**Homework Assignment #8**

**ECE 597UU/697UU, Fall 2009**

(Posted on the course website on Fri, 20 Nov; due in class on Tue, 1 Dec)

**Problem 1 (8 pts.):** *Uncorrelated noise*

Fast-response sensors such as ultrasonic anemometers/thermometers are characterized by uncorrelated noise. At frequencies where the external signal has a low spectral density, the measured spectral density may be dominated by the noise spectral density. At high frequencies, the spectral density of turbulent signals decreases with increasing frequency, such that the "noise floor" of the sensor can often be recognized as a flattening of the measured spectral density at high frequencies, and the uncorrelated noise r.m.s. value can be determined from that noise floor.

Now, let $x[n]$, $n = 0...N-1$ be a discrete-time sequence of zero-mean, stationary noise with variance $\sigma_{xx}^2$. Let the sampling frequency be $T_s$. Suppose that the samples $x[n]$ are equidistant samples of a continuous-time signal $x_c(t)$, which is the output of an ideal anti-aliasing filter, such that $x[n]$ contains no energy from frequencies above the Nyquist frequencies. Suppose further that the sampled noise is uncorrelated, such that

$$\langle x[n]x^*[m]\rangle = \sigma_{xx}^2 \delta[n-m], \tag{1}$$

where $\delta[n]$ is the discrete-time unit impulse.

**(a, 4 pts.)** Show that $\langle |X[k]|^2 \rangle = N\sigma_{xx}^2$, where $X[k], k = 0...N-1$ is the DTFS of $x[n]$, $n = 0...N-1$.

**(b, 4 pts.)** Now, let $S_{xx}(f_k)$ be the one-sided noise spectrum sampled at the discrete frequencies $f_k = k\frac{2\pi}{N}$, $k = 1...N/2$. Show that

$$S_{xx}(f_k) = \frac{2T_s}{N}\left\langle |X[k]|^2 \right\rangle. \tag{2}$$

**Problem 2 (8 pts.):** *Taylor's hypothesis and temperature structure parameter*

Let $S_{TT}(f)$ be the *one*-sided temperature frequency spectrum, such that

$$\int_0^\infty S_{TT}(f)\mathrm{d}f = \sigma_{TT}^2, \tag{3}$$

where $f = \Omega/2\pi$ is the frequency and $\Omega$ is the angular frequency, and where $\sigma_{TT}^2$ is the temperature variance. Now, suppose that the turbulence is locally homogeneous and isotropic and that Taylor's hypothesis is valid. Let $F_{TT}(k_1)$ be the *one*-sided, one-dimensional wave-number spectrum, such that

$$\int_0^\infty F_{TT}(k_1)\mathrm{d}k_1 = \sigma_{TT}^2. \tag{4}$$

**(a, 4 pts.)** Use Taylor's hypothesis to express $S_{TT}(f)$ in terms of $F_{TT}(k_1)$. Suppose the mean wind speed is $U$.

**(b, 4 pts.)** Suppose that $F_{TT}(k_1)$ follows the inertial-range law, such that $F_{TT}(k_1) = 0.249 C_T^2 k_1^{-5/3}$, where $C_T^2$ is the so-called temperature structure parameter. (The exact value of the coefficient 0.249 is $\Gamma(2/3)/\sqrt{3}\pi$, where $\Gamma(\cdot)$ is the gamma function.)

Find $S_{TT}(f)$ in terms of $C_T^2$, $U$ and $f$. Present the final result in a way that it contains only a single numerical coefficient.

**Problem 3 (8 pts.):** *Temperature structure parameter retrieved from sonic data*

The attached .mat file contain 48 hours of equidistant samples of the three wind vector components $u$, $v$ and $w$ and of temperature $T$, all measured with the same ultrasonic anemometer/thermometer. The sampling period of the sonic data is $T_s = 0.1$ s.

**(a, 4 pts.)** Determine the r.m.s. noise values of $u$, $v$, $w$ and $T$. (Select episodes with very little turbulence.)

**(b, 4 pts.)** Calculate and plot time-series of 30-min averages of $U$, $T$ and $C_T^2$ for these 48 hours. Explain what you see.