11. The atmospheric surface layer

The atmospheric surface layer (ASL) is the lowest part of the ABL.

Def.: The ASL is the near-surface air layer within which the turbulent fluxes do not deviate much (say, 10% or less) from the surface fluxes. The ASL is also called the constant-flux layer or the PRANDTL layer.

Turbulent momentum flux (turbulent drag):

\[ \tau = -S \langle u'w' \rangle = S \mu_*^2, \]  

where

\[ \mu_* = -\sqrt{\frac{T}{S}} \lim_{z \to 0} \text{(surface limit)} \]

is the friction velocity.

PRANDTL's mixing-length theory (1925):

\[ \tau = S \ell^2 \left( \frac{\partial \mu}{\partial z} \right)^2 \]
Similarity theory (von KARMAN 1930):

\[ l = \alpha z, \]  

where \( l = \) "mixing length",
\( z = \) height above ground,
\( \alpha = 0.4 \) (von-Karman constant)

(1), (5)

\[ \mu_*^2 = \frac{1}{l} \left( \frac{\partial \langle u \rangle}{\partial z} \right)^2. \]

\[ \Rightarrow \left| \frac{\partial \langle u \rangle}{\partial z} \right| = \frac{\mu_*}{l} \tag{5} \]

(4)

\[ \Rightarrow \left| \frac{\partial \langle u \rangle}{\partial z} \right| = \frac{\mu_*}{\alpha z}, \text{ for neutral stratification} \tag{6} \]

\[ \Rightarrow \langle u(z) \rangle = \frac{U_0}{\alpha} \cdot \int_0^z \frac{z}{z_0}, \text{ logarithmic law or law of the wall} \tag{7} \]

where \( z_0 \) is the roughness length.

<table>
<thead>
<tr>
<th>Surface type</th>
<th>( z_0 [\text{m}] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Open field, terrain, grass</td>
<td>3</td>
</tr>
<tr>
<td>Low crops</td>
<td>10</td>
</tr>
<tr>
<td>High crops</td>
<td>25</td>
</tr>
<tr>
<td>Parkland, bushes</td>
<td>50</td>
</tr>
<tr>
<td>Suburbs, forest</td>
<td>50 - 100</td>
</tr>
</tbody>
</table>
Example: Surface wind speed and $u_*$

Suppose $\langle u(z=2\,m) \rangle = 1 \frac{m}{s}$, how crops ($z_o=0.1\,m$)

$\Rightarrow u_* = \frac{\alpha \langle u(z) \rangle}{\ln \frac{z}{z_o}} = \frac{0.4 \cdot 1 \frac{m}{s}}{\ln \frac{1\,m}{0.1\,m}} = 0.17 \frac{m}{s}$

Drag: $u_* = \sqrt{\frac{f}{5}} \Rightarrow T = 5\,u_*^2$

$\Rightarrow T = 1 \frac{kN}{m} \cdot (0.17 \frac{m}{s})^2 = 0.019 \frac{N}{m^2}$

$= 28,900 \frac{N}{kN^2}$

Note: $T \propto u_* \Rightarrow T \propto \langle u \rangle^2$

Energy dissipation rate in ASL:

$\varepsilon(z) = \frac{u_*^3}{2z}$

Example: $\varepsilon$ at $z=1\,m$

Let $u_* = 0.17 \frac{m}{s} \Rightarrow \varepsilon = \frac{(0.17 \frac{m}{s})^3}{0.4 \cdot 1\,m} = 0.012 \frac{m^2}{s^3}$

$= 12 \frac{m^2}{kg}$
Note: The typical thickness of the ASL is a few tens of meters.

Example: Energy dissipation in ASL:

\[ P = A \cdot \int_{z_0}^{l} \frac{\delta E(z)}{d z} \, d z = A \cdot \int_{z_0}^{l} \frac{\mu_*^3}{\delta z} \, d z \]

\[ = A \frac{\mu_*^3}{\delta} \frac{h}{z_0} \]

\[ \Rightarrow \frac{P}{A} = \frac{1}{\frac{\mu_*^3}{\delta}} \left( 0.17 \frac{u}{\delta} \right)^3 \frac{\mu}{\delta z_0} \approx 1.2 \frac{W}{m^2} \]

That is, the turbulent dissipation per unit area is typically small compared to \( S_0 \).

But because \( \frac{P}{A} \propto \mu^3 \), an increase of \( \langle u \rangle \) by a factor of 10 would lead to an increase of \( \frac{P}{A} \) by \( 10^3 = 1000 \)!

That is, hurricanes dissipate huge amounts of energy.

The MO theory accounts for effects due to non-neutral temperature stratification.

\[
\frac{\partial \langle u \rangle}{\partial z} = \frac{u_*}{\alpha z} \phi_m \left( \frac{Z}{L_*} \right) ,
\]

(10)

where

\[
L_* = -\frac{\langle \theta' \mu_* \rangle}{\alpha g \langle \theta' \omega' \rangle}.
\]

(11)

is the Monin - Obukhov length, \( \theta \) = pot. temp., \( g \) = acceleration due to gravity.

Def.: \( \Theta_* = -\frac{\langle \theta' \omega' \rangle}{u_*} \) is the scaling temperature.

(12)

MO theory:

\[
\frac{\partial \langle \theta \rangle}{\partial z} = \Theta_* \frac{u_*}{\alpha z} \phi_n \left( \frac{Z}{L_*} \right) .
\]

(13)

Note: According to MO theory, the similarity function for momentum, \( \phi_m \left( \frac{Z}{L_*} \right) \), and the similarity function for heat, \( \phi_n \left( \frac{Z}{L_*} \right) \), are universal functions of \( \frac{Z}{L_*} \).
The similarity functions $\Phi_m\left(\frac{z}{L_*}\right)$ and $\Phi_h\left(\frac{z}{L_*}\right)$ have been measured in numerous field experiments since the 1960s.

\[
\Phi_m\left(\frac{z}{L_*}\right) = \begin{cases} 
(1 + 16\left|\frac{z}{L_*}\right|)^{-\frac{1}{4}}, & -2 \leq \frac{z}{L_*} \leq 0 \\
1 + 5\frac{z}{L_*}, & 0 \leq \frac{z}{L_*} \leq 1
\end{cases}
\]

\[
\Phi_h\left(\frac{z}{L_*}\right) = \begin{cases} 
(1 + 16\left|\frac{z}{L_*}\right|)^{-\frac{1}{2}}, & -2 \leq \frac{z}{L_*} \leq 0 \\
1 + 5\frac{z}{L_*}, & 0 \leq \frac{z}{L_*} \leq 1
\end{cases}
\]