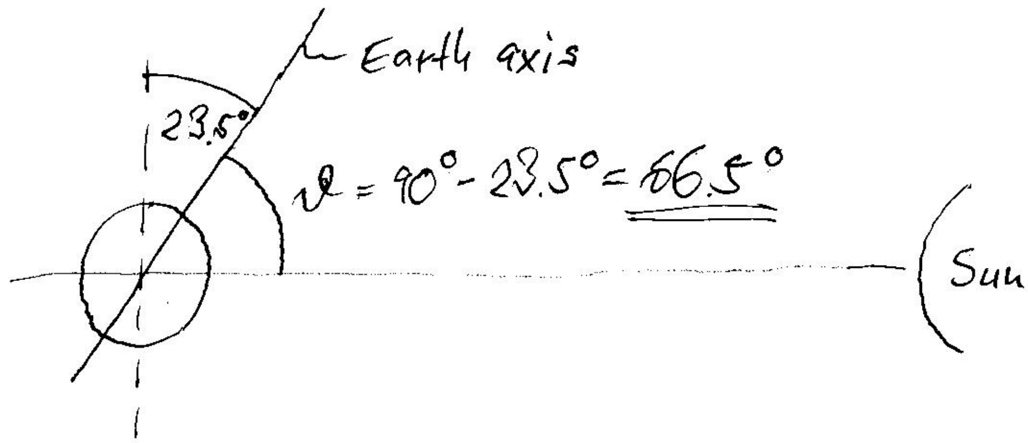
Solutions to HW # 3

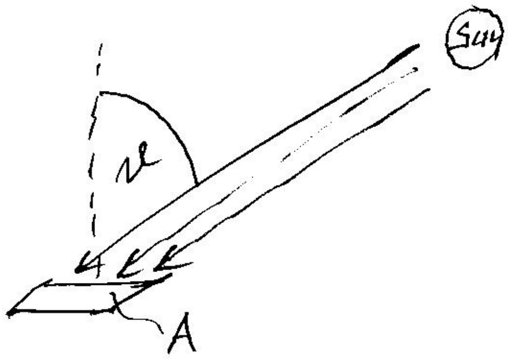
Problem 1

(a)



$$\vartheta = 90° - 23.5° = \underline{\underline{66.5°}}$$

(b) Intensity of solar radiation for horizontal unit area with albedo a:



$$S = (1-a)\, S_0 \cos\vartheta$$

Outgoing long-wave radiation: $M = \underset{= 0.97 \text{ for ice}}{\varepsilon}\, \sigma\, T_e^4$

Equilibrium temperature T_e (no greenhouse effect):

$$S = M \Rightarrow (1-a)\, S_0 \cos\vartheta = \varepsilon \sigma T_e^4$$

$$\Rightarrow T_e = \left(\frac{(1-a)\, S_0 \cos\vartheta}{\varepsilon \sigma} \right)^{\frac{1}{4}} \overset{!}{=} \left(\frac{(1-0.5) \cdot 1366 \frac{W}{m^2} \cdot \cos(66.5°)}{0.97 \cdot 5.97 \cdot 10^{-8}\, W/m^2K^4} \right)^{\frac{1}{4}}$$

$$\overset{!}{=} \boxed{265\ K = -8°C}$$

(c) Now, let $a = 0.05$, $\varepsilon = 0.93$ (asphalt)

$$\Rightarrow T_e = \left(\frac{(1-0.05)\cdot 1366\,W/m^2 \cdot \cos(66.5^\circ)}{0.93\cdot 5.67\cdot 10^{-8}\,W/m^2K^4}\right)^{\frac{1}{4}} \approx \boxed{315\,K = 42\,^\circ C}$$

(d) The values of T_e for a flat surface are higher than T_e for a sphere because the factor 4 in the denominator is missing. Note that ϑ has to be fairly close to 90° in order to overcompensate the missing $\frac{1}{4}$:

$$\frac{1}{4} = \cos\vartheta \Rightarrow \vartheta \approx 75^\circ$$

(e) Moon, $\vartheta = 0^\circ$, $\varepsilon = 1$, $a = 0.12$

$$\Rightarrow T_e = \left(\frac{(1-0.12)\cdot 1366\,W/m^2 \cdot 1}{1\cdot 5.67\cdot 10^{-8}\,W/m^2K^4}\right)^{\frac{1}{4}} = \boxed{382\,K = 109\,^\circ C}$$

(f) The ice around the North Pole is only sea ice and only ca. 3 meters thick, floating on an ocean that is 4300 m deep at the North Pole. (The first manned visit of the North Pole ocean bottom was in 2007!)

Problem 2

(a) Pressure $p = \frac{F}{A} = \frac{M \cdot g}{A}$,

where F = force acting on area A,
M = mass of air,
g = acceleration due to gravity

Now, $A = 4\pi R_e^2$,
$R_e = 6371\ \mathrm{km}$ = Earth radius

$$\Rightarrow M = \frac{pA}{g} = \frac{p \cdot 4\pi R_e^2}{g}$$

$$= \frac{1.013 \cdot 10^5\,\mathrm{Pa} \cdot 4\pi \cdot (6.371 \cdot 10^6\,\mathrm{m})^2}{10\ \mathrm{m/s^2}}$$

$$= \boxed{5.2 \cdot 10^{18}\ \mathrm{kg}}$$ is (approximately) the mass of Earth's atmosphere

(b) $$\frac{M_a}{M_e} = \frac{5.2 \cdot 10^{18}\ \mathrm{kg}}{6.0 \cdot 10^{24}\ \mathrm{kg}} = \boxed{8.7 \cdot 10^{-7}}$$

$\Rightarrow$ M_a is about one millionth of M_e.

Problem 3

(a) $p_d = \rho_d R_d T \implies \rho_d = \dfrac{p_d}{R_d T}$

$$T = 0^\circ C = 273\,K \implies \rho_d = \frac{10^5\,Pa}{287\,\frac{J}{kg\,K} \cdot 273\,K} = 1.28\,\frac{kg}{m^3}$$

$$T = 20^\circ C = 293\,K \implies \rho_d = 1.19\,\frac{kg}{m^3}$$

(b) For $p_d = 800\,hPa = 8 \cdot 10^4\,Pa$

$$T = 0^\circ C \implies \rho_d = 1.02\,\frac{kg}{m^3}$$

$$T = 20^\circ C \implies \rho_d = 0.95\,\frac{kg}{m^3}$$

Problem 4

(a) From Lecture 8: $R_h = \dfrac{\rho_d}{\rho_d + \rho_w} R_d + \dfrac{\rho_w}{\rho_d + \rho_w} R_w$

From Lecture 9: $q = \dfrac{\rho_w}{\rho_d + \rho_w}$

$$\implies \boxed{R_h = (1-q) R_d + q\,R_d}$$

<u>Note:</u> R_h depends only on q!

(b) $q = 1\,g/kg = 10^{-3}\,\frac{kg}{kg} = 10^{-3}$

$$\implies R_h = (1-10^{-3}) \cdot 287\,\frac{J}{kg\,K} + 10^{-3} \cdot 459\,\frac{J}{kg\,K}$$

$$\overset{!}{=} \boxed{287.2\,J/kg \cdot K}$$, very close to R_d