Homework Assignment #1

ECE 597UU/697UU, Fall 2009

(Posted on course website and handed out in class on Thu, 17 Sep; due in class on Tue, 22 Sep)

Problem 1 (10 pts. total, 2 pts. per subproblem): *Dimensionally correct or incorrect physical expressions and equations*

In the following, m is a mass, g is the acceleration due to gravity, h is a height, ρ is a mass density, v is a velocity, k_B is the Boltzmann constant, T is a temperature, and κ is a wave number. Assume that all these quantities are different from zero.

Find out whether or not the following physical expressions or equations are valid. Justify your answer.

(a) mgh; (b) $mg + \rho v$; (c) $\sin(\kappa h)$; (d) $\exp(-k_B T/mg)$; (e) $v = \sqrt{k_B T/3m}$.

Problem 2 (6 pts. total): *Dimensional analysis: The Stefan-Boltzmann law*

The Stefan-Boltzmann law[1] , $M = \sigma T^4$, describes the intensity of black-body radiation. Here, M is the total outgoing radiative power per unit area (unit: $\mathrm{W\,m^{-2}}$) of a black surface, T is the temperature of that surface, and σ is a universal constant (the "Stefan-Boltzmann constant").

(a, 2 pts.) Find the unit of σ.

(a, 4 pts.) Because σ is a universal constant that is independent of any specific material, it can be related only to other universal constants, such as the Planck constant ($h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$), the speed of light in the vacuum ($c = 2.998 \times 10^8\,\mathrm{m\,s^{-1}}$), and the Boltzmann constant $k_B = 1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$. Based on the "educated guess" $\sigma = a\,h^\alpha c^\beta k_B^\gamma$, where a is a universal dimensionless (i.e., numerical) coefficient and the exponents α, β and γ are also dimensionless, find the dimensionally correct physical equation for M in terms of a, T, h, γ, c, and k_B.

Problem 3 (4 pts. total): *Using the Stefan-Boltzmann law for energy considerations*

From experiments and Planck's law, it is known that $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}K^{-4}}$.

(a, 2 pts.) Consider an area of 1 m^2 of fresh snow on the ground at high elevation (say, at the top of Pike's Peak in Colorado) in a cold, clear winter night (20 degrees Celsius below freezing). What is the loss of energy (in watts), assuming (1) that fresh snow radiates like a black body (which is actually a valid assumption!) and (2) that the downward infrared radiation from the sky is negligible.

(b, 2 pts.) The surface temperature of the sun is about 6000 K. What area A of sun surface emits as much power as a typical nuclear power plant (1000 MW)?

[1] Named after the two Austrian physicists Josef Stefan (1835-1893) and Ludwig Boltzmann (1844-1906).