

Homework Assignment #3

ECE 314, Spring 2006

(Posted on Tue, 21 Feb; due in class on Mon, 27 Feb)

Problem 1:

Consider families with 5 children. Consider the R.V. $X(s) = x$, where x is the number of girls among the children in a family.

- (a) Why is X a R.V.? (Describe the experiment and the function that maps outcomes to real numbers.)
- (b) Is X a discrete R.V.? A finite R.V.? Justify your answer.
- (c) Write down the set S_X of possible values of X .
- (d) Determine the PMF. List your assumptions carefully.

Problem 2:

Consider a randomly chosen group of n people.

- (a) How large must that group be so that the probability that at least one of them was born on a 29th of February exceeds 50%? Justify your answer.
- (b) How large must that group be so that the probability that at least two of them have birthday on the same day exceeds 99%? (For this experiment, make sure that nobody in your party was born on a 29th of February.)

Problem 3:

Suppose that there are 6000 stars that are bright enough to be visible with the naked eye. The full moon has an angular diameter of about half a degree. Assume that the positions of these 6000 stars are independent of each other.

- (a) What is the probability that the moon covers one of those stars at a randomly chosen time?
- (b) What is the probability that the moon covers two of those stars at a randomly chosen time?
- (c) What is the probability that the moon covers none of those stars at a randomly chosen time?

Problem 4:

Use Theorem 1.7 to prove the following facts:

- (a) $P[A \cup B] \geq P[A]$
- (b) $P[A \cup B] \geq P[B]$
- (c) $P[A \cap B] \leq P[A]$
- (d) $P[A \cap B] \leq P[B]$.

Problem 5:

Deer ticks can carry both Lyme disease and human granulocytic ehrlichiosis (HGE). In a study of ticks in the Midwest, it was found that 16% carried Lyme disease, 10% had HGE, and that 10% of the ticks that had either Lyme disease or HGE carried both diseases.

- (a) What is the probability $P[LH]$ that a tick carries both Lyme disease (L) and HGE (H)?
- (b) What is the conditional probability that a tick has HGE given that it has Lyme disease?

Problem 6:

Suppose you play 52-card poker. What is the probability that in your 5-card poker hand you have

- (a) at least one pair;
- (b) exactly one pair;
- (c) two pairs;
- (d) no pair;
- (e) a triple;
- (f) a full house (a triple and a pair)?

Justify your answers in detail.