Midterm Exam Solutions

Problem 1

(a) $p = \rho R_d T$, $R_d = 287 \frac{J}{kg\,K}$ (assuming dry air)

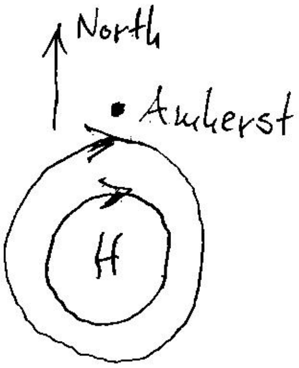
$$\Rightarrow \rho = \frac{p}{R_d T} = \frac{9 \cdot 10^4\,Pa}{287 \frac{J}{kg\,K} \cdot 283\,K} = \underline{\underline{1.11 \frac{kg}{m^3}}}$$

(b) Hydrostatic equation : $dp = -\rho g\, dz$

$$\Rightarrow h = -\frac{\Delta p}{\rho g} = -\frac{-1\,Pa}{1.11 \frac{kg}{m^3} \cdot 10 \frac{m}{s^2}} = 0.090\,m = \underline{\underline{9\,cm}}$$

Problem 2

(a)



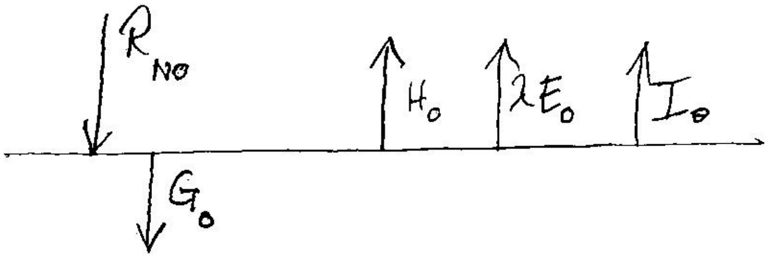
$\Rightarrow$ The wind comes from the west.

(b) Geostrophic wind speed : $v_g = \frac{1}{\rho f} |\underline{\nabla}_h p|$

$$\Rightarrow v_g = \frac{1}{1 \frac{kg}{m^3} \cdot 10^{-4} \frac{1}{s}} \cdot \frac{200\,Pa}{80 \cdot 10^3\,m} = \underline{\underline{25 \frac{m}{s}}}$$

Problem 3

(a)



$$\Rightarrow \underbrace{R_{N0} - G_0}_{\text{available energy}} = H_0 + \lambda E_0 + I_0 ,$$

where I_0 is the "ice-melting energy flux".

$$I_0 = \frac{\Delta M \cdot \lambda_f}{\Delta A \, \Delta t} = \frac{\Delta h \cdot \rho_{ice} \cdot \lambda_f}{\Delta t}$$

$$= \frac{0.1\,\text{m} \cdot 920\,\frac{\text{kg}}{\text{m}^3} \cdot 0.334 \cdot 10^6\,\frac{\text{J}}{\text{kg}}}{10 \cdot 86400\,\text{s}}$$

$$= \underline{\underline{35{,}6\,\frac{\text{W}}{\text{m}^2}}}$$

(b) The available energy is typically on the order of a few hundred W/m², of which I_0 is a small but not negligible fraction. So 35 W/m² is a realistic value. The scenario of 10 cm of ice melting away within 10 days is also realistic for Amherst in March.

Problem 4

(a)
$$S_{\text{Sirius}} = \frac{25\, \overset{= 3.8 \cdot 10^{26}\,\mathrm{W}}{L_{\text{sun}}}}{4\pi \cdot (8.6 \cdot \underbrace{365 \cdot 86400\,\mathrm{s} \cdot 3 \cdot 10^{8}\,\frac{\mathrm{m}}{\mathrm{s}}}_{1\ \text{light-year}})^2}$$

$$= \underline{\underline{1.14 \cdot 10^{-7}\,\mathrm{W}}}$$

(b)
$$P = \underbrace{\pi R_E^2}_{\text{Earth's cross-section}} \cdot S_{\text{Sirius}} = \underline{\underline{14.5\ \mathrm{MW}}}$$

(c) Sirius has <u>no significant effect</u> on Earth's climate because S_{Sirius} is about 10 orders of magnitude smaller than the solar constant.

Problem 5

(a)
$$F = a\, r^{\alpha} \rho^{\beta} v^{\gamma} \Rightarrow [F] = [r]^{\alpha} [\rho]^{\beta} [v]^{\gamma}$$

$$\Rightarrow \frac{\mathrm{kg\,m}}{\mathrm{s}^2} = \mathrm{m}^{\alpha} \left(\frac{\mathrm{kg}}{\mathrm{m}^3}\right)^{\beta} \left(\frac{\mathrm{m}}{\mathrm{s}}\right)^{\gamma}$$

$$\Rightarrow \begin{cases} \mathrm{kg}: & 1 = \beta \Rightarrow \boxed{\beta = 1} \\ \mathrm{m}: & 1 = \alpha - 3\beta + \gamma \Rightarrow \alpha = 1 + 3\beta - \gamma \Rightarrow \boxed{\alpha = 2} \\ \mathrm{s}: & -2 = -\gamma \Rightarrow \boxed{\gamma = 2} \end{cases}$$

$$\Rightarrow \boxed{F = a\, r^2 \rho\, v^2}$$

Problem 5 (cont'd)

(b) $\alpha = 2 > 0$ makes sense because F should increase if r increases.

$\beta = 1 > 0$ makes sense because F should increase if s increases.

$\gamma = 2 > 0$ makes sense because F should increase if v increases.