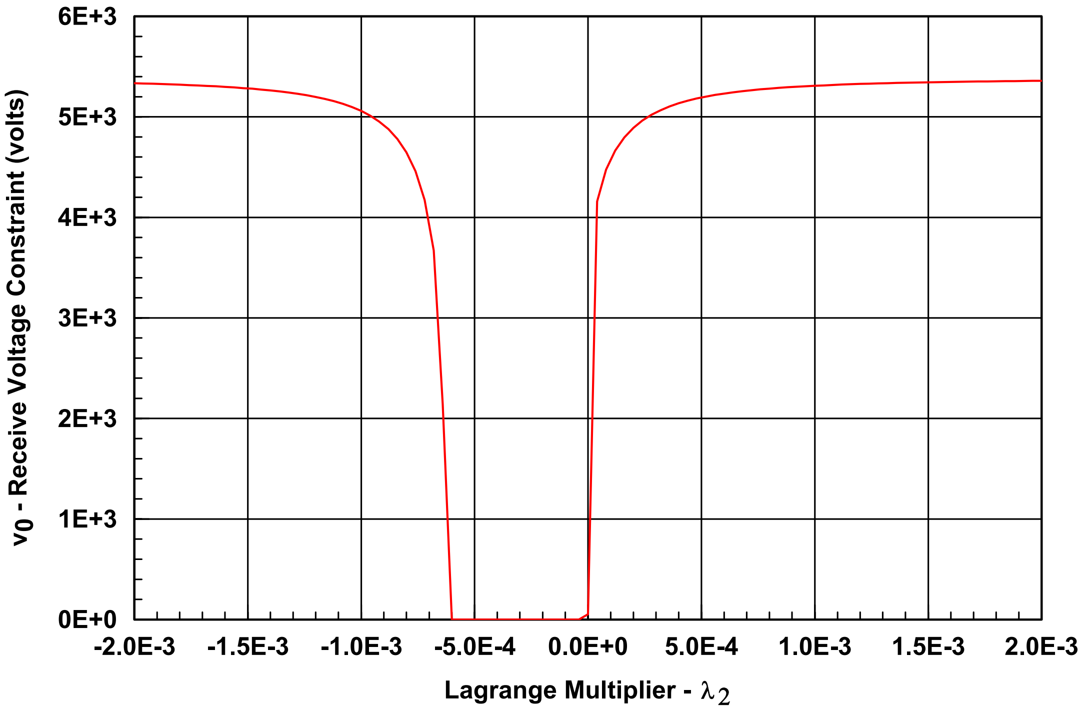

6E+3
5E+3
4E+3
3E+3
2E+3
1E+3
0E+0
v0 - Receive Voltage Constraint (volts)
-2.0E-3
-1.5E-3
-1.0E-3
-5.0E-4
0.0E+0
5.0E-4
1.0E-3
1.5E-3
2.0E-3
Lagrange Multiplier - λ2

Figure 5.

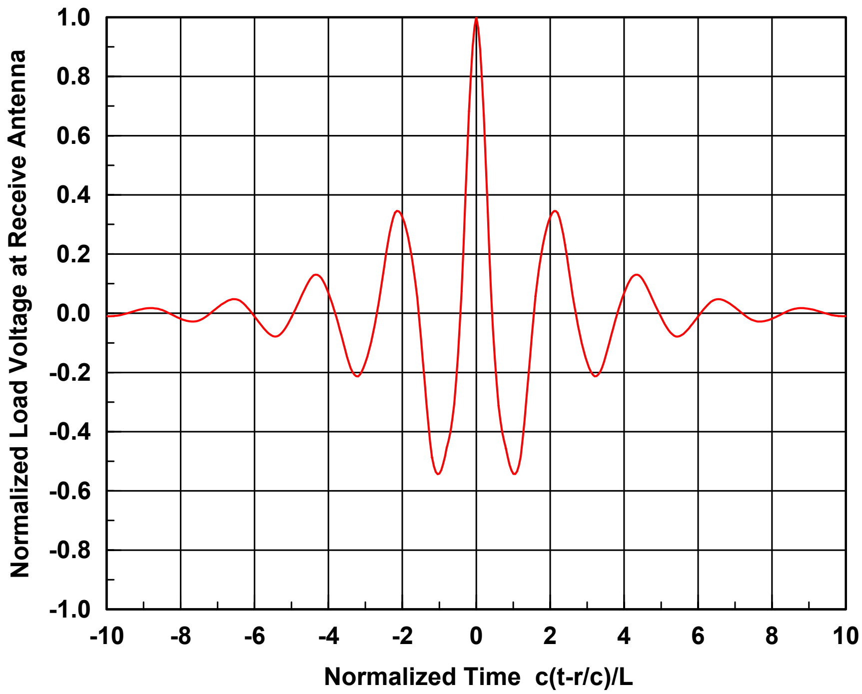

Normalized Load Voltage at Receive Antenna
1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
-10
-8
-6
-4
-2
0
2
4
6
8
10
Normalized Time c(t-r/c)/L

Figure 6a.

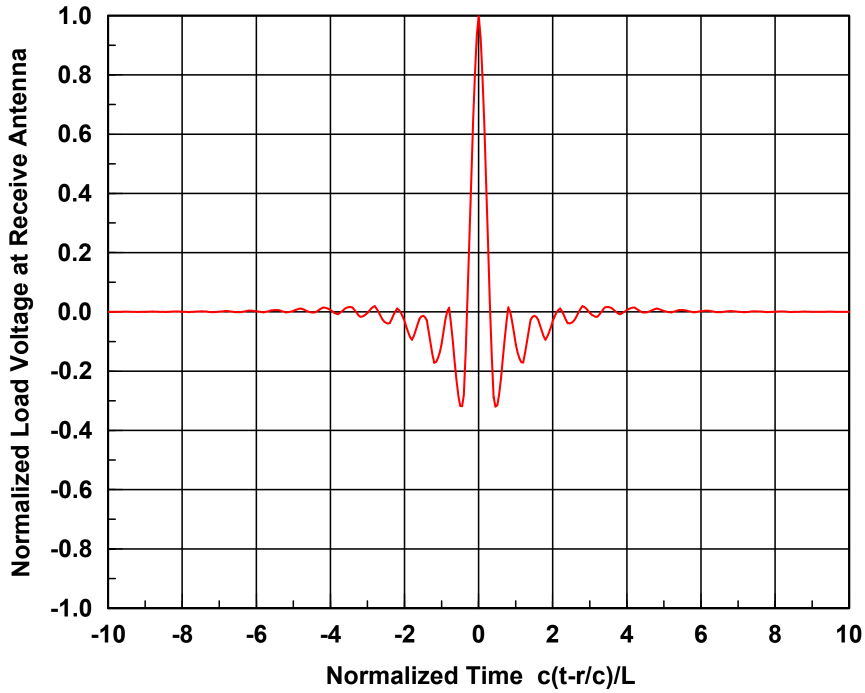

1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
Normalized Load Voltage at Receive Antenna
-10
-8
-6
-4
-2
0
2
4
6
8
10
Normalized Time c(t-r/c)/L

Figure 6b.

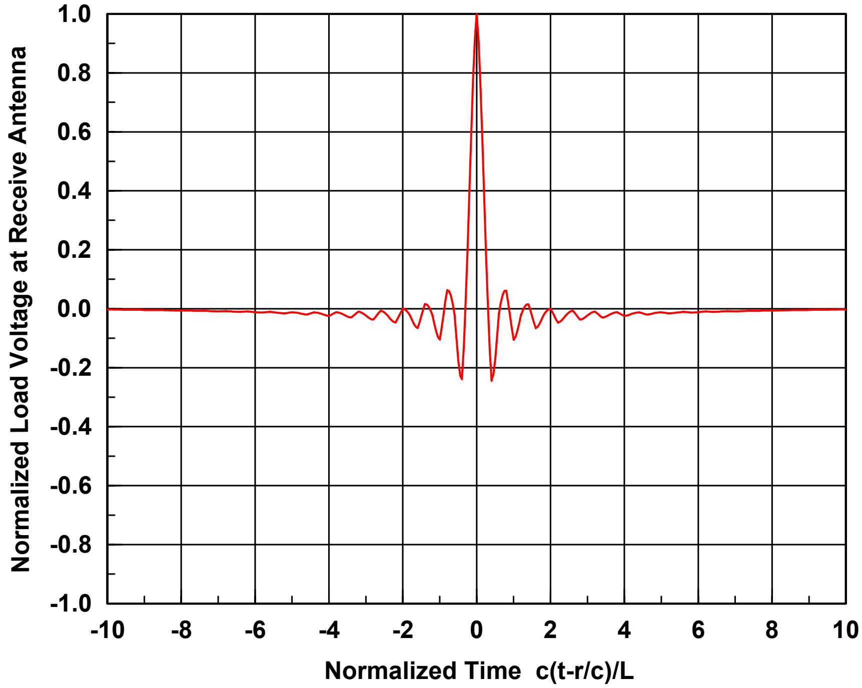

1.0
0.8
0.6
0.4
0.2
0.0
-0.2
-0.4
-0.6
-0.8
-1.0
Normalized Load Voltage at Receive Antenna
-10
-8
-6
-4
-2
0
2
4
6
8
10
Normalized Time c(t-r/c)/L

Figure 6c.

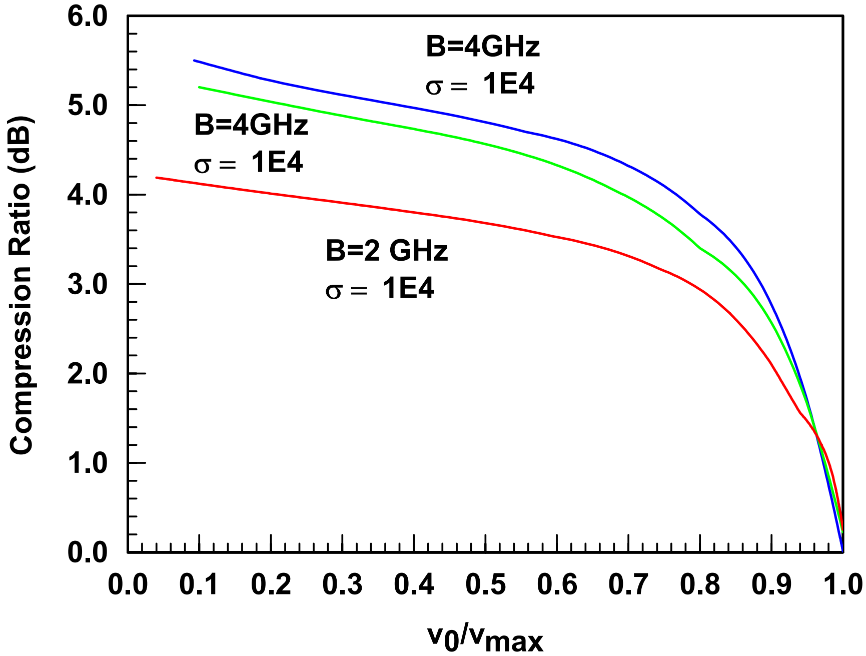

B=4GHz
σ = 1E4
B=4GHz
σ = 1E4
B=2 GHz
σ = 1E4
Compression Ratio (dB)
6.0
5.0
4.0
3.0
2.0
1.0
0.0
0.0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1.0
v0/vmax

Figure 7.