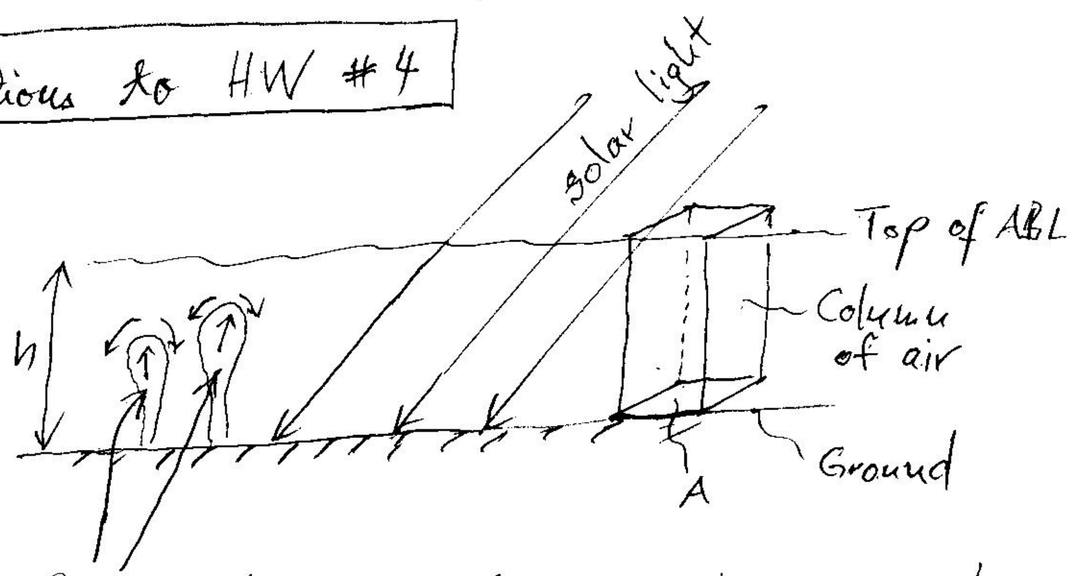
Solutions to HW #4

Problem 1:



(a) Consider a column of air with height h above area A.

Surface heat flux: $H = \frac{\Delta E_t}{A \cdot \Delta t} = \frac{c_p \cdot M \cdot \Delta T}{A \cdot \Delta t}$

$$\Rightarrow \frac{\Delta T}{\Delta t} = \frac{H \cdot A}{c_p M} = \frac{H \cdot A}{c_p \rho A h} = \frac{H}{c_p \rho h},$$

where ΔT = temperature increase during time Δt,
$c_p = 1004 \,\mathrm{J/kg \cdot K}$ = specific heat of air for p = const.,

Now, let $\Delta t \to 0$

$$\Rightarrow \lim_{t \to 0} \frac{\Delta T}{\Delta t} = \boxed{\frac{dT}{dt} = \frac{H}{c_p \rho h}}$$

The additional parameter is c_p.

(b) $$\left[\frac{dT}{dt}\right] = \frac{[dT]}{[dt]} = \frac{[T]}{[t]} = \frac{K}{s}$$

$$\left[\frac{H}{c_p \rho h}\right] = \frac{W/m^2}{(J/kg\,K)(kg/m^3)\,m} = \frac{W\,K}{J} = \frac{(J/s)\,K}{J} = \frac{K}{s} \quad \checkmark$$

Problem 1 (cont'd):

(c) $$\frac{dT}{dt} = \frac{300\,W/m^2}{1004\,J/kg\,K \cdot 1\,kg/m^3 \cdot 10^3\,m} = 3 \cdot 10^{-4}\,\frac{K}{s}$$
$$\approx 1\ \text{K per hour},$$

which is a realistic value.

Problem 2:

(a) The physics is the same as in Problem 1, just replace $H \to \Delta S_0$, $h \to D$, $\rho \to \rho_{lw}$, $c_p \to c_{lw}$, where $\rho_{lw} = 1000\,\frac{kg}{m^3}$ = density of liquid water, $c_{lw} = 4180\,\frac{J}{kg\,K}$ = spec. heat of liquid water

$$\Rightarrow \frac{dT}{dt} = \frac{\Delta S_0}{c_{lw}\,\rho_{lw}\,D}$$

(b) Because $[\Delta S_0] = [H]$, $[h] = [D]$, $[\rho] = [\rho_{lw}]$, $[c_p] = [c]$, we have $\left[\frac{\Delta S_0}{c_{lw}\,\rho_{lw}\,D}\right] = \left[\frac{H}{c_p\,\rho\,h}\right]$, which is evaluated in 1 (b).

(c) $$\frac{dT}{dt} = \frac{1.4\,W/m^2}{4180\,J/kg\,K \cdot 10^3\,kg/m^3 \cdot 4000\,m} = \frac{1.4}{4.18 \cdot 4}\,10^{-3-3-3}\,\frac{K}{s}$$
$$= 0.084 \cdot 10^{-9}\,\frac{K}{s} = 8.4 \cdot 10^{-11}\,\frac{K}{s} = \frac{1\,K}{380\ \text{years}}$$

This is small compared to heating rates currently discussed for global warming ($\sim 1\,K$ in a few decades)

Problem 3

Kinetic energy E_k is converted to thermal energy E_t:

$$\langle E_k \rangle = \left\langle \frac{1}{2} M v^2 \right\rangle = \frac{1}{2} M \langle v^2 \rangle = \frac{M}{2} \sigma_v^2 ,$$

where $\langle \cdot \rangle$ = expected value of,
M = mass,
v = speed (magnitude of velocity),
σ_v = r.m.s. value of v

$$\langle E_k \rangle = E_t \Rightarrow \frac{M}{2} \sigma_v^2 = M c_{lw} \Delta T$$

$$\Rightarrow \Delta T = \frac{\sigma_v^2}{2 c_{lw}} = \frac{\left(1 \frac{m}{s}\right)^2}{2 \cdot 4180 \ \mathrm{J/kg\,K}} = \boxed{0.12 \text{ mK (millikelvins)}}$$

Problem 4

From Lecture 9: $2^{\frac{\vartheta - \vartheta_d}{10\,K}} = \frac{1}{r} \Rightarrow r = 2^{\frac{\vartheta_d - \vartheta}{10\,K}}$

Here: $\vartheta_d < 0°C$, $\vartheta = 40°C \Rightarrow \vartheta_d - \vartheta < -40\,K$

$$\Rightarrow r < 2^{\frac{-40\,K}{10\,K}} = \frac{1}{16} = 6.25\,\%$$

That is, the relative humidity is $\boxed{\text{below } 6.25\,\%.}$