A Study of the Impact of Computational Delays in Missile Interception Systems

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Abstract: Most publications discussing missile interception systems assume a zero computer response time. This paper studies the impact of computer response time on single-missile single-target and multiple- missile multiple-target systems. Simulation results for the final miss distance as the computer response time increases are presented. A simple online cooperative adjustment model for multiple-missile multiple-target system is presented for the purpose of studying the computer delay effect.

1 INTRODUCTION

Computational delay is a key determinant of performance in a cyber-physical system. The computer is in the feedback loop of the controlled plant; any additional delay may degrade the quality of control. The impact of computational delays has not been adequately addressed in the literature. In this paper, we use as an example a tracking system for missile control. The dynamics of such systems have been well-studied in the literature (under the tacit assumption that the computational delays are zero); we focus on the impact of computational delays in such a problem.

In this work, computational delay (or computer response time) refers to the time that elapses from the point the control algorithm is triggered to the point that the control signal is generated.

The paper is organized as follows: Section 2 provides the basic background including the widely used guidance laws, and presents the simulation results for a single-missile single-target system. In Section 3, a simple but still effective model for an online adjustment algorithm is described. Section 4 presents simulation results for the multiple-missile multiple-target system, focusing on two aspects: the effectiveness and advantages over the single-missile single-target system, and its ability to handle the computational delay during flight time. Section 5 presents conclusions and future work.

2 SINGLE MISSILE SINGLE TARGET SYSTEM

This section briefly summarizes the principles of some classic guidance laws, and presents simulation results for a single-missile single-target system.

The typical guidance laws that are implemented in missile guidance systems are the Proportional Navigation Guidance (PNG), and its more advanced counterpart, the Augmented Proportional Navigation Guidance (APNG). PNG is one of the most widely used guidance laws in homing air target missile systems. The main underlying assumption is that if two vehicles are on a collision course, their direct Line-of-Sight (LOS) does not change direction or value. Generally speaking, PNG indicates that the missile velocity direction should rotate at a rate proportional to the turn rate of the LOS, and should be in the same direction.

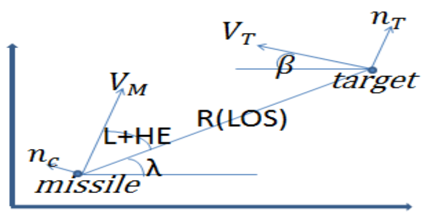

Figure 1: Missile-Target Intercept Geometry.

Typical parameters in this geometry (see Figure 1) can be found in (P. Zarchan, 2007):

L+HE: where L is the Missile Lead Angle and HE is the Heading Error.

R: Length of the line of sight (LOS)

n_c: Missile Acceleration; V_M: Missile Velocity

λ: Angle of Line of Sight (LOS)

β: Target Heading; V_T: Target Velocity

n_T: Target Maneuver (i.e., target acceleration normal to its velocity.)

The Proportional Navigation Guidance law (PNG) can be stated as (P. Zarchan, 2007):

$$n_C = N'V_c\dot{\lambda} \qquad (1)$$

There are more advanced guidance laws that can yield smaller miss distances against a highly maneuvering target, where the miss distance is measured at the point of closest approach of the missile and target. One of these advanced guidance laws is the Augmented Proportional Navigation (APNG) (P. Zarchan, 2007):

$$n_C = N'V_c\dot{\lambda} + N'n_T/2 \qquad (2)$$

APNG is a proportional navigation with an extra term to account for the maneuvering target.

2.1 Model Assumptions

The missile and target model assumptions used in this paper (for both single and multiple cases) are:

- No noise
- Target is a mass point
- Missile is a rigid body, influenced by aerodynamic forces (has velocity reduction), and the missile airframe is a 1st order system with a time constant of 1 and the transfer function: $\frac{1}{s+1}$
- Target escape range: once the relative distance at this time between the target and missile is less than this range, the target starts to escape
- Target maneuvering: 10g
- Missile maneuvering saturation: 3 times as much as target maneuvering.
- Initial Target velocity: 1000m/s
- Missile Initial Velocity: 4000 m/s.

2.2 Simulation Results for Single - Missile Single - Target System

Below are several simulation results based on the above assumptions; note that the problem of computational delay is made more acute by the fact that, quite often, relative slow processors are used in control functions. Figures 2 and 3 show that with a higher computer response time, the final miss distance would get worse. Figure 4 shows the relationship between the final miss distance and the computer response time.

As Figure 4 shows, the final miss distance would increase but at slower rate as the computer response time keeps increasing.

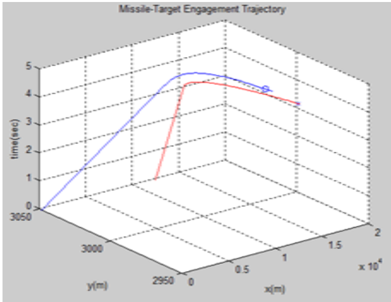

Figure 2: The trajectories of the missile and the target for a computer response time of 0.01s.

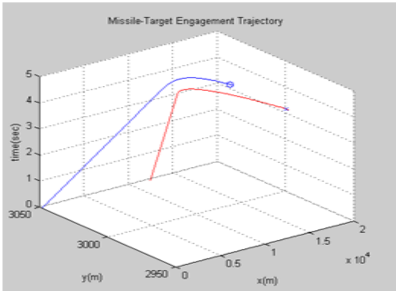

Figure 3: The trajectories of the missile and the target for a computer response time of 0.1s.

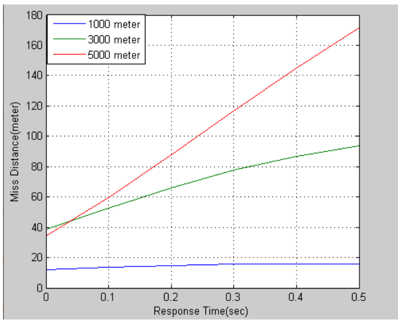

Figure 4: The miss distance as a function of the computer response time for different relative ranges at which the target starts to escape.

Moreover, the miss distance will reach a “saturation region” if the response time increases to a very large value. If the computer is significantly slow, the missile would never make any adjustments online, and would follow a straight line flight path as it is launched at ground. Thus, once beyond the point at which the computer is too slow to generate any command signal, the final miss distance would stay the same.

3 MULTIPLE TARGETS COOPERATIVE SYSTEM

The analysis above is for a single-missile single-target system. In this section we present a simple but still effective online adjustment algorithm for the case of a multiple-missile multiple-target system for the purpose of studying the impact of the computer response time. Since there are only very few prior publications about multiple-missile multiple-target systems in the open literature, we had to create our own simple model to study the effect of computer response time.

Our algorithm enables cooperation among the missiles to control the following three parameters during the flight time: (*i*) which target would be engaged by each missile; (*ii*) which guidance law to employ; and (*iii*) which guidance parameters should be used by the guidance law. The guidance laws considered here are PNG and APNG.

3.1 Online Cooperation

The proposed Online Cooperative Adjustment Algorithm (OCAA) has three sub tasks. The solution vector $sol[]$ includes three elements: the index of the target that this missile should engage, the guidance law this missile should employ, and the guidance law parameter the missile should use. We have used in our experiments two guidance laws, PNG and APNG, with index 0 (1) indicating the use of PNG (APNG).

The first sub task is a periodic task, passing the current missile's last solution vector to the missile computer, and evaluating this solution. The on-board computer would check two conditions at this time: (1) whether the final estimated miss distance using the last solution vector would be within the missile's explosion range, (2) whether other missiles sent a *switch target request* (see below). If both are false, the missile continues using the last solution vector.

The second subtask is: if during the online evaluation the missile finds out that the last solution vector cannot guarantee a hit, the computer would generate a *switch target request* signal to other missiles. At the same time, the computer would also generate new solution vectors for each missile target pair, and save all the available solution vectors for later use, as well as generate the information vector showing which target the current missile can hit, to be sent to the other missiles.

The third sub task is: after every missile gets the information vectors from all other missiles, all missiles would combine these vectors into an information table. Then, each missile's on board computer would make a missile-target assignment using the same assignment algorithm. This finishes the online adjustment and the missile computer would return to sub task one.

4 RESULTS FOR MULTIPLE MISSILE-TARGET SYSTEM

In order to study the computational delay impact for the multiple missile case, it is helpful to compare it with traditional PNG. Below are several results showing a 3-missile and 3-target engagement system. Suppose that the target and missile models are the same as in the single missile system and the target escape times (after which the target starts to escape) for targets 1, 2 and 3, are 15s, 14s, and 15s, respectively; and the target maneuvering levels are 15g, 20g and 10g, respectively. The missile explosion range is 30 meters.

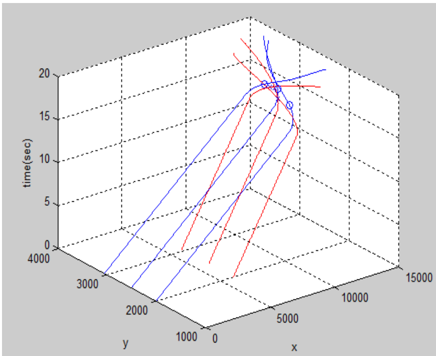

Figure 5: Simulation results for PNG, with a computer response of 0.01s. Blue (Red) lines are missile (target) trajectories. Once the missile enters its explosion range, it explodes, and this missile and its target stop moving.

Figure 5 shows that all of the missiles and targets never stop moving, which means none of the missiles ever enters their explosion range. The detailed final miss distance for each missile target pair is shown in Table 1. As we can see in Figures 6 and 7 a multiple-missile multiple-target system using communication and online cooperation, could achieve a better performance than a single-missile single-target system where each missile employs its initial algorithm without online cooperation.

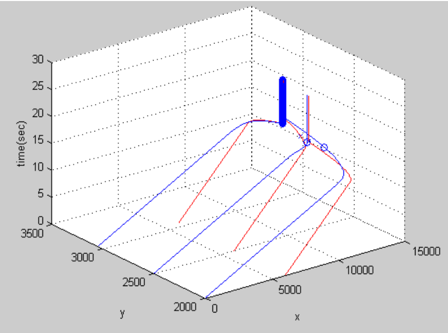

Figure 6: Simulation results for OCAA with a computer response time of 0.01s. Around 17s to 18s all three missile-target pairs stop moving, i.e., all three missiles entered their explosion range and hit their targets.

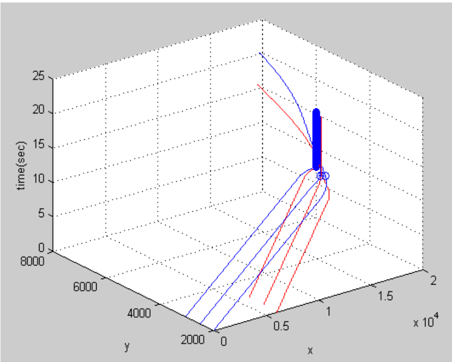

Figure 7: Simulation results for OCAA with computer response time of 0.05s. Two missile-target pairs stop moving, while one pair never stops, i.e., two missiles hit their targets, and one missile misses its target.

Table 1: Performance in terms of computer response time; “hit” (“miss”) means that the final miss distance is within (outside) the missile explosion range.

Computer response time (seconds)	Final result for all targets
0.01	all hits
0.02	all hits
0.03	all hits
0.04	all hits
0.05	2 hits, 1 miss
0.1	2 hits, 1 miss
0.2	2 hits, 1 miss
0.3	2 hit, 1 misses
0.4	1 hit, 2 misses
0.5	three misses

Moreover, Table 1 also shows that a multiple missile and target system can handle the problem of computational delay well. All three missiles are within their explosion range (of 30 meters) even for a delay time of 0.05s. Also, not all targets are missed until the computer response time is as large as 0.5 seconds, an unlikely case for modern computers

5 CONCLUSIONS

The computer response time for missile guidance (and other computer algorithms in the missile airframe) is generally ignored in previous papers in the open literature. This paper studies the impact of computational delay for both single-missile single-target system and multiple-missile multiple -target system.

Future work may include a more realistic missile and target model, noise in the sensor, a more sophisticated online cooperation algorithm, and additional physical limitations. For the latter, this paper uses a simplified missile and target model, assuming that the actuators are perfect without internal mechanical delay, and assuming that the missile and target airframe model are first order systems. A more realistic model could be used for more accurate results.

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