ECE 665, Computer Algorithms, Spring 2005

Homework 5

Due: Friday, May 13

The first three problems are taken from Homework 4. You are now asked to formulate and solve these problems using Linear Programming (LP) or Integer Linear Programming (ILP) method, whichever is appropriate, and compare the results to those obtained with graph algorithms in Homework 4.
You can use LINGO or Mathematica to solve the LP problems.

1. **Assignment Problem**
   Consider the assignment problem from Homework 4, Problem 4.

   (a) Formulate and solve the assignment problem as an LP and compare the result with that obtained using graph-theoretic approach in Homework 4. Clearly show your formulation, including the objective function and the constraints. Is the constraint matrix totally unimodular, and what are the consequences of this fact?

   (b) Formulate and solve the *dual LP problem* for the above primal problem. Compare the primal and dual results. How can you interpret the dual problem?

   (c) Now consider a *Weighted Assignment Problem*, where each assignment of worker to job has an integer cost represented by the following matrix. The goal is to maximize the total assignment cost. Solve this problem using LP and compare it to the solution you obtained in Homework 4.

	1	2	3	4	5	6
1				3		
2		4	1			2
3				1		
4			2		1	
5	1					2
6						1

   (d) Can you model this problem as a flow network problem? Explain why or why not.

2. **Multi-commodity flow problem**.
Consider the multi-commodity flow problem given in Homework 4, Problem 5.

   (a) Formulate and solve the multi-commodity problem as an LP problem and compare the results with that obtained using graph-theoretic approach in Homework 4. Clearly show your formulation, including the objective function and the constraints.

   (b) Comment on the structure of the constraint matrix. Is it totally unimodular, and if so, what are the consequences of this fact?

3. Solve the **Flight Scheduling** problem (given as programming assignment in Homework 4, Problem 6), using Linear Programming approach. You do not need to write a new program, just solve the LP problem for the given data. Clearly show the objective function, all the constraints and the solution.

4. The **Transportation Problem** is defined as follows. Quantities $a_1, a_2, ..., a_m$ of a product are being shipped from each of m locations and received in amounts $b_1, b_2, ..., b_n$ at each of n destinations. Associated with the shipping of a unit of a product from origin i to destination j is a unit shipping cost c_{ij}. It is desired to determine the amounts x_{ij} to be shipped between each origin-destination pair (i, j), $i = 1, 2, ..., m, j = 1, 2, ..., n$, so as to satisfy the shipping requirements and minimize the total cost of transportation of the product.

   (a) Formulate the transportation problem as an LP problem and solve it for the following data:

$$\mathbf{a} = [25\ 25\ 50]^T, \quad \mathbf{b} = [15\ 20\ 30\ 35]^T$$

$$C = \begin{bmatrix} 10 & 5 & 6 & 7 \\ 8 & 2 & 7 & 6 \\ 9 & 3 & 4 & 8 \end{bmatrix}$$

   (b) Show that the constraint matrix is totally unimodular and demonstrate that the optimal solution to this problem is integer for every integer vector **a** and **b**.

   (c) Formulate and solve the *dual LP problem* for the above primal problem. Compare the primal and dual results; how can you interpret the dual problem?