

## Homework Assignment #7

ECE 597UU/697UU, Fall 2009

(Posted on the course website on Fri, 13 Nov; due in class on Thu, 19 Nov)

### Problem 1 (10 pts.): *Gaussian probability density function*

Consider the probability density function (p.d.f.)

$$p_x(x) = a \exp\left(-\frac{(x-b)^2}{c^2}\right), \quad (1)$$

where  $a$ ,  $b$  and  $c$  are real numbers and  $x$  is a real-valued random variable (R.V.).

(a, 2 pts.) Find  $a$  as a function of  $b$  and  $c$ . (Hint: Remember that the integral of a p.d.f. over all possible values of the R.V. is always 1.)

(b, 2 pts.) Find the ensemble average of  $x$ , based on the definition given in Lecture 17.

(c, 2 pts.) Find the standard deviation of  $x$ , based on the definition given in Lecture 17.

(d, 2 pts.) Find the skewness of  $x$ , based on the definition given in Lecture 17.

(e, 2 pts.) Find the kurtosis of  $x$ , based on the definition given in Lecture 17.

### Problem 2 (6 pts.): *Some basic questions*

(a, 2 pts.) Suppose  $p_q(q)$  is the p.d.f. of some R.V.  $q$ . What is the probability that a random sample of  $q$  is larger than  $\langle q \rangle$ ?

(b, 2 pts.) Why is  $p_q(q)$  nonnegative for any  $q$ ?

(c, 2 pts.) What is the probability for the event  $q > \langle q \rangle$  if the skewness of  $q$  is zero?

### Problem 3 (6 pts.): *Air temperature as a random variable*

Suppose  $p_T(T)$  is the p.d.f. of the continuous random variable  $T$ , where  $T$  is the air temperature 1.5 m above ground level at the center of the engineering quad.

(a, 2 pts.) What is the meaning of the quantity  $P = \int_{T_1}^{T_2} p_T(T) dT$ , where  $T_1 = 5$  degrees Celsius and  $T_2 = 10$  degrees Celsius?

(b, 2 pts.) Estimate the value of  $P$  for  $T_1 = -40$  degrees Celsius and  $T_2 = +40$  degrees Celsius.

(c, 2 pts.) Estimate the probability that  $T$  is exactly 20 degrees Celsius.