CONSTRUCTIVE PLACEMENT OF GENERAL BLOCKS IN VLSI UNDER UNCERTAINTIES IN THE POSITION OF PORTS

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ABSTRACT

Placement problems are in many cases solved at a very early stage of the design cycle when only a rough estimate of the blocks' geometry is available. Hence, the placement can be viewed as a problem of optimization under uncertainties. One type of uncertainties that may arise in the physical design process is in the position of ports. We first present a mathematical framework to model these uncertainties, and then solve the associated stochastic optimization problem. The solution is obtained by proving that the stochastic optimization problem can be reduced to an equivalent deterministic one. Then a normal law of distribution of the placement cost is proved. Finally, a post optimization phase that resolves the uncertainties is suggested.

Index Terms - IC layout, Constructive placement, Physical design, Stochastic programming.

1. INTRODUCTION

Many algorithms have been proposed to solve the placement problem of *general blocks*. Among them are the *constructive* algorithms in which *son blocks* are *selected* and then *located* one at a time, within the area of a *father block*. All the blocks are assumed to have a rectangular shape and they communicate with each other through *ports* on their boundaries.

Typically, the solution of the general block placement problem is attempted at a very early stage of the chip design process. At this stage there is usually some uncertainty regarding the final geometry of the blocks. The uncertainty can be in the exact area of a son block, its aspect ratio or in the position of its ports. Most existing models for the placement problem are deterministic; it is desirable however, to consider the above uncertainties when making decisions at the early stages of the design cycle.

In this paper we limit our discussion to the uncertainty that may arise in the position of ports. We attempt to find the optimal block configuration (under some constructive placement algorithm) when this uncertainty is considered.

2. OUTLINE OF THE DETERMINISTIC PLACEMENT PROCESS

Given the geometry of the father block B_o and its b son blocks B_i, $1 \le i \le b$, and the logical interconnections between their ports through several *nets*, the problem of the optimal placement of general blocks is to minimize a nonnegative real function $C(\Phi_1, x_1, y_1, \ldots, \Phi_b, x_b, y_b)$, where Φ_i is the position transformation of the i-th son block (generally, there are eight possible transformations resulting from two reflections and four rotations), and (x_i, y_i) is the location of its center within the coordinates of B_o. The function C is the cost of a complete valid placement (i.e., blocks do not overlap). In constructive placement the solution is obtained incrementally. At every step we first select a yet unplaced block and then we attempt to locate it optimally within the available free area. This free area is decomposed into maximal rectangles called *prime free rectangles* (PFRs), whose number is $O(b^2)$. Obviously, for every valid location of a new block there exists at least one PFR that must contain that block. Therefore, it has been suggested to solve the minimization problem for each PFR separately, and then select the solution that yields the lowest cost [4].

Assume that a block B_k can be located successfully in a PFR, and let R denote the feasible rectangle (within this PFR) for positioning the center of B_k, $R = \{(x,y) | a \le x \le b,\ c \le y \le d\}$. Assume further that B_k should be connected to p ports of the father and the already placed blocks, and let (x_i, y_i), $1 \le i \le p$, be their positions in the father block (B_o) coordinate system. Let (u_i, v_i), $1 \le i \le p$, denote the positions of the corresponding p ports of B_k in the B_k coordinate system, where (0,0) is its center position. We are looking for $(x,y) \in R$ such that if we locate there the center of B_k, the contribution $f(x,y)$ to the cost of the current partial placement is minimized, that is, we attempt to solve the following problem:

$$\text{minimize } f(x,y) = \sum_{i=1}^{p} w_i[(x + u_i - x_i)^2 + (y + v_i - y_i)^2], \quad (1)$$

$$\text{subject to: } a \le x \le b, \quad c \le y \le d, \quad (2)$$

where w_i is a weight assigned to the net containing the ports located at (x_i, y_i) and (u_i, v_i). (Weights are assigned to nets according to their relative significance, e.g., [4] .) Equations (1)-(2) are a convex program and can be solved analytically [1] .

3. *UNCERTAINTY IN THE POSITION OF PORTS*

Let us now assume that the positions of ports are not determined in advance but can be varied within given rectangles. Since ports are usually located on the boundary of blocks, the rectangles are degenerated into intervals. This uncertainty may arise in the early stages of a top-down design process, when the final location of the ports may be affected by the relative position of the son blocks within the father block, or by the block's internal layout which is not determined yet. Another motivation for introducing these uncertainties is that, a port corresponding to a wide bus can be considered as uniformly distributed in a finite interval on the block's edge.

To model the above uncertainty we assume that the x and y coordinates of a port are independent random variables and consequently, $f(x,y)$ in (1) is a random variable too. The program given in (1)-(2) is now a stochastic programming problem [3] and we may therefore minimize the expected value of $f(x,y)$. The solution of the stochastic program is given by the following theorem:

Theorem: Let x_i, u_i, y_i and v_i that appear in (1) be distributed in the intervals $[\alpha_i^1, \alpha_i^2]$, $[\beta_i^1, \beta_i^2]$, $[\gamma_i^1, \gamma_i^2]$ and $[\delta_i^1, \delta_i^2]$, $1 \le i \le p$, respectively, with some probability density functions (vanishing outside the intervals). Then, minimizing the expected value of the stochastic function $f(x,y)$ in (1) is reduced to the minimization of the deterministic function $f(x,y)$ where x_i, u_i, y_i and v_i are replaced by their expected values.

Proof: Since the expected value is a linear functional, we obtain from (1)

$$E[f(x,y)] = \sum_{i=1}^{p} w_i E[(x + u_i - x_i)^2 + (y + v_i - y_i)^2]. \quad (3)$$

After some manipulations we obtain

$$E[f(x,y)] = \sum_{i=1}^{p} w_i \quad (4)$$

$$\{[x^2 + E(u_i)^2 + E(x_i)^2 + \sigma^2(u_i) + \sigma^2(x_i) + 2xE(u_i) - 2xE(x_i) - 2E(u_i)E(x_i)] + [y^2 + E(v_i)^2 + E(y_i)^2 + \sigma^2(v_i) + \sigma^2(y_i) + 2yE(v_i) - 2yE(y_i) - 2E(v_i)E(y_i)]\},$$

where σ^2 denotes the variance. To solve the stochastic program we have to differentiate (4) with respect to x and y. That is

$$\frac{\partial E[f(x,y)]}{\partial x} = 2\sum_{i=1}^{p} w_i[x + E(u_i) - E(x_i)], \quad (5)$$

and

$$\frac{\partial E[f(x,y)]}{\partial y} = 2\sum_{i=1}^{p} w_i[y + E(v_i) - E(y_i)]. \quad (6)$$

The partial derivatives of $E[f(x,y)]$ have the same form as those of $f(x,y)$, except that x_i, u_i, y_i and v_i are replaced by their expected values. Hence the assertion is satisfied. ■

The above theorem says that though the location of ports within blocks is unknown, an optimal placement can be performed by considering the mean of each location as if it was the exact location of the port. Note that this is similar to the situation that occurs in stochastic optimal control. In case of certain stochastic linear plant equation, the optimal control policy is obtained by solving the related deterministic system, where the random variables are replaced by their expected values. This is known as the *certainty equivalence principle* [5] .

We are also interested in the distribution law of the stochastic function $f(x,y)$. The importance of this distribution stems from the fact that $f(x,y)$ reflects the utilization of a placement and one might wish to know the "risk" that he/she takes when letting the position of ports be undetermined. We claim that as the number of the nets increases, it approaches a normal distribution. This can be proved by showing that the contributions of nets to the cost of a complete placement are independent random variables that satisfy the conditions of the Central Limit Theorem [2] . Fig. 1 depicts the optimal placement of a 16 blocks and 16 nets example where ports are uniformly distributed in their given intervals. This placement was obtained deterministically (following the Theorem). The ports were then allowed to distribute uniformly in their intervals and the resulting probability density function of the placement cost (obtained after drawing a sample of size 10000) is shown in Fig. 2. The mean and standard deviation that were calculated from the observations are indicated in the figure.

The moments of the cost distribution function have also been derived analytically (based on the parameters of the ports' distribution functions), yielding $E(COST) = 1.52071 \times 10^9$ and $\sigma^2(COST) = 3.28146 \times 10^{15}$, which are very close to the moments derived from the random sampling in Fig. 2.

4. POST OPTIMIZATION

As the design proceeds, the positions of some ports are determined by design constraints, while others may be located arbitrarily in their intervals of distribution. The optimal positioning of the latter is called *post optimization*. Since two different nets have no common ports and the x and the y coordinates do not interfere in the objective function, the post optimization can be accomplished separately for the x and y coordinates of every net. Consider a net whose ports' coordinates x_i, $1 \le i \le n$, are distributed in the intervals $[\alpha_i, \beta_i]$. Then, the post optimization problem is given by:

(7) minimize $f(x_1, ..., x_n) = \sum_{i,j=1}^{n} (x_i - x_j)^2$,

(8) subject to: $\alpha_i \le x_i \le \beta_i$, $1 \le i \le n$,

which is a quadratic programming problem and can be solved [1] .

ACKNOWLEDGEMENT: A part of this research was performed while S. Wimer was with the Design Center of National Semiconductor in Tel-Aviv. The Design Center support of this research is gratefully acknowledged.

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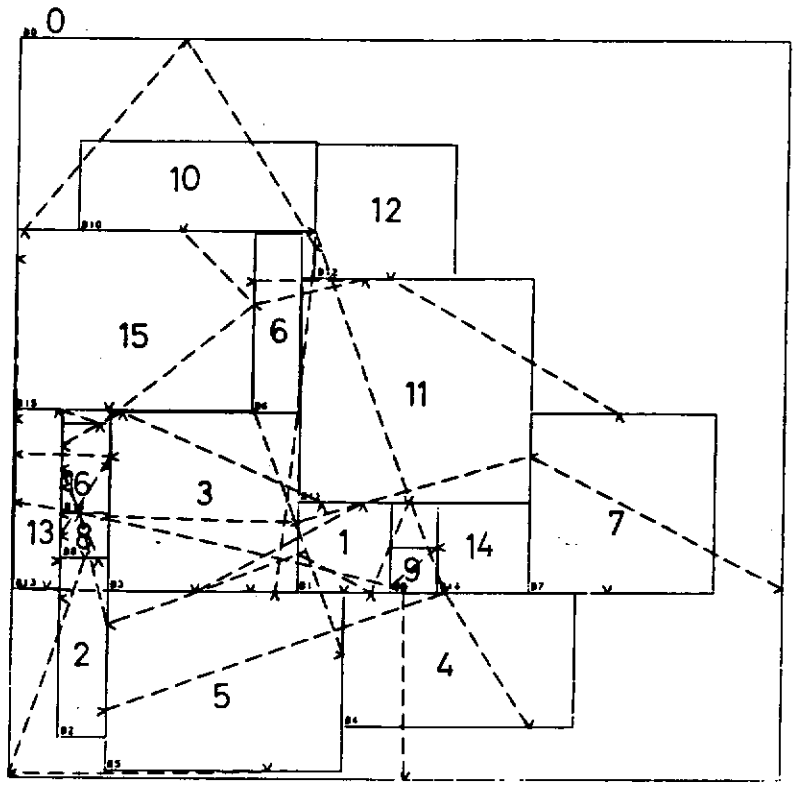

SED cost· 1.46E+09

Figure 1: Optimal placement of a 16 blocks and 16 nets example, obtained by deterministic optimization.

mean: 1.52060×10^9

variance: 3.27108×10^{15}

lowest observed value: 1.36868×10^9

highest observed value: 1.74197×10^9

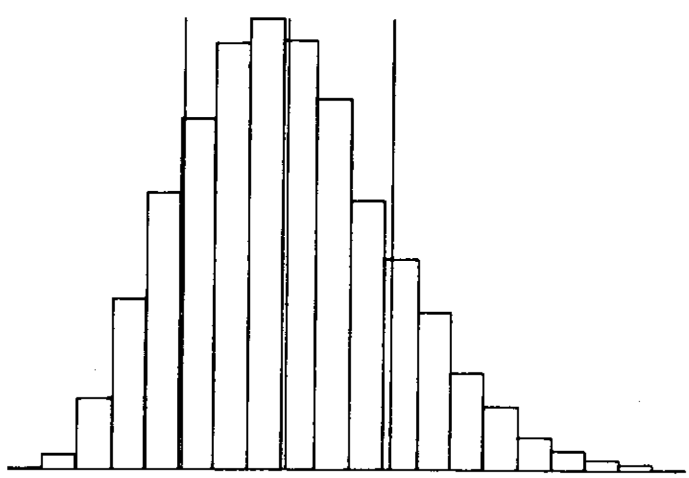

Figure 2: Distribution of placement cost of 16 blocks and 16 nets example.