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The difference between Doppler velocity and real wind velocity in single scattering from refractive index fluctuations

V. I. Tatarskii and A. Muschinski

University of Colorado, Cooperative Institute for Research in Environmental Sciences
and NOAA Environmental Technology Laboratory, Boulder, Colorado, USA

Abstract. Scattering of electromagnetic waves by turbulence is considered. It is shown that Doppler velocity (the ratio of Doppler shift and Bragg wave number) includes two terms: The first is the mean radial velocity in the scattering volume, and the second is what we call the "correlation velocity," caused by a correlation between the fluctuations of velocity and those of refractive index. The correlation velocity is proportional to the imaginary part of the Bragg wave vector component of the three-dimensional spatial cross spectrum of refractive index and radial wind velocity. In the electromagnetic case in the atmosphere the correlation velocity provides information about the combination of humidity and heat fluxes. In the acoustic case it provides information about the flux of sensible heat in the atmosphere or about some combination of salinity and heat fluxes in water. In situ measurements of the temperature, humidity, and wind fluctuations in the atmosphere are analyzed to estimate the magnitude of the correlation velocity. It appears that the correlation velocity, which cannot be directly measured with existing radar wind profilers, could be separated from the total Doppler shift by means of bistatic radars. A potential application is the profiling of fluxes using bistatic systems.

1. Introduction

Electromagnetic or acoustic single scattering from a continuous medium is caused by the corresponding Bragg component of refractive index $n(\mathbf{R})$ inhomogeneities [*Tatarskii*, 1961, 1971]. The instantaneous scattered signal is proportional to the random spatial Fourier component of $n(\mathbf{R})$ that corresponds to the difference between the incident and scattered wave vectors. Thus the measured complex amplitude of a scattered field provides information about a specific spatial spectral component of a random medium. On the other hand, the temporal (Doppler) spectrum of a scattered field provides information about the velocities of scatterers. *Woodman and Guillen* [1974] were the first to show empirically that the Doppler shift obtained from clear-air, radio wave echoes at meter-scale wavelengths can be used for wind measurements. Since then, the basic working hypothesis of radar wind profiling has been that the ensemble average of the Doppler velocity is identical to the ensemble average of the spatial mean radial wind velocity within the scattering volume. In this paper, we analyze this problem starting with rigorous scattering theory and find that in a general case this hypothesis is not true. We analyze the correlation between the signal envelope and its temporal derivative. This correlation corresponds to the first moment of the Doppler spectrum (see Appendix B) and provides the Doppler velocity. There exists a difference between mean Doppler velocity W_D and mean velocity component along the scattering vector $\bar{W}$ (in the case of monostatic radar this velocity coincides with the radial velocity). The leading term describing that difference is what we call the "correlation velocity" $\tilde{W}$. The correlation velocity is a result of the correlation between the refractive index fluctuations and radial velocity fluctuations w. It follows from scattering theory that the existence of the correlation velocity is rather general and occurs in any scattering phenomenon. The value of $\tilde{W}$, however, may significantly vary in different types of scattering and different situations. In this paper, we analyze the correlation velocity from a general point of view and estimate its possible value for wind profilers.

Clear-air Doppler radars have been used for ~25 years to remotely measure wind velocities in the troposphere, stratosphere, and mesosphere [e.g.,

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Woodman and Guillen, 1974; *Balsley and Gage*, 1982; *Gage*, 1990; *Rottger and Larsen*, 1990; *Weber and Wuertz*, 1990; *Clifford et al.*, 1994]. The working hypothesis upon which the retrieval of wind velocities relies is that the radial wind velocity may be estimated by the Doppler velocity

$$v_D = -\omega_D / k_B, \tag{1}$$

where ω_D is the Doppler shift and

$$k_B = 4\pi/\lambda \tag{2}$$

is the Bragg wave number with radar wavelength λ.

For more than a decade it has been known that there is often a downward bias in the mean vertical velocity observed with vertically looking clear-air radars in the free troposphere. The bias is typically on the order of several centimeters per second, which is 1 or 2 orders of magnitude larger than what one would expect for the mean synoptic-scale vertical motion. Several mechanisms have been suggested to explain that bias. *Nastrom and VanZandt* [1994] show that in a field of upward propagating gravity waves there is a negative correlation between radar reflectivity and vertical velocity, which leads to a downward bias in the observed Doppler velocity. *Muschinski* [1996] suggested that in regions of vertical shear of horizontal wind one should expect a correlation between Doppler velocity and the tilting angle of Fresnel-scattering layers, which results in a downward bias in the troposphere, where the wind speed increases with height, and results in an upward bias in the stratosphere, where the wind speed often decreases with height. *Worthington et al.* [2001] show that wind profilers are usually situated in valleys or in the vicinity of mountains, rather than on mountain tops, which leads to a preferred phase of mountain waves above wind profiler sites, affecting the mean vertical wind above the radar site in a characteristic way.

In the convective boundary layer, and at decimeter-scale UHF wavelengths, *Angevine* [1997] observed an even larger downward bias of up to 30 cm s^{-1}. The mechanisms suggested by *Nastrom and VanZandt* [1994], *Muschinski* [1996], and *Worthington et al.* [2001] offer no explanation of a bias in the mean vertical velocity because in the convective boundary layer there are neither gravity waves, mountain waves, nor Fresnel-scattering layers. *Angevine* [1997] suggested that Rayleigh-scattering, downward moving insects may be the cause for the large downward bias in the convective boundary layer. *Muschinski et al.* [1999] simulated wind profiler observations of the vertical wind in the convective boundary layer on the basis of spatiotemporally evolving refractive index fields generated with the large eddy simulation (LES) technique. Since the grid spacing was 8 m in the vertical direction and 16 m in the horizontal directions, a direct evaluation of the scattering integral, which would have required a grid spacing of a fraction of 16 cm (the Bragg wavelength of the simulated 915 MHz boundary layer profiler), was not possible, and the Bragg-scale correlation between refractive index and radial velocity that is investigated in the present paper could not be simulated in the *Muschinski et al.* [1999] study. In other words, the LES used by *Muschinski et al.* [1999] was not suitable for investigating the correlation velocity because it was too coarse. Although the *Muschinski et al.* [1999] study did not provide evidence for biases caused by the correlation velocity, it did show that Doppler velocities and radial wind velocities can significantly differ from each other. *Muschinski et al.* [1999] showed that the differences between (1) the Doppler velocities estimated from 1-s-long wind profiler signal time series and (2) the instantaneous, true radial wind velocity averaged over the radar's resolution volume and over the same 1-s-long time interval can amount to several tens of centimeters per second [*Muschinski et al.*, 1999, Figure 6]. Those differences are to be attributed to inhomogeneities of field of radar reflectivity at length scales smaller than the size of the radar's resolution volume and large compared to the Bragg wavelength, and at timescales comparable to 1 s. If those inhomogeneities are persistent, then that mechanism causes a bias also in the ensemble-averaged Doppler velocities. That bias is a third-order bias, in contrast to the correlation velocity, which is a second-order bias. As we will see, however, it seems that often the magnitude of the third-order bias is larger than the correlation velocity.

In the present paper, we show that the relation between $\bar{W}$ and $\tilde{W}$ depends on many factors, such as the mutual orientation of the mean wind vector and the radar beam, radar wavelength, intensity of turbulence, and stability of the atmosphere. Guided by in situ measurements of turbulence in the lower troposphere, we estimate the magnitude of the correlation velocity. Depending on the atmospheric conditions and the magnitude of the scattering vector, the value of $\tilde{W}$ may vary from less than 1 mm s^{-1} to tens of centimeters per second (see Figure 3). For a 50 MHz wind profiler the correlation velocity can be estimated as several cm s^{-1}, and for a 1000 MHz wind profiler it is less than 1 mm s^{-1}. Using existing wind profilers, it is not possible to measure $\tilde{W}$ directly, i.e., to separate $\tilde{W}$ from the radial wind velocity. However, in the future it may be possible to measure $\tilde{W}$ directly using bistatic atmospheric radars and thus obtain information about turbulent fluxes in the atmosphere.

2. Theory: Correlation Between the Signal Time Derivative and Its Quadrature Component

In the atmosphere, refractive index fluctuations for electromagnetic waves, $n'(\mathbf{R}, t)$, are related to temperature fluctuations $T'(\mathbf{R}, t)$ and absolute humidity $a'(\mathbf{R}, t)$ by the linear relation

$$n'(\mathbf{R}, t) = K_1 T'(\mathbf{R}, t) + K_2 a'(\mathbf{R}, t), \tag{3}$$

where K_1 and K_2 depend on the mean temperature $\bar{T}$, mean pressure $\bar{p}$, and mean humidity $\bar{a}$. For acoustic waves in the atmosphere a similar relation is true, but turbulent velocity $\mathbf{u}(\mathbf{R}, t)$ replaces $a'(\mathbf{R}, t)$. For acoustic waves in water, $n'(\mathbf{R}, t)$ is some linear combination of velocity, temperature, and salinity (see details given by *Ostashev* [1997]). For ionospheric plasma, n' is proportional to the fluctuations in a concentration of free electrons.

In all these cases, however, the complex amplitude of the scattered field in the far (Fraunhofer) zone can be presented by a single formula [*Tatarskii*, 1971]. We consider only single scattering, i.e., the Born approximation, and a nonabsorbing medium:

$$\mathcal{E}_{sc}(\mathbf{n}, t) = \frac{A_0 k^2 e^{ikR}}{4\pi R} \iiint_V e^{ik(\mathbf{n}_0 - \mathbf{n})\mathbf{R}'} n'(\mathbf{R}', t)\, d^3R'. \tag{4}$$

Here, $k = \omega_0/c$ is the wave number, ω_0 is the carrier frequency of the incident wave, c is the phase velocity of the wave, R is the distance from the center of the scattering volume V to the point of observation, $\mathbf{n}_0$ is the unit vector indicating the propagation direction of the incident wave, $\mathbf{n}$ is the unit vector indicating the propagation direction of the scattered wave, and A_0 is the amplitude of the incident wave in the scattering volume. Integration in (4) is performed within the scattering volume V; a more rigorous consideration would use the weighting functions related to antenna diagrams [*Tatarskii*, 1971, see section 28], but it is enough for our goals to use the simpler description. The scattering angle θ is the angle between the vectors $\mathbf{n}_0$ and $\mathbf{n}$.

The real scattered field $E_{sc}(t)$ (we omit the second argument, $\mathbf{n}$) can be expressed in terms of $\varepsilon_{sc}(\mathbf{n}, t)$ as follows:

$$E_{sc}(t) = \mathrm{Re}\,[\exp(-i\omega_0 t)\mathcal{E}_{sc}(\mathbf{n}, t)] \tag{5}$$

or

$$E_{sc}(t) = I(t)\cos(\omega_0 t) + Q(t)\sin(\omega_0 t),$$

$$I(t) = A_{sc} \iiint_V \cos[\mathbf{K}_{sc}\mathbf{R}' + \psi] n'(\mathbf{R}', t)\, d^3R', \tag{6}$$

$$Q(t) = A_{sc} \iiint_V \sin[\mathbf{K}_{sc}\mathbf{R}' + \psi] n'(\mathbf{R}', t)\, d^3R',$$

where we denoted that $A_{sc} = A_0 k^2/4\pi R$, $\psi = kR$, and $\mathbf{K}_{sc}$ is the scattering vector,

$$\mathbf{K}_{sc} = k(\mathbf{n}_0 - \mathbf{n}), \qquad K_{sc} = |\mathbf{K}_{sc}| = 2k\sin(\theta/2). \tag{7}$$

Both functions, in-phase envelope $I(t)$ and quadrature envelope $Q(t)$, can be measured simultaneously.

For statistically stationary signals the functions $I(t)$ and $Q(t)$ satisfy the relations

$$\langle I^2(t)\rangle = \langle Q^2(t)\rangle = \mathrm{const}, \qquad \langle I(t)Q(t)\rangle = 0. \tag{8}$$

The values $\langle I^2(t)\rangle = \langle Q^2(t)\rangle$ depend only on the strength of refractive index fluctuations and are independent of velocities in the scattering volume.

To obtain information about velocities, we consider the time derivative $\dot{I}(t)$. From (6) we obtain

$$\dot{I}(t) = A_{sc} \iiint_V \cos[\mathbf{K}_{sc}\mathbf{R}' + kR] \frac{\partial n'(\mathbf{R}', t)}{\partial t}\, d^3R'. \tag{9}$$

The time derivative of the refractive index according to (3) is determined by the equations for $\partial T'/\partial t$ and $\partial a'/\partial t$. The temperature T satisfies the heat conductivity equation,

$$\frac{\partial T}{\partial t} + \mathbf{U}\nabla T = \chi \Delta T, \tag{10}$$

where $\mathbf{U}(\mathbf{R}, t)$ is the velocity in the point $\mathbf{R}$, χ is the temperature conductivity coefficient, and $\Delta = \nabla^2$. Let us compare the two terms $\chi \Delta T$ and $\mathbf{U}\nabla T$. We have

$$\chi \Delta T \sim \frac{\chi T'}{l^2}, \qquad \mathbf{U}\nabla T \sim \frac{UT'}{l},$$

where l is the length scale of the temperature fluctuation T'. The ratio of these terms is of the order of magnitude

$$\frac{\chi \Delta T}{\mathbf{U}\nabla T} \sim \frac{\chi}{lU} = \frac{\nu}{lU}\frac{\chi}{\nu} = \frac{1}{Re_l}\frac{1}{Pr}.$$

Here, Re_l is the Reynolds number at the length scale l, and Pr is the Prandtl number. We have $Pr \approx 1$ and $Re_l \gg 1$ for all scales $l \gg l_0$ (l_0 is the Kolmogorov scale of turbulence). The scale of interest is $l = \lambda/2 \sin(\theta/2)$, where λ is the wavelength. Thus the ratio $\chi \Delta T/\mathbf{U}\nabla T$ is small if

$$\lambda \gg 2 l_0 \sin(\theta/2),$$

in which case we can neglect the term $\chi \Delta T$ in (10).

If we neglect the term $\chi \Delta T$ in (10), we obtain

$$\frac{\partial T}{\partial t} + \mathbf{U}\nabla T = \frac{dT[\mathbf{r}(\mathbf{t}), t]}{dt} = 0. \tag{11}$$

This relation expresses the conservation of temperature in a fluid parcel moving along the trajectory $\mathbf{r} = \mathbf{r}(t)$.

The quantity a' in (3) requires similar examination, except that the diffusion coefficient D replaces the coefficient χ. If we neglect the diffusion effect (for $Re_l \gg 1$), we obtain for the quantity a an equation of the same form (11) as for T. Hence the refractive index n also satisfies a similar continuity equation in the inertial subrange

$$\frac{\partial n(\mathbf{R}, t)}{\partial t} + \mathbf{U}(\mathbf{R}, t)\nabla n(\mathbf{R}, t) = 0. \tag{12}$$

Let us present n and $\mathbf{U}$ as sums of mean values and fluctuations (we denote the mean values corresponding to the ensemble averaging by angle brackets):

$$\begin{aligned} n(\mathbf{R}, t) &= \langle n(\mathbf{R})\rangle + n'(\mathbf{R}, t), \\ \mathbf{U}(\mathbf{R}, t) &= \langle \mathbf{U}(\mathbf{R})\rangle + \mathbf{u}(\mathbf{R}, t). \end{aligned} \tag{13}$$

Here,

$$\langle n'(\mathbf{R}, t)\rangle = 0, \qquad \langle \mathbf{u}(\mathbf{R}, t)\rangle = 0, \tag{14}$$

and the mean values, $\langle n(\mathbf{R})\rangle$ and $\langle \mathbf{U}(\mathbf{R})\rangle$, may vary in the scattering volume. Then,

$$\frac{\partial \langle n\rangle}{\partial t} + \frac{\partial n'}{\partial t} = -\langle \mathbf{U}\rangle \nabla \langle n\rangle - \mathbf{u}\nabla \langle n\rangle - \langle \mathbf{U}\rangle \nabla n' - \mathbf{u}\nabla n', \tag{15}$$

and after averaging, we obtain

$$\frac{\partial \langle n\rangle}{\partial t} = -[\langle \mathbf{U}\rangle \nabla \langle n\rangle + \langle \mathbf{u}\nabla n'\rangle]. \tag{16}$$

Subtracting (16) from (15), we obtain the equation for the time derivative of the refractive index fluctuations

$$\frac{\partial n'}{\partial t} = -\langle \mathbf{U}(\mathbf{R})\rangle \nabla n'(\mathbf{R}, t) - \mathbf{u}(\mathbf{R}, t)\nabla \langle n(\mathbf{R})\rangle - [\mathbf{u}\nabla n' - \langle \mathbf{u}\nabla n'\rangle]. \tag{17}$$

Let us discuss this formula. The first term in the right-hand side of (17), $\langle \mathbf{U}(\mathbf{R})\rangle \nabla n'(\mathbf{R}, t)$, describes the transport of refractive index inhomogeneities by the mean velocity $\langle \mathbf{U}(\mathbf{R})\rangle$. This term corresponds to the classical Taylor frozen turbulence hypothesis. We will see later (equations (34) and (41)) that the first term causes the standard Doppler shift of the signal. The second term, $\mathbf{u}(\mathbf{R}, t)\nabla \langle n(\mathbf{R})\rangle$, describes the mixing of the mean profile $\langle n(\mathbf{R})\rangle$ by the fluctuations of velocity $\mathbf{u}(\mathbf{R}, t)$. This term exists only if the mean gradient $\nabla \langle n(\mathbf{R})\rangle$ is nonzero. The third term, $[\mathbf{u}\nabla n' - \langle \mathbf{u}\nabla n'\rangle]$, describes the mixing of the local random profile of refractive index by the random component of velocity $\mathbf{u}$. Both the second and third terms in the right-hand side of (17) are small in comparison with the first term. Because we are interested in corrections to the standard interpretation of the Doppler shift, we must include the most important term which causes the deviation. Let us compare the second and third terms. For $r \ll l_0$ the structure function of refractive index has the form $D_{nn}(r) \approx C_n^2 l_0^{-4/3} r^2$ [see, e.g., *Tatarskii*, 1971, formula (7), p. 76]. Thus, if we express the variance of the gradient of n' in terms of $D_{nn}(r)$, we obtain $\langle (\nabla n')^2\rangle = \lim_{r\to 0} [D_{nn}(r)/r^2] = C_n^2 l_0^{-4/3}$. The outer scale of turbulence, L_0, is defined [see, e.g., *Tatarskii*, 1971, p. 73] as the length scale for which the variance of the random component n' is equal to the square of the systematic difference of n, i.e., $D_{nn}(L_0) = (\nabla \langle n\rangle)^2 L_0^2$, or $C_n^2 L_0^{2/3} = (\nabla \langle n\rangle)^2 L_0^2$. Thus $(\nabla \langle n\rangle)^2 = C_n^2 L_0^{-4/3}$.

Therefore, for the ratio of random and mean gradients we obtain

$$\langle(\nabla n')^2\rangle/(\nabla\langle n\rangle)^2 = (L_0/l_0)^{4/3} \gg 1.$$

Thus, a priori we can expect that the third term in the right-hand side of (17) is more important than the second term. Nevertheless, in Appendix A it is shown that this term provides negligible contribution to the Doppler shift because of a small numerical coefficient. Because of this, we will neglect in (17) the second-order term $[\mathbf{u}\nabla n' - \langle \mathbf{u}\nabla n'\rangle]$ and will use the equation

$$\frac{\partial n'(\mathbf{R}, t)}{\partial t} = -\langle \mathbf{U}(\mathbf{R})\rangle \nabla n'(\mathbf{R}, t) - \mathbf{u}(\mathbf{R}, t)\nabla\langle n(\mathbf{R})\rangle. \tag{18}$$

We will show that the term $\mathbf{u}(\mathbf{R}, t)\nabla\langle n(\mathbf{R})\rangle$ in (18) contributes to that part of the Doppler shift that is related to the correlation velocity.

If we substitute (18) into (9), we obtain

$$\dot{I}(t) = -A_{sc} \iiint_V \cos[\mathbf{K}_{sc}\mathbf{R} + \psi] \cdot [\langle \mathbf{U}(\mathbf{R})\rangle \nabla n'(\mathbf{R}, t) + \mathbf{u}(\mathbf{R})\nabla\langle n(\mathbf{R})\rangle]\, d^3R. \tag{19}$$

Let us consider the correlation between two synchronous values $\dot{I}(t)$ and $Q(t)$:

$$\langle \dot{I}(t)Q(t)\rangle = -A_{sc}^2 \iiint_V d^3R \iiint_V d^3R' \sin[\mathbf{K}_{sc}\mathbf{R}' + \psi] \cos[\mathbf{K}_{sc}\mathbf{R} + \psi]\left[\langle \mathbf{U}(\mathbf{R})\rangle \frac{\partial\langle n'(\mathbf{R}, t)n'(\mathbf{R}', t)\rangle}{\partial \mathbf{R}} + \frac{\partial\langle n(\mathbf{R})\rangle}{\partial \mathbf{R}} \langle \mathbf{u}(\mathbf{R})n'(\mathbf{R}', t)\rangle\right]. \tag{20}$$

(The quantity $\langle \dot{I}(t)Q(t)\rangle$ can be easily related to the time derivative of the complex autocorrelation function of the signal at zero lag or to the first moment of the Doppler spectrum of the scattered signal (see Appendix B).)

Two spatial correlation functions appear in (20). The first one is the refractive index autocorrelation function that has the spectral representation

$$\langle n'(\mathbf{R}, t)n'(\mathbf{R}', t)\rangle = B_{n,n}(\mathbf{R} - \mathbf{R}') = \iiint d^3p \cos[\mathbf{p}(\mathbf{R} - \mathbf{R}')]\Phi_{n,n}(\mathbf{p}). \tag{21}$$

If we differentiate (21) with respect to $\mathbf{R}$, we obtain the derivative that enters in (20):

$$\frac{\partial\langle n'(\mathbf{R}, t)n'(\mathbf{R}', t)\rangle}{\partial \mathbf{R}} = -\iiint d^3p \sin[\mathbf{p}(\mathbf{R} - \mathbf{R}')]\mathbf{p}\Phi_{n,n}(\mathbf{p}). \tag{22}$$

The second correlation function in (20) is the mutual correlation function between refractive index and velocity. In contrast to the autocorrelation function the mutual correlation function has both an even and an odd part with respect to $\mathbf{R} - \mathbf{R}'$. Because of this, its spectral representation consists of two terms, even and odd:

$$\langle u_j(\mathbf{R}, t)n'(\mathbf{R}', t)\rangle = B_{u_j,n}(\mathbf{R} - \mathbf{R}') = \iiint d^3p\{F_{u_j,n}^{(+)}(\mathbf{p}) \cos[\mathbf{p}(\mathbf{R} - \mathbf{R}')] + F_{u_j,n}^{(-)}(\mathbf{p}) \sin[\mathbf{p}(\mathbf{R} - \mathbf{R}')]\}. \tag{23}$$

Here,

$$F_{u_j,n}^{(+)}(-\mathbf{p}) = F_{u_j,n}^{(+)}(\mathbf{p}); \qquad F_{u_j,n}^{(-)}(-\mathbf{p}) = -F_{u_j,n}^{(-)}(\mathbf{p}). \tag{24}$$

Because of (24) we can present the real mutual correlation function $B_{u_j,n}$ in a more compact complex notation:

$$B_{u_j,n}(\mathbf{R} - \mathbf{R}') = \iiint F_{u_j,n}(\mathbf{p}) \exp[i\mathbf{p}(\mathbf{R} - \mathbf{R}')]\, d^3p. \tag{25}$$

Here,

$$F_{u_j,n}(\mathbf{p}) = F_{u_j,n}^{(+)}(\mathbf{p}) - iF_{u_j,n}^{(-)}(\mathbf{p}), \tag{26}$$

and the integral in the right-hand side of (25) is real because of (24).

In terms of $B_{n,n}(\mathbf{R} - \mathbf{R}')$ and $B_{u_j,n}(\mathbf{R} - \mathbf{R}')$ we obtain for $\langle \dot{I}(t)Q(t)\rangle$:

$$\langle \dot{I}(t)Q(t)\rangle = M_1 + M_2 + M_3 + M_4$$
$$= -\frac{A_{sc}^2}{2}\iiint_V d^3R \iiint_V d^3R' \cdot \sin[\mathbf{K}_{sc}(\mathbf{R}'-\mathbf{R})]\langle\mathbf{U}(\mathbf{R})\rangle \frac{\partial B_{n,n}(\mathbf{R}-\mathbf{R}')}{\partial \mathbf{R}}$$
$$-\frac{A_{sc}^2}{2}\iiint_V d^3R \iiint_V d^3R' \cdot \sin[\mathbf{K}_{sc}(\mathbf{R}'-\mathbf{R})]\frac{\partial\langle n(\mathbf{R})\rangle}{\partial \mathbf{R}}\langle \mathbf{u}(\mathbf{R})n'(\mathbf{R}', t)\rangle$$
$$-\frac{A_{sc}^2}{2}\iiint_V d^3R \iiint_V d^3R' \sin[\mathbf{K}_{sc}(\mathbf{R}'+\mathbf{R}) + 2\psi]\langle\mathbf{U}(\mathbf{R})\rangle \frac{\partial B_{n,n}(\mathbf{R}-\mathbf{R}')}{\partial \mathbf{R}}$$
$$-\frac{A_{sc}^2}{2}\iiint_V d^3R \iiint_V d^3R' \sin[\mathbf{K}_{sc}(\mathbf{R}'+\mathbf{R}) + 2\psi]\frac{\partial\langle n(\mathbf{R})\rangle}{\partial \mathbf{R}}\langle \mathbf{u}(\mathbf{R})n'(\mathbf{R}', t)\rangle. \quad (27)$$

All these integrals can be evaluated using the spectral representations (22) and (23). We show here how the first term, M_1, can be evaluated. All other terms can be obtained in a similar way. Substituting (22) into the formula for M_1, we obtain

$$M_1 = -\frac{A_{sc}^2}{2}\iiint_V d^3R \iiint_V d^3R' \sin[\mathbf{K}_{sc}(\mathbf{R}' - \mathbf{R})]\langle\mathbf{U}(\mathbf{R})\rangle \frac{\partial B_{n,n}(\mathbf{R}-\mathbf{R}')}{\partial \mathbf{R}}$$
$$= \frac{A_{sc}^2}{2}\iiint_V \langle\mathbf{U}(\mathbf{R})\rangle\, d^3R \iiint_V d^3R' \cdot \sin[\mathbf{K}_{sc}(\mathbf{R}'-\mathbf{R})] \cdot \iiint d^3p\,\mathbf{p}\Phi_{n,n}(\mathbf{p}) \sin[\mathbf{p}(\mathbf{R}-\mathbf{R}')]. \quad (28)$$

Changing the order of integration and presenting the product of sine in terms of the difference of cosine, we obtain

$$M_1 = -\frac{A_{sc}^2}{4}\iiint \mathbf{p}\Phi_{n,n}(\mathbf{p})\, d^3p \iiint_V \langle\mathbf{U}(\mathbf{R})\rangle\, d^3R \cdot \iiint_V d^3R' \{\cos[(\mathbf{p}-\mathbf{K}_{sc})(\mathbf{R}'-\mathbf{R})] - \cos[(\mathbf{p}+\mathbf{K}_{sc})(\mathbf{R}'-\mathbf{R})]\}. \quad (29)$$

In the case of infinite volume V the integrals over $\mathbf{R}'$ can be expressed in terms of δ functions

$$\iiint d^3R' \{\cos[(\mathbf{p}-\mathbf{K}_{sc})(\mathbf{R}'-\mathbf{R})] - \cos[(\mathbf{p}+\mathbf{K}_{sc})(\mathbf{R}'-\mathbf{R})]\} = 8\pi^3[\delta(\mathbf{p}-\mathbf{K}_{sc}) - \delta(\mathbf{p}+\mathbf{K}_{sc})].$$

For the finite volume V having the scales (L_1, L_2, L_3) instead of δ functions we obtain "spread" δ functions having the widths $(2\pi/L_1, 2\pi/L_2, 2\pi/L_3)$ [see *Tatarskii*, 1971, p. 112]. We denote this function as

$$\delta_V(\mathbf{p} \pm \mathbf{K}_{sc}) = \frac{1}{8\pi^3}\iiint_V \cos[(\mathbf{p} \pm \mathbf{K}_{sc})(\mathbf{R}' - \mathbf{R})]\, d^3R'.$$

Thus (29) takes the form

$$M_1 = -2\pi^3 A_{sc}^2 \iiint \mathbf{p}\Phi_{n,n}(\mathbf{p})\, d^3p \cdot \iiint_V \langle\mathbf{U}(\mathbf{R})\rangle\, d^3R[\delta_V(\mathbf{p}-\mathbf{K}_{sc}) - \delta_V(\mathbf{p}+\mathbf{K}_{sc})]. \quad (30)$$

We define the averaged spectrum, $\overleftrightarrow{\Phi}_{n,n}(\mathbf{K}_{sc})$, as a mean value of the function $\Phi_{n,n}(\mathbf{p})$ over the volume of **p**-space which is centered in the point $\mathbf{K}_{sc}$ and has dimensions

$$(2\pi/L_1, 2\pi/L_2, 2\pi/L_3).$$

See analogous definition given by *Tatarskii* [1971, p. 112]:

$$\mathbf{K}_{sc}\overleftrightarrow{\Phi}_{n,n}(\mathbf{K}_{sc}) \equiv \iiint \mathbf{p}\Phi_{n,n}(\mathbf{p})\delta_V(\mathbf{p}-\mathbf{K}_{sc})\, d^3p. \quad (31)$$

Using (31), we present the integral over **p** in formula (30) as follows:

$$\iiint \mathbf{p}\Phi_{n,n}(\mathbf{p})[\delta_V(\mathbf{p}-\mathbf{K}_{sc}) - \delta_V(\mathbf{p}+\mathbf{K}_{sc})]\, d^3p$$
$$= \mathbf{K}_{sc}\overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{sc}) - (-\mathbf{K}_{sc})\overset{\leftrightarrow}{\Phi}_{n,n}(-\mathbf{K}_{sc})$$
$$= 2\mathbf{K}_{sc}\overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{sc}). \qquad (32)$$

If the function $\Phi_{n,n}(\mathbf{p})$ is smooth in the volume of averaging, then $\overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{sc}) \approx \Phi_{n,n}(\mathbf{K}_{sc})$; if it changes significantly, $\overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{sc})$ may differ from $\Phi_{n,n}(\mathbf{K}_{sc})$. Substituting (32) in (30), we obtain

$$M_1 = -4\pi^3 A_{sc}^2 \overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{sc})\mathbf{K}_{sc} \iiint_V \langle \mathbf{U}(\mathbf{R})\rangle\, d^3R. \qquad (33)$$

If we introduce the projection of the mean velocity in the point **R** on the scattering vector direction $\mathbf{K}_{sc}/K_{sc}$, i.e., $W(\mathbf{R}) = \langle \mathbf{U}(\mathbf{R})\rangle \mathbf{K}_{sc}/K_{sc}$, we may present (33) in the form

$$M_1 = -4\pi^3 A_{sc}^2 \overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{sc})K_{sc} \iiint_V W(\mathbf{R})\, d^3R. \qquad (34)$$

We denote this projection as W, but it is not necessary that the direction of the scattering vector $\mathbf{K}_{sc}/K_{sc}$ be vertical.

In a similar way all other terms M_2, M_3, and M_4 can be calculated. The terms M_3 and M_4 are negligible because they are proportional to the magnitude of the Fourier transform of the very smooth scattering volume shape at the spatial wave number $2\mathbf{K}_{sc}$. These two terms also contain the phase shift $\psi = kR$, where R is the distance from the observation point to the center of the scattering volume. This distance cannot be defined with the accuracy of the wavelength λ, and because of this the averaging over ψ must be performed. This averaging also zeroes the terms M_3 and M_4.

The term M_2 can be found in the same way and is given by the formula

$$M_2 = 4\pi^3 A_{sc}^2 \overset{\leftrightarrow}{F}{}^{(-)}_{u_j,n}(\mathbf{K}_{sc}) \iiint_V \frac{\partial\langle n(\mathbf{R})\rangle}{\partial R_j}\, d^3R, \qquad (35)$$

where $\overset{\leftrightarrow}{F}{}^{(-)}_{u_j,n}(\mathbf{K}_{sc})$ is the **p**-space average of the quadrature component of the n, u_j cross spectrum. Here and in the following, the summation over the repeated index j is implied. Note that it follows from (27) that the same integral

$$\iiint_V d^3R' \sin[\mathbf{K}_{sc}(\mathbf{R}' - \mathbf{R})]$$

determines the averaging in **p**-space and enters in both terms M_1 and M_2; thus the averaging in both terms M_1 and M_2 has the same meaning.

The average over the scattering volume mean velocity $\bar{W}$ (we denote the averaging over the scattering volume by the overbar) enters in M_1:

$$\bar{W} = \frac{1}{V}\iiint_V W(\mathbf{R})\, d^3R. \qquad (36)$$

The average over the scattering volume gradient of the mean refractive index, $\bar{G}_j$, enters in M_2:

$$\bar{G}_j = \frac{1}{V}\iiint_V \frac{\partial\langle n(\mathbf{R})\rangle}{\partial R_j}\, d^3R. \qquad (37)$$

Note that by using the Gauss theorem the volume integral in (37) can be presented in the form of an integral over the surface wrapping the volume V. In the simplest case when $\langle n(\mathbf{R})\rangle$ depends only on the vertical coordinate, and if the scattering volume can be approximated by a rectangular box, we can present $\bar{G}_z$ in the form

$$\bar{G}_z = \frac{\langle n(z_2)\rangle - \langle n(z_1)\rangle}{z_2 - z_1},$$

where $\langle n(z_2)\rangle$ and $\langle n(z_1)\rangle$ are the mean values of n at the top and bottom of the scattering volume.

In terms of $\bar{W}$ and $\bar{\mathbf{G}}$ the correlation $\langle \dot{I}(t)Q(t)\rangle$ takes the form

$$\langle \dot{I}(t)Q(t)\rangle = -4\pi^3 A_{sc}^2 V K_{sc} \overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{sc})\bar{W}$$
$$+ 4\pi^3 A_{sc}^2 V \overset{\leftrightarrow}{F}{}^{(-)}_{u_j,n}(\mathbf{K}_{sc})\bar{G}_j. \qquad (38)$$

If the scattering vector $\mathbf{K}_{sc}$ is perpendicular to the mean wind $\bar{\mathbf{U}}$, the value $\bar{W} = 0$. In this simplest case, we obtain from (38)

$$\langle \dot{I}(t)Q(t)\rangle = 4\pi^3 A_{sc}^2 \overset{\leftrightarrow}{F}{}^{(-)}_{u_j,n}(\mathbf{K}_{sc}) V\bar{G}_j, \qquad (39)$$

and by measuring the value $\langle \dot{I}(t)Q(t)\rangle$ we obtain the quadrature spatial Bragg component $F^{(-)}_{u_j,n}(\mathbf{K}_{\rm sc})$. In the general case we have $\bar{W} \neq 0$, and we must take into account both terms in (38).

Using the standard approximations, we can easily obtain from (6) the following well-known formula:

$$\langle I^2(t)\rangle = \langle Q^2(t)\rangle = 4\pi^3 A^2_{\rm sc} V \overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{\rm sc}). \qquad (40)$$

By dividing (38) by $\langle Q^2(t)\rangle$ we can present the relation (38) in another form that does not contain an uncertain factor $A^2_{\rm sc}V$:

$$W_D \equiv -\frac{\langle \dot{I}(t)Q(t)\rangle}{\langle Q^2(t)\rangle K_{\rm sc}} = \bar{W} + \frac{\bar{G}_j}{K_{\rm sc}} \frac{\overset{\leftrightarrow}{F}{}^{(-)}_{u_j,n}(\mathbf{K}_{\rm sc})}{\overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{\rm sc})}. \qquad (41)$$

The left-hand side of this equation is the Doppler velocity W_D, i.e., the ratio of the normalized first moment of the signal Doppler spectrum and the Bragg wave number $K_{\rm sc}$ (see Appendix B). To the best of our knowledge, (41) was first derived by *Muschinski* [1998, equations (6.42) and (6.45), pp. 59 and 60].

Our main result, equation (41), does not assume that the atmospheric spectra $F^{(-)}_{u_j,n}(\mathbf{p})$ and $\Phi_{n,n}(\mathbf{p})$ are smooth in the environment of $\mathbf{K}_{\rm sc}$, i.e., that $\overset{\leftrightarrow}{F}{}^{(-)}_{u_j,n}(\mathbf{K}_{\rm sc}) \approx F^{(-)}_{u_j,n}(\mathbf{K}_{\rm sc})$ and $\overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{\rm sc}) \approx \Phi_{n,n}(\mathbf{K}_{\rm sc})$. It is in the simplest case of smooth spectra, however, that the interpretation of results is clearest. There may be situations in which $F^{(-)}_{u_j,n}(\mathbf{p})$ and $\Phi_{n,n}(\mathbf{p})$ are similarly sharp as $\delta_V(\mathbf{p} - \mathbf{K}_{\rm sc})$, as *Doviak and Zrnic* [1984] have convincingly demonstrated for $\Phi_{n,n}(\mathbf{p})$. In that case, (41) remains correct, but the averaged spectra $\overset{\leftrightarrow}{F}{}^{(-)}_{u_j,n}(\mathbf{K}_{\rm sc})$ and $\overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{\rm sc})$ may differ from the non-averaged values. In order to keep the formalism simple we do not explicitly discuss the effect of the spectral sampling function $\delta_V(\mathbf{p} - \mathbf{K}_{\rm sc})$ in this paper.

The sum of two terms appears in the right-hand side of (41). The first term, $\bar{W}$, is the standard mean radial velocity. We call the second term,

$$\tilde{W} = \frac{\bar{G}_j}{K_{\rm sc}} \frac{\overset{\leftrightarrow}{F}{}^{(-)}_{u_j,n}(\mathbf{K}_{\rm sc})}{\overset{\leftrightarrow}{\Phi}_{n,n}(\mathbf{K}_{\rm sc})}, \qquad (42)$$

the correlation velocity. The physical meaning of $\tilde{W}$ is rather simple. The mean value of the random component of the radial wind, $\langle w\rangle$, is equal to zero, but the contribution to the Doppler spectrum depends on the refractive index fluctuation associated with the scatterer. If the magnitudes of the refractive index perturbations associated with recessive and approaching scatterers are different, then the Doppler spectrum becomes asymmetric and the effective nonzero mean Doppler shift (velocity) appears. Using statistical terminology, it can be said that if the correlation between velocity and refractive index exists, it provides an effective nonzero mean Doppler shift, even if $\langle w\rangle$ is zero. Because of this we call the value $\tilde{W}$ the "correlation" velocity. The second term in the right-hand side of (41) can be presented in terms of effective correlation velocity.

The relation between the mean velocity in the direction of the scattering vector, $\bar{W}$, and the correlation velocity $\tilde{W}$ depends on the atmospheric (oceanic) conditions. For example, in the surface layer of the atmosphere over a plain surface we have, with good accuracy, $\bar{W} = 0$. In this case, the term $\tilde{W}$ becomes dominant in (41). In other cases, such as convective conditions, or because of wave motions in the free atmosphere, $\bar{W}$ is typically nonzero and may be greater than $\tilde{W}$. In such cases, (41) takes the form

$$-\frac{\langle \dot{I}(t)Q(t)\rangle}{\langle Q^2(t)\rangle K_{\rm sc}} = \bar{W} \qquad |\tilde{W}| \ll |\bar{W}|$$

and is used in a wind profiler technique for determination of $\bar{W}$ from the radar observations.

It follows from our analysis (formula (41)) that in general, the Doppler velocity W_D is the sum of the mean radial velocity and the correlation velocity. We are unable to say a priori which term is dominant in this combination. In section 3 we will show how $\tilde{W}$ can be estimated from one-dimensional (1-D) turbulence spectra obtained from airborne in situ turbulence measurements, and we will derive a simple equation that relates $\tilde{W}$ to the radial wind standard deviation σ_w.

3. Magnitude of the Correlation Velocity

For estimation purposes we will consider the simplest case of a vertically oriented scattering vector. In this case, $\mathbf{K}_{\rm sc} = -K_{\rm sc}\mathbf{e}$, where $\mathbf{e}$ is the unit vector pointing vertically upward, and $u_3 = w$. We also will neglect the difference between $\overset{\leftrightarrow}{\Phi}_{nn}$ and Φ_{nn} and between $\overset{\leftrightarrow}{F}{}^{(-)}_{wn}$ and $F^{(-)}_{wn}$ and use for the correlation velocity (equation (42)) the formula

$$\tilde{W} = -\frac{1}{K_{\rm sc}} \frac{\partial\langle n\rangle}{\partial z} \frac{F^{(-)}_{wn}(K_{\rm sc}\mathbf{e})}{\Phi_{nn}(K_{\rm sc})}. \qquad (43)$$

Here, $F_{wn}^{(-)}(\mathbf{p})$ is the odd part of the three-dimensional spatial cross spectrum of w and n, $\Phi_{nn}(\mathbf{p})$ is the three-dimensional spatial power spectrum of n, and we substituted $F_{wn}^{(-)}(-K_{sc}\mathbf{e}) = -F_{wn}^{(-)}(K_{sc}\mathbf{e})$.

3.1. Correlation Velocity in Terms of 1-D Spectra Measured With In Situ Sensors

We will estimate the magnitude of $\tilde{W}$ starting with in situ measurements of turbulent fluctuations of T, a, and w in the troposphere. These measurements provide one-dimensional spatial spectra along the line of flight. To estimate $\tilde{W}$, however, we need the three-dimensional spectra $F_{wn}^{(-)}(\mathbf{K}_{sc})$ and $\Phi_{nn}(\mathbf{K}_{sc})$. Therefore we need to find the relations between the one-dimensional and three-dimensional spectra of turbulence in the case of nonisotropic turbulence. The mathematical consideration of this problem is presented in Appendix C. Here we apply these results to estimates of $\tilde{W}$.

We assume that K_{sc} belongs to the inertial subrange of turbulence. In this subrange the anisotropy creates the need for small corrections to the relation between the 3-D and 1-D spectra of n'. In the first approximation we neglect the anisotropy and use the relation that is accurate for the isotropic case (see Appendix C):

$$\Phi_{nn}(\mathbf{p}) = \Phi_{nn}(p) = -\frac{1}{2\pi p}\frac{dp_{nn}(\mathbf{m}, p)}{dp}$$

$$= -\frac{1}{2\pi p}\frac{dp_{nn}(p)}{dp}, \tag{44}$$

regardless of the direction of the vector $\mathbf{m}$. Here, $p_{nn}(\mathbf{m}, p)$ is the one-dimensional power spectrum of the refractive index fluctuations along an arbitrary direction $\mathbf{m}$.

The situation is more complicated in the case of the cross spectra of velocity and refractive index. Here, in the isotropic case the correlation vanishes because of the incompressibility of turbulence. Because of this it is impossible to consider the anisotropy as a small correction to the isotropic case. The relation between the three-dimensional quadrature spectrum $F_{wn}^{(-)}(p, \eta)$ and the one-dimensional quadrature spectrum $q_{wn}(p)$ of w and n measured in the direction $\mathbf{m}$ is derived in the Appendix C. In the case of a power spectrum of the form

$$q_{wn}(p) = q_0 p^{-b}$$

we can use (105):

$$F_{wn}^{(-)}(p, \eta) = \frac{\eta}{2\pi\cos\beta_0}\frac{b^2+4b+3}{b+(2-b)\eta^2}\frac{q_{wn}(p)}{p^2}. \tag{45}$$

Here, $\eta = \mathbf{ne} = \cos\gamma$, and γ is the angle between vectors $\mathbf{p}$ and $\mathbf{e}$ (see Figure 4).

Now, we consider the ratio $F_{wn}^{(-)}(K_{sc}\mathbf{e})/\Phi_{nn}(K_{sc})$, which appears in (43) for the correlation velocity:

$$\frac{F_{wn}^{(-)}(K_{sc}\mathbf{e})}{\Phi_{nn}(K_{sc})} = -\frac{\eta}{\cos\beta_0}$$

$$\left.\frac{(b^2+4b+3)q_{wn}(p)}{[b+(2-b)\eta^2]p\dfrac{d}{dp}p_{nn}(p)}\right|_{p=K_{sc}}. \tag{46}$$

For $p_{nn}(p)$ in the inertial range we also assume the power law:

$$p_{nn}(p) = p_0 p^{-a}.$$

Then we have

$$p\frac{d}{dp}p_{nn}(p) = -ap_0p^{-a} = -ap_{nn}(p),$$

$$\frac{F_{wn}^{(-)}(K_{sc}\mathbf{e})}{\Phi_{nn}(K_{sc})} = \frac{\eta}{\cos\beta_0}\frac{(b^2+4b+3)}{a[b+(2-b)\eta^2]}\frac{q_{wn}(K_{sc})}{p_{nn}(K_{sc})}. \tag{47}$$

Formula (43) for the correlation velocity takes the form

$$\tilde{W} = -\frac{\eta}{\cos\beta_0}\frac{(b^2+4b+3)}{a[b+(2-b)\eta^2]}\frac{\partial\langle n\rangle}{\partial z}\frac{q_{wn}(K_{sc})}{K_{sc}p_{nn}(K_{sc})}. \tag{48}$$

Similarity theory leads to $a = 5/3$ and $b = 7/3$ [see, e.g., *Sorbjan*, 1989]. These values of a and b lead to

$$\tilde{W} = -\frac{32}{7-\eta^2}\frac{\eta}{\cos\beta_0}\frac{\partial\langle n\rangle}{\partial z}\frac{q_{wn}(K_{sc})}{K_{sc}p_{nn}(K_{sc})}. \tag{49}$$

This equation can be used to estimate $\tilde{W}(K_{sc})$ from turbulence measurements taken with airborne sensors.

Figure 1 shows the clear-air refractive index profile along the line of flight as a function of elevation. Refractive index measurements were performed using the helicopter-borne turbulence measurement system HELIPOD [*Muschinski and Wode*, 1998; *Muschinski et al.*, 2001] in the marine boundary layer between 300 m msl and 480 m msl. Data were collected on November 8, 1997, during the Profiler-HELIPOD Intercomparison Experiment (PHELIX),

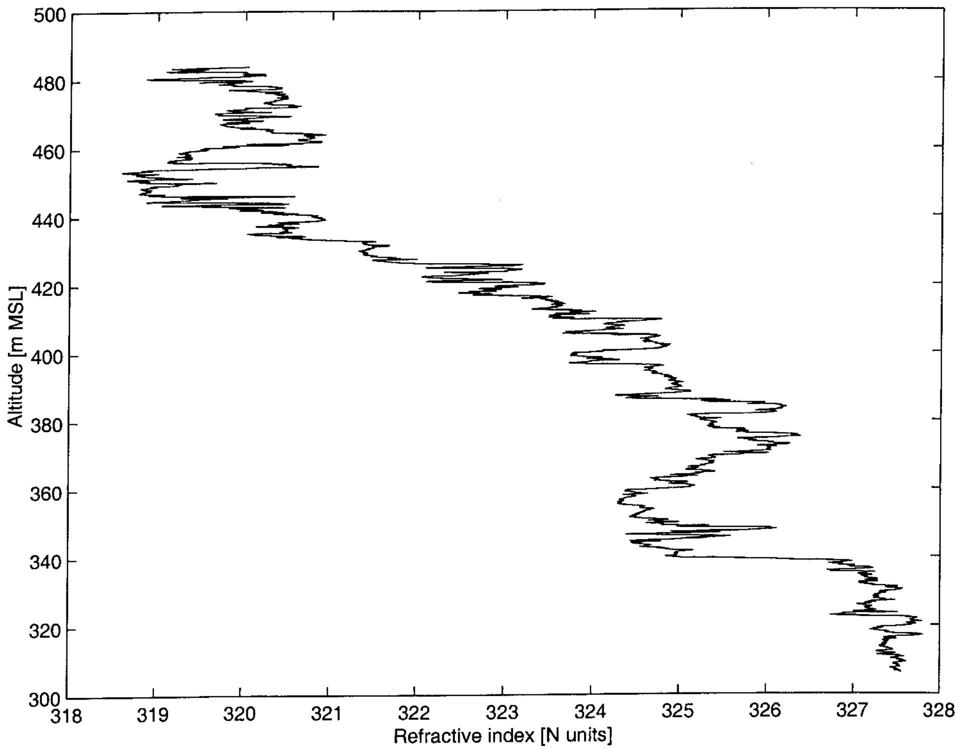

Figure 1. Refractive index profile along the line of flight as a function of elevation.

which was carried out at Vandenberg Air Force Base on the Pacific Coast near Santa Maria, California. We chose an 180-m-altitude interval because 180 m is a typical range resolution for a standard boundary layer wind profiler. The profile shown in Figure 1, however, is not a true vertical profile because the data were collected during a slanted flight, where the elevation angle ($\pi/2 - \beta_0$) was ~7 deg.

Figure 2 is the magnitude of the imaginary part of the frequency cross spectrum $S_{wn}(f)$, calculated from the same data segment as that shown in Figure 1. Figure 2 shows that $|\text{Im}\ [S_{wn}(f)]|$ follows a 7/3 power law, as expected.

From Im $[S_{wn}(f)]$ we calculated the one-dimensional spatial quadrature spectrum along the flight path, $q_{wn}(p)$. Using (49), we calculated the magnitude of the correlation velocity as a function of the scattering wave number K_{sc}. Up to wave numbers of $\sim 1\ \text{m}^{-1}$, $\tilde{W}(K_{sc})$ is well approximated by a 5/3 power law. At higher wave numbers there is a roll-off caused by an as yet unidentified calibration problem or a filter effect in the high-frequency component of the humidity data measured with HELIPOD's Lyman-α hygrometer. Figure 3 shows that in that specific meteorological situation, $\tilde{W}(K_{sc})$ may be approximated by the power function $\tilde{W}(K_{sc}) \approx W_0(K_{sc}/K_0)^{-5/3}$ (straight line) where $W_0 = 10\ \text{cm s}^{-1}$ and $K_0 = 1\ \text{m s}^{-1}$. For instance, if $K_{sc} = 2\ \text{m}^{-1}$, $\tilde{W}$ (2 m^{-1}) = 3 cm s^{-1}. In the backscattering case, $K_{sc} = 2\ \text{m}^{-1}$ corresponds to a radar wavelength of ~6 m. In the region $K_{sc} \sim 0.25\ \text{m}^{-1}$ which can be realized by using bistatic scattering, the $\tilde{W}(K_{sc})$ may reach 1 m s^{-1}.

3.2. Stably Stratified Turbulence

Here we derive an expression for $\tilde{W}(K_{sc})$ under the assumption that the atmosphere is stably stratified and assuming that **m** points vertically upward (in this case, sin $\gamma = 0$, cos $\gamma = 1$). First, we write $q_{wn}(p)$ in terms of the power spectra of w and n and a coherence function $\varphi_{wn}(p)$ (which is an analog of the correlation coefficient for the spectral components of n and w):

$$q_{wn}(p) = \sqrt{p_{ww}(p)p_{nn}(p)}\varphi_{wn}(p). \quad (50)$$

In terms of $\varphi_{wn}(p)$ we obtain

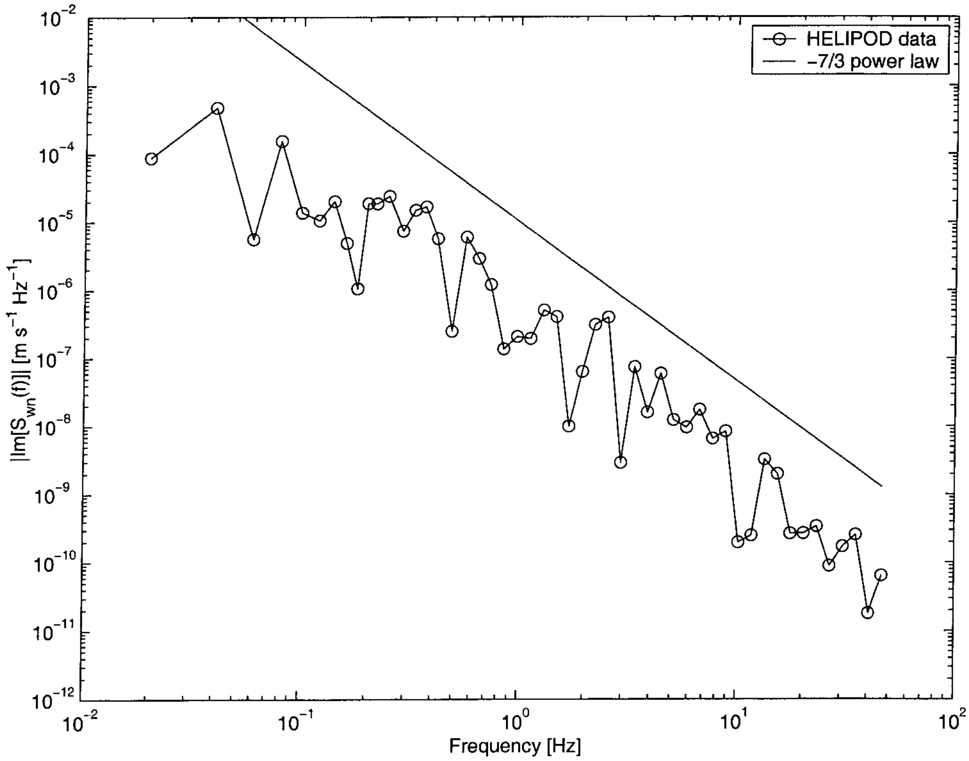

Figure 2. Magnitude of the imaginary part of cross spectrum $S_{wn}(f)$, measured in the marine boundary layer on the California coast.

$$\frac{q_{wn}(p)}{p_{nn}(p)} = \sqrt{\frac{p_{ww}(p)}{p_{nn}(p)}}\,\varphi_{wn}(p) = \sqrt{\frac{cC_w^2 p^{-5/3}}{cC_n^2 p^{-5/3}}}\,\varphi_{wn}(p)$$

$$= \sqrt{\frac{C_w^2}{C_n^2}}\,\varphi_{wn}(p) = \sqrt{\frac{C_w^2 L_0^{2/3}}{C_n^2 L_0^{2/3}}}\,\varphi_{wn}(p). \quad (51)$$

Here, L_0 is an outer scale of turbulence. We assumed that outer scales for velocity and refractive index are of the same order of magnitude and use the relations

$$\sigma_w^2 = C_w^2 L_0^{2/3}, \quad (52)$$

where σ_w is the RMS vertical velocity, and

$$C_n^2 L_0^{2/3} = \sigma_n^2 = L_0^2 \left(\frac{d\langle n\rangle}{dz}\right)^2. \quad (53)$$

The second formula defines the outer scale L_0 as a scale for which the RMS of the random difference of n is equal to its systematic difference [see *Tatarskii*, 1961, equation (3.29″)].

Substitution of (52) and (53) into (51) leads to

$$\frac{q_{wn}(p)}{p_{nn}(p)} = \frac{\sigma_w}{L_0 \left|\frac{d\langle n\rangle}{dz}\right|}\,\varphi_{wn}(p). \quad (54)$$

Our estimations of $\varphi_{wn}(p)$ described in section 3.1 show that $b - a = 2/3$; that is, $\varphi_{wn}(p)$ in the inertial subrange is proportional to $p^{-2/3}$. Thus, because $\varphi_{wn}(p)$ is a dimensionless quantity, we can approximate it in the range $pL_0 \geq 1$ by the dependence

$$\varphi_{wn}(p) = \frac{\varphi_0}{(pL_0)^{2/3}}, \quad (55)$$

where φ_0 is some constant on the order of unity.

Substituting (55) and (54) into (49), we obtain

$$\tilde{W} \sim -5.3\,\frac{d\langle n\rangle/dz}{|d\bar{n}/dz|}\,\frac{\varphi_0 \sigma_w}{(K_{sc}L_0)^{5/3}} \quad (56)$$

and arrive at the following order-of-magnitude estimate for $\tilde{W}$ as a function of the nondimensional scattering wave number $K_{sc}L_0$:

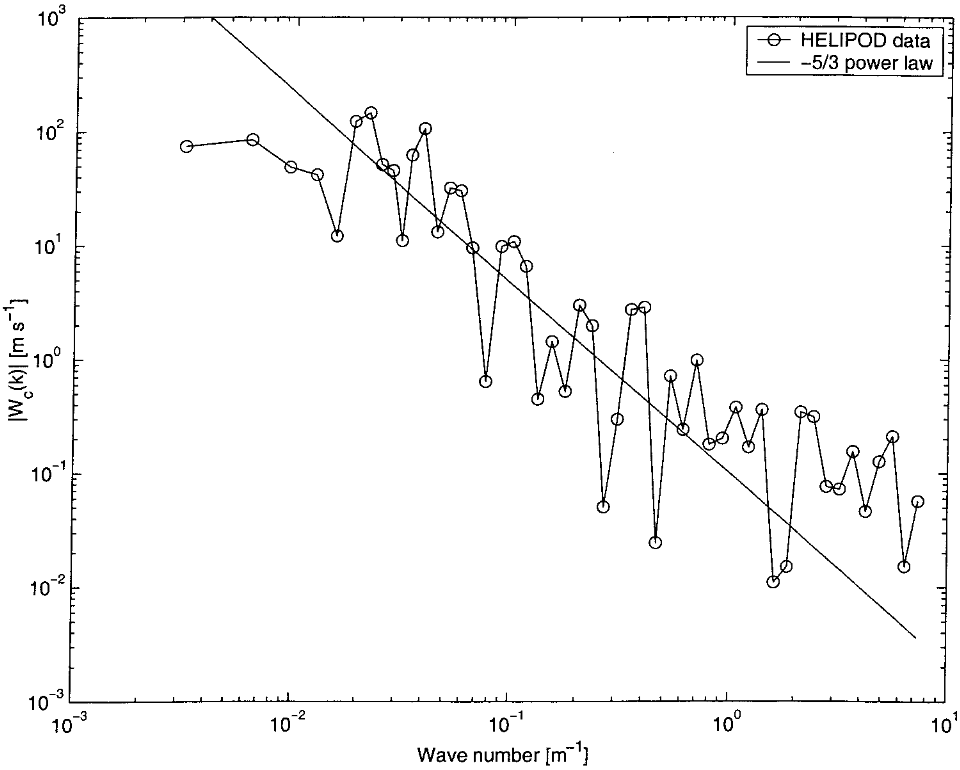

Figure 3. Correlation velocity versus magnitude of the scattering vector.

$$|\tilde{W}(K_{sc})| \sim \frac{\sigma_w}{(K_{sc}L_0)^{5/3}}. \tag{57}$$

That is, if $K_{sc}L_0 = 2L_0 \sin(\theta/2)/\lambda$ is of the order of unity, then the magnitude of the correlation velocity is comparable to the standard deviation of w. Note that this line of argument is not valid in the convective boundary layer. On the other hand, the empirical results obtained from the HELIPOD measurements taken in the marine boundary layer as shown above indicate that (57) may be quite a robust result.

4. Summary and Conclusions

We have shown that there is generally a difference between the ensemble average of the Doppler velocity (the velocity obtained from the Doppler shift measured by scattering from random refractive index fluctuations) and the ensemble average of the mean fluid velocity (in the direction of the scattering vector) in the scattering volume. We call the leading term of this difference the "correlation velocity," which stems from a correlation between velocity and refractive index fluctuations within the scattering volume. More precisely, the imaginary part of the spatial Bragg wave vector component of the three-dimensional cross spectrum of refractive index and velocity (along the scattering vector) causes a bias in the standard measurements of the velocity. On the one hand, this affects wind profiler measurements; on the other hand, the correlation velocity, if identified, could be used to obtain information about turbulent fluxes in the atmosphere.

Our estimations show that the correlation velocity may be important in the surface layer of the atmosphere, where there is no systematic vertical wind component (measurements in the surface layer can be performed using bistatic sodars; see *Tatarskii* [1971]). In the atmospheric boundary layer, usually the nonzero vertical wind component exists. In this case, our estimations show that a significant correlation velocity (of the order of tens of cm s^{-1}; see Figure 3 and formula (57)) may appear in the case where $K_{sc}L_0 \sim 1$. The large value of K_{sc} can be obtained either by increasing the wavelength or by decreasing the scattering angle in bistatic scattering.

For radar observations the refractive index fluctuations consist of a combination of temperature and humidity fluctuations. Thus the information about the cross spectrum of velocity and these quantities can be obtained. For sound scattering in the atmosphere the refractive index fluctuations consist of a combination of velocity and temperature fluctuations. Thus the information about turbulent stress and turbulent heat flux can be obtained from sodar signals. In the case of sound backscattering only the temperature fluctuations are important [*Tatarskii*, 1971]. Thus it may be possible to separate the temperature and humidity contributions from simultaneous radar and sodar observations.

We must stress that the quadrature component of the cross spectrum does not directly contribute to the turbulent fluxes. Only if there exists some relation between the in-phase and quadrature cross spectra and if that relation is known, can it be used for estimations of turbulent fluxes. This problem can be studied on the basis of in situ turbulence measurements or computer simulations.

Appendix A: Contribution of the Term $[u\nabla n' - \langle u\nabla n'\rangle]$ to the Doppler Shift

We show in this appendix that with respect to the Doppler shift we can neglect the term $[\mathbf{u}\nabla n' - \langle \mathbf{u}\nabla n'\rangle]$ in (17). We denote $\dot{I}_3$ as the contribution of this term to $\dot{I}$:

$$\dot{I}_3(t) = -A_{\rm sc} \iiint_V d^3R' \cos[\mathbf{K}_{\rm sc}\mathbf{R}' + \psi] \cdot \left[\mathbf{u}(\mathbf{R}')\frac{\partial n'(\mathbf{R}')}{\partial \mathbf{R}'} - \langle \mathbf{u}\nabla n'\rangle\right]. \tag{58}$$

Multiplying (58) by $Q(t)$ and averaging the product, we obtain

$$\langle \dot{I}_3(t)Q(t)\rangle = -A_{\rm sc}^2 \iiint_V d^3R' \cos[\mathbf{K}_{\rm sc}\mathbf{R}' + \psi] \cdot \iiint_V d^3R'' \sin[\mathbf{K}_{\rm sc}\mathbf{R}'' + \psi] \cdot \frac{\partial}{\partial \mathbf{R}'}\langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}')n'(\mathbf{R}'')\rangle. \tag{59}$$

The mixed third moment of the velocity and refractive index,

$$\langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}')n'(\mathbf{R}'')\rangle,$$

appears in this equation. Moments of this type were first considered by *Yaglom* [1949]. It was shown in this paper (the derivation of Yaglom's equation is given by *Tatarskii* [1971, pp. 61–63]) that in the case of locally isotropic turbulence the mixed third-order structure function has the form

$$\langle [\mathbf{u}(\mathbf{R}') - \mathbf{u}(\mathbf{R}'')][n'(\mathbf{R}') - n'(\mathbf{R}'')]^2\rangle = \frac{\mathbf{r}}{r}D_{rnn}(r), \tag{60}$$

where

$$D_{rnn}(r) = -\frac{4}{3}Nr + 2\chi D'_{nn}(r). \tag{61}$$

In (61),

$$D_{nn}(r) = \langle [n'(\mathbf{R}') - n'(\mathbf{R}'')]^2\rangle = \begin{cases} C_n^2 r^{2/3} & r \gg l_0 \\ C_n^2 l_0^{-4/3} r^2 & r \ll l_0 \end{cases} \tag{62}$$

is the refractive index structure function, and the parameter N is related to C_n^2, l_0, and χ by the formula

$$N = 3\chi C_n^2 l_0^{-4/3}. \tag{63}$$

Using the property of the incompressible isotropic turbulence [see, e.g., *Tatarskii*, 1971, equation (12), p. 42]

$$\langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}'')\rangle = 0, \tag{64}$$

it is easy to find that the value $(\partial/\partial\mathbf{R}')\langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}')n'(\mathbf{R}'')\rangle$ entering in (59) can be expressed in terms of $D_{rnn}(r)$ as follows:

$$\frac{\partial}{\partial \mathbf{R}'}\langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}')n'(\mathbf{R}'')\rangle = -\frac{1}{4}\frac{\partial}{\partial \mathbf{R}'}\langle [\mathbf{u}(\mathbf{R}') - \mathbf{u}(\mathbf{R}'')] \cdot [n'(\mathbf{R}') - n'(\mathbf{R}'')]^2\rangle = -\frac{1}{4}\frac{\partial}{\partial \mathbf{r}}\left[\frac{\mathbf{r}}{r}D_{rnn}(r)\right] = -\frac{1}{4r^2}\frac{d}{dr}[r^2 D_{rnn}(r)]. \tag{65}$$

If we substitute in (65) *Yaglom*'s [1949] formula (61), we obtain

$$\frac{\partial}{\partial \mathbf{R}'} \langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}')n'(\mathbf{R}'')\rangle = \frac{1}{2r^2}\frac{d}{dr}\left[\frac{2}{3}Nr^3 - \chi r^2 D'_{nn}(r)\right]. \tag{66}$$

Using formula (66) in the inertial subrange, where $r \gg l_0$ and $D_{nn}(r) = C_n^2 r^{2/3}$, we obtain

$$\frac{\partial}{\partial \mathbf{R}'} \langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}')n'(\mathbf{R}'')\rangle = N - \frac{5}{9}C_n^2 \chi r^{-4/3}$$

$$= N\left[1 - \frac{5}{27}\left(\frac{l_0}{r}\right)^{4/3}\right]. \tag{67}$$

The second term is small for $r \gg l_0$, and we can neglect it. Thus, for the inertial subrange we obtain the formula

$$\frac{\partial}{\partial \mathbf{R}'} \langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}')n'(\mathbf{R}'')\rangle = N. \tag{68}$$

Note that R. J. Hill (Structure-function equations for scalars, submitted to *Physics of Fluids*, 2000) (hereinafter referred to as Hill, submitted manuscript, 2000) obtained a generalization of *Yaglom*'s [1949] equation (61) that does not use the assumptions of statistical homogeneity, isotropy, and stationarity. It follows from the equation obtained by R. J. Hill (submitted manuscript, 2000) that formula (68) is true even in the case of nonisotropic turbulence. It is important that the right-hand side of (68) does not depend on $\mathbf{R}' - \mathbf{R}''$.

If we substitute (68) into (59), we obtain

$$\langle \dot{I}_3(t)Q(t)\rangle = -A_{sc}^2 N \iiint_V d^3R'$$

$$\cdot \cos[\mathbf{K}_{sc}\mathbf{R}' + \psi] \iiint_V d^3R'' \sin[\mathbf{K}_{sc}\mathbf{R}'' + \psi]. \tag{69}$$

To estimate the integrals over the scattering volume V in the right-hand side of (69), we introduce the weighting function

$$Y(\mathbf{R}) = \frac{V}{\pi\sqrt{\pi}L^3}\exp\left(-\frac{\mathbf{R}^2}{L^2}\right),$$

$$\iiint_{-\infty}^{\infty} Y(\mathbf{R})\, d^3R = V$$

and replace the integrals as follows:

$$\iiint_V d^3R' \cos[\mathbf{K}_{sc}\mathbf{R}' + \psi]$$

$$\to \iiint_{-\infty}^{\infty} Y(\mathbf{R}')\cos(\mathbf{K}_{sc}\mathbf{R}' + \psi)\, d^3R',$$

$$\iiint_V d^3R'' \sin[\mathbf{K}_{sc}\mathbf{R}'' + \psi]$$

$$\to \iiint_{-\infty}^{\infty} Y(\mathbf{R}'')\sin(\mathbf{K}_{sc}\mathbf{R}'' + \psi)\, d^3R''.$$

Using the integral

$$\iiint_{-\infty}^{\infty} Y(\mathbf{R})\exp(i\mathbf{K}_{sc}\mathbf{R} + i\psi)\, d^3R = V$$

$$\cdot \exp\left(-\frac{K_{sc}^2L^2}{4} + i\psi\right), \tag{70}$$

we obtain

$$\left[\iiint_{-\infty}^{\infty} Y(\mathbf{R}')\cos(\mathbf{K}_{sc}\mathbf{R}' + \psi)\, d^3R\right]$$

$$\cdot \left[\iiint_{-\infty}^{\infty} Y(\mathbf{R}')\sin(\mathbf{K}_{sc}\mathbf{R}' + \psi)\, d^3R\right]$$

$$= V^2 \sin\psi\cos\psi\exp\left(-\frac{K_{sc}^2L^2}{2}\right).$$

Formula (69) takes the form

$$\langle \dot{I}_3(t)Q(t)\rangle = -\frac{A_{sc}^2NV^2}{2}\sin 2\psi\exp\left(-\frac{K_{sc}^2L^2}{2}\right). \tag{71}$$

The corresponding contribution to the Doppler velocity is given by the formula (we use formula (40) for $\langle Q^2(t)\rangle$)

$$W_3 = -\frac{\langle \dot{I}_3(t)Q(t)\rangle}{\langle Q^2(t)\rangle K_{sc}} = \frac{NV \sin 2\psi}{8\pi^3 K_{sc}\Phi_{n,n}(\mathbf{K}_{sc})} \exp\left(-\frac{K_{sc}^2 L^2}{2}\right). \tag{72}$$

Let us substitute in (72) the value $\Phi_{n,n}(\mathbf{K}_{sc}) = 0.033C_n^2 K_{sc}^{-11/3}$ for the inertial subrange [see *Tatarskii*, 1961], and $N = 3\chi C_n^2 l_0^{-4/3}$, $V = L^3$. We obtain

$$W_3 = 0.37\,\frac{\chi}{l_0}\left(\frac{L}{l_0}\right)^{1/3}(K_{sc}L)^{8/3}\exp\left(-\frac{K_{sc}^2L^2}{2}\right)\sin 2\psi. \tag{73}$$

The maximum of the function $x^{8/3}\exp(-x^2/2)$ is equal to 0.97 in the point $x = \sqrt{8/3}$. Thus, for any K_{sc} and any $\sin 2\psi$ we obtain the estimation

$$W_3 < 0.36\left(\frac{L}{l_0}\right)^{1/3}\frac{\chi}{l_0}. \tag{74}$$

For typical values $L = 100$ m, $l_0 = 10^{-2}$ m, and $\chi/l_0 = 0.2$ cm s^{-1} (this value is of the order of magnitude of Kolmogorov velocity) we obtain $W_3 < 1.6$ cm s^{-1}. The additional factor

$$(K_{sc}L)^{8/3}\exp\left(-\frac{K_{sc}^2L^2}{2}\right)\sin 2\psi$$

significantly reduces the obtained estimation. The additional factor 0.01 appears in the obtained estimation if $K_{sc}L = 4.1$ (this factor is much less for the wind profiler working at a frequency of 915 MHz).

We conclude from this consideration that in the context of a possible effect on the Doppler velocity it is always possible to neglect the term $[\mathbf{u}\nabla n' - \langle \mathbf{u}\nabla n'\rangle]$ in (17).

Appendix B: Relation Between $\langle Q(dI/dt)\rangle$ and Doppler Spectrum

If we define the complex signal

$$C(t) = I(t) + iQ(t), \qquad \dot{C}(t) = \dot{I}(t) + i\dot{Q}(t), \tag{75}$$

we find that

$$\langle C^*(t)\dot{C}(t)\rangle = \langle I(t)\dot{I}(t)\rangle + \langle Q(t)\dot{Q}(t)\rangle + i\langle I(t)\dot{Q}(t)\rangle - i\langle \dot{I}(t)Q(t)\rangle. \tag{76}$$

For any stationary random process it follows from (8) by differentiating with respect to t that

$$\langle I(t)\dot{I}(t)\rangle = \langle Q(t)\dot{Q}(t)\rangle = 0,$$
$$\langle I(t)\dot{Q}(t)\rangle + \langle \dot{I}(t)Q(t)\rangle = 0. \tag{77}$$

Thus

$$\langle C^*(t)\dot{C}(t)\rangle = -2i\langle \dot{I}(t)Q(t)\rangle. \tag{78}$$

On the other hand, we have

$$\frac{d\langle C(t)C^*(t')\rangle}{dt} = \langle C^*(t')\dot{C}(t)\rangle, \tag{79}$$

and if we set $t' = t$,

$$\langle C^*(t)\dot{C}(t)\rangle = \left.\frac{d\langle C(t)C^*(t')\rangle}{dt}\right|_{t'=t}. \tag{80}$$

Thus we can express $\langle \dot{I}(t)Q(t)\rangle$ in terms of $\langle C(t)C^*(t')\rangle \equiv B_{CC^*}(t - t')$ as follows:

$$\langle \dot{I}(t)Q(t)\rangle = \frac{i}{2}\left.\frac{d\langle C(t)C^*(t')\rangle}{dt}\right|_{t'=t} = \frac{i}{2}\left.\frac{dB_{CC^*}(\tau)}{d\tau}\right|_{\tau=0}. \tag{81}$$

In practice, the Fourier analysis of signals is used. The random function $C_T(t)$, which coincides with $C(t)$ in the interval of observation $(-T/2, T/2)$ and is equal to zero outside this interval, can be presented in terms of the Fourier integral

$$C_T(t) = \int_{-\infty}^{\infty}\tilde{C}(\omega)e^{i\omega t}\,d\omega;$$

$$\cdot\,\tilde{C}_T(\omega) = \frac{1}{2\pi}\int_{-T/2}^{T/2}C(t)e^{-i\omega t}\,dt \tag{82}$$

with the random spectrum $\tilde{C}_T(\omega)$. The power spectrum $S(\omega)$ of the complex signal $C(t)$ is determined by the relation

$$S(\omega) = \lim_{T\to\infty}\frac{2\pi}{T}\langle|\tilde{C}_T(\omega)|^2\rangle \tag{83}$$

and according to the Wiener-Khinchin theorem $S(\omega)$ coincides with the Fourier transform of the correlation function $\langle C(t)C^*(t + \tau)\rangle$, i.e.,

$$B_{CC^*}(\tau) = \int_{-\infty}^{\infty}S(\omega)\exp(i\omega\tau)\,d\omega. \tag{84}$$

Thus

$$\left.\frac{dB_{CC^*}(\tau)}{d\tau}\right|_{\tau=0} = \int_{-\infty}^{\infty} i\omega S(\omega)\, d\omega, \tag{85}$$

$$\langle \dot{I}(t)Q(t)\rangle = -\frac{1}{2}\int_{-\infty}^{\infty} \omega S(\omega)\, d\omega. \tag{86}$$

For the quantity $\langle C(t)C^*(t)\rangle$ that enters in (41) we obtain

$$\langle C(t)C^*(t)\rangle = \langle I^2(t)\rangle + \langle Q^2(t)\rangle = 2\langle Q^2(t)\rangle; \tag{87}$$

thus

$$\langle Q^2(t)\rangle = \frac{1}{2}\langle C(t)C^*(t)\rangle = \frac{1}{2}B_{CC^*}(0) = \frac{1}{2}\int_{-\infty}^{\infty} S(\omega)\, d\omega. \tag{88}$$

Substituting (86) and (88) into (41), we obtain

$$W_D \equiv -\frac{\langle \dot{I}(t)Q(t)\rangle}{\langle Q^2(t)\rangle K_{sc}} = \frac{\int_{-\infty}^{\infty} \omega S(\omega)\, d\omega}{K_{sc}\int_{-\infty}^{\infty} S(\omega)\, d\omega}. \tag{89}$$

Appendix C: Relation Between 3-D and 1-D Spectra for Correlation Between Velocity and Scalar

In many measurements we obtain the one-dimensional spectra. For a scalar field in isotropic turbulence the relation between the one-dimensional spectrum $p(p)$ and the three-dimensional spectrum $\Phi(p)$ is known [see, e.g., *Tatarskii*, 1961, 1971]:

$$\Phi(p) = -\frac{1}{2\pi p}\frac{dp(p)}{dp}. \tag{90}$$

In this paper we are interested in the three-dimensional quadrature spectrum of velocity and refractive index. In isotropic turbulence in an incompressible liquid there is no correlation between the velocity and any scalar. Therefore the nonzero correlation between velocity and scalar quantity (turbulent flux) may exist only in anisotropic turbulence. Here, even a small anisotropy can cause the finite nonzero correlation. We consider here the simplest case of axially symmetric turbulence in the case of free convection. If a small finite shear exists, it provides a deviation from the axial symmetry, which causes small deviations from an axis-symmetrical case.

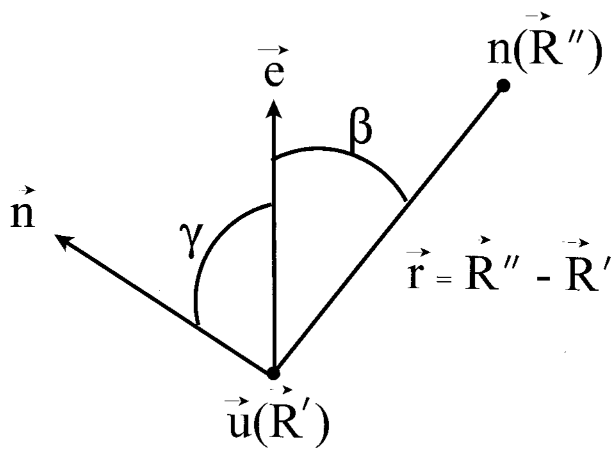

Figure 4. Geometry of anisotropic turbulence.

The correlation between the velocity $\mathbf{u}(\mathbf{R}')$ and the scalar $n'(\mathbf{R}'')$ is a vector function $\mathbf{B}^{(\mathbf{u},n)}(\mathbf{r})$ that in the case of statistical homogeneity depends on the vector $\mathbf{r} = \mathbf{R}'' - \mathbf{R}'$ (see Figure 4). The vector $\mathbf{B}^{(\mathbf{u},n)}(\mathbf{r})$ may depend also on $\mathbf{e}$, where $\mathbf{e}$ is the unit vector in the flux direction. The general form of $\mathbf{B}^{(\mathbf{u},n)}(\mathbf{r})$ is as follows [see, e.g., *Batchelor*, 1970, formula (3.3.8), p. 43]:

$$\langle \mathbf{u}(\mathbf{R}')n'(\mathbf{R}'')\rangle = \mathbf{B}^{(\mathbf{u},n)}(\mathbf{r}) = \frac{\mathbf{r}}{r}B_1\left(r, \frac{\mathbf{er}}{r}\right) + \mathbf{e}B_2\left(r, \frac{\mathbf{er}}{r}\right) \tag{91}$$

(note that $\mathbf{r}$ may have the component along the vector $\mathbf{e}$).

From the incompressibility condition, div $\mathbf{B}^{(\mathbf{u},n)}(\mathbf{r}) = 0$, it is easy to derive the following relation between the functions B_1 and B_2:

$$\frac{1}{r^2}\frac{\partial}{\partial r}[r^2B_1(r, \xi)] + \xi\frac{\partial B_2(r, \xi)}{\partial r} + \frac{\partial B_2(r, \xi)}{\partial \xi}\frac{1-\xi^2}{r} = 0. \tag{92}$$

Here, $\xi \equiv \mathbf{er}/r = \cos\beta$, where β is the angle between the vectors $\mathbf{e}$ and $\mathbf{r}$. From (92) it follows that the function B_1 can be expressed in terms of B_2:

$$B_1(r, \xi) = -\frac{1}{r^2}\int_0^r \left[\rho\frac{\partial B_2(\rho, \xi)}{\partial \xi} + \xi\rho^2\frac{\partial B_2(\rho, \xi)}{\partial \rho} - \xi^2\rho\frac{\partial B_2(\rho, \xi)}{\partial \xi}\right] d\rho. \tag{93}$$

In the case of isotropic turbulence the term B_2 in (91) vanishes, and from (93) we obtain $B_1 = 0$, i.e., $\mathbf{B}^{(\mathbf{u},n)} = 0$.

It follows from (91) that

$$\mathbf{eB}^{(\mathbf{u},n)}(\mathbf{r}) = \xi B_1(r, \xi) + B_2(r, \xi). \tag{94}$$

The function $B_2(r, \xi)$ can be presented as the sum of two terms, even and odd, with respect to ξ. We consider here only the odd term that is responsible for the imaginary (odd) part of the cross spectrum. The odd component of $B_2(r, \xi)$ can be presented in the form of a Taylor series:

$$B_2(r, \xi) = A_1(r)\xi + A_3(r)\xi^3 + \cdots. \tag{95}$$

We will consider the case of small and smooth anisotropy and will take into account only the first term of this expansion, $A_1(r)\xi$. In other words, we assume that the angular dependence of B_2 is rather smooth and that the higher harmonics are negligible.

It is convenient to present $A_1(r)$ in terms of another arbitrary function, $K(r)$, as follows:

$$A_1(r) = \frac{1}{r}\frac{d}{dr}[r^2K(r)]. \tag{96}$$

Using (96) in (93), we obtain

$$B_1(r, \xi) = -K(r) - \xi^2 r^2 \frac{d}{dr}\left[\frac{K(r)}{r}\right],$$
$$B_2(r, \xi) = \frac{\xi}{r}\frac{d}{dr}[r^2K(r)]. \tag{97}$$

The 3-D cross spectrum of velocity and n' is defined by the formula

$$\mathbf{F}^{(\mathbf{u},n)}(\mathbf{p}) = \frac{1}{8\pi^3}\int\int\int \exp(-i\mathbf{pr})\mathbf{B}^{(\mathbf{u},n)}(\mathbf{r})\, d^3r. \tag{98}$$

In the case of incompressible fluid the vector $F_j^{(\mathbf{u},n)}(\mathbf{p})$ has the form

$$F_j^{(\mathbf{u},n)}(\mathbf{p}) = F^{(\mathbf{u},n)}(p, \eta)[\delta_{jk} - n_j n_k]e_k, \tag{99}$$

where

$$\mathbf{p} = p\mathbf{n}, \qquad \eta = \mathbf{ne} = \cos\gamma,$$

and γ is the angle between vectors $\mathbf{p}$ and $\mathbf{e}$. It follows from (99) that

$$n_j F_j^{(\mathbf{u},n)}(\mathbf{p}) = 0,$$

which corresponds to the equation div $\mathbf{B}^{(\mathbf{u},n)} = 0$.

If we multiply (98) by $\mathbf{e}$, substitute (94) and (97), and use the spherical coordinates with the polar axis along the vector $\mathbf{n}$, we can perform the integration over the angular variables and receive the following formula for the scalar function $F_{wn}^{(-)}(p, \eta)$ (we use the notation for the odd part of the spectrum because the assumption of oddness (95) was used):

$$F_{wn}^{(-)}(p, \eta) = -\frac{i\eta}{2\pi^2 p^2}\left(p^2\frac{d^2}{dp^2} - 3p\frac{d}{dp} + 3\right) \cdot \int_0^\infty K(r)\sin pr\, dr. \tag{100}$$

Note that because we assumed that $B_2(r, \xi) = A_1(r)\xi$, we obtain $F_{wn}^{(-)}(p, \eta) \sim \eta = \cos\gamma$.

We also consider the one-dimensional mutual correlation function between the velocity component $w = \mathbf{eu}(\mathbf{R}')$ and $n'(\mathbf{R}'')$ along the line $\mathbf{R}'' = \mathbf{R}' + \mathbf{m}_0 l$. Here, $-\infty < l < \infty$, $\mathbf{r} = \mathbf{m}_0 l$, $r = |l|$, $\xi_0 = \mathbf{m}_0\mathbf{e}$, and $\xi = \xi_0 l/|l|$. We obtain

$$\langle w(\mathbf{R}')n'(\mathbf{R}' + \mathbf{m}_0 l)\rangle = \langle \mathbf{eu}(\mathbf{R}')n'(\mathbf{R}' + \mathbf{m}_0 l)\rangle = \mathbf{eB}^{(\mathbf{u},n)} \cdot (\mathbf{m}_0 l) = \xi B_1(r, \xi) + B_2(r, \xi) = (\xi_0 l/|l|)B_1(|l|, \xi_0 l/|l|) + B_2(|l|, \xi_0 l/|l|). \tag{101}$$

The function $\langle w(\mathbf{R}')n'(\mathbf{R}' + \mathbf{m}_0 l)\rangle$ is an odd function of l, because $B_2(r, -\xi) = -B_2(r, \xi)$ and $B_1(r, -\xi) = B_1(r, \xi)$ (see equation (97)). Thus the 1-D Fourier transform of $\langle w(\mathbf{R}')n'(\mathbf{R}' + \mathbf{m}_0 l)\rangle$ with respect to l is an odd function of p:

$$q_{wn}(p) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(-ipl)\langle w(\mathbf{R}')n'(\mathbf{R}' + \mathbf{m}_0 l)\rangle\, dl = -\frac{i}{\pi}\int_0^\infty \sin(pl)\langle w(\mathbf{R}')n'(\mathbf{R}' + \mathbf{m}_0 l)\rangle\, dl. \tag{102}$$

Using (101) and (97) in (102), integrating some terms by parts, and substituting $\xi_0 = \cos\beta_0$, where β_0 is the angle between the line of measurement and flux direction, we obtain

$$q_{wn}(p) = \frac{i\cos\beta_0}{\pi}\left[\sin^2\beta_0 p\frac{d}{dp} - 2\cos^2\beta_0\right] \cdot \int_0^\infty K(r)\sin(pr)\, dr. \tag{103}$$

Using the pair of formulae, formulae (100) and (103), we can express the 3-D and 1-D cospectra in terms of the same function,

$$\int_0^\infty K(r) \sin(pr)\, dr.$$

This means that we found the parametric dependence between $F_{wn}^{(-)}(p, \eta)$ and $q_{wn}(p)$.

If the 1-D spectrum has the power form, $q_{wn}(p) \sim p^{-b}$, it means that

$$\int_0^\infty K(r) \sin pr\, dr = Cp^{-b}. \tag{104}$$

In this case,

$$p \frac{dCp^{-b}}{dp} = -bCp^{-b}, \qquad p^2 \frac{d^2Cp^{-b}}{dp^2} = b(b+1)Cp^{-b},$$

$$q_{wn}(p) = -\frac{i \cos \beta_0}{\pi} (b \sin^2 \beta_0 + 2 \cos^2 \beta_0) Cp^{-b},$$

$$F_{wn}^{(-)}(p, \eta) = -\frac{i \cos \gamma}{2\pi^2 p^2} [b(b+1) + 3b + 3] Cp^{-b}.$$

We remind that β_0 is the angle between the line of measurements and the flux direction, while γ is the angle between the direction of the argument of the 3-D spectrum and the flux direction. Thus, for the case of power cross spectra the relation between $F_{wn}^{(-)}(p, \eta)$ and $q_{wn}(p)$ takes the form

$$F_{wn}^{(-)}(p, \eta) = \frac{\cos \gamma}{2\pi \cos \beta_0} \frac{b^2 + 4b + 3}{\sin^2 \gamma + 2 \cos^2 \gamma} \frac{q_{wn}(p)}{p^2}.$$

If we substitute $\cos \gamma = \eta$ and $\sin^2 \gamma = 1 - \eta^2$, we obtain

$$F_{wn}^{(-)}(p, \eta) = \frac{\eta}{2\pi \cos \beta_0} \frac{b^2 + 4b + 3}{b + (2-b)\eta^2} \frac{q_{wn}(p)}{p^2}. \tag{105}$$

For $b = 7/3$, $\cos \beta_0 = 0.12$ (these values correspond to measurements we used), and $\eta = 1$ (vertical orientation of the scattering vector) the numerical coefficient in (105) in front of $q_{wn}(p)/p^2$ is ~ 12.

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A. Muschinski and V. Tatarskii, NOAA Environmental Technology Laboratory, 325 Broadway, R/ET4, Boulder, CO 80305-3328, USA. (amuschinski@etl.noaa.gov; valeryan.i.tatarskii@noaa.gov)

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