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A Continuous-Parameter Markov Model and Detection Procedures for Intermittent Faults

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Abstract—A continuous-parameter Markov model for intermittent faults in digital systems is presented. This continuous model is more realistic than discrete-parameter models previously presented by other authors. The results obtained using the proposed model can be reduced to those obtained from the previous models, by using appropriate approximations.

Two testing strategies are considered, continuous testing for combinational networks and repetitive testing for both combinational and sequential networks while only the latter strategy was considered in earlier studies. Next, optimal detection experiments for both testing strategies are developed and the optimization problem is shown to be equivalent to a nonlinear programming problem.

Index Terms—Continuous-parameter Markov model, continuous testing, intermittent faults, fault detection, optimal testing experiments, repetitive testing.

I. INTRODUCTION

Most of the faults occurring in digital systems are intermittent faults [1], [2], yet very little theoretical work has been done on modeling these faults and designing detection experiments for them.

An intermittent fault is a fault which, when existing in the system, may be *active* at one instant of time causing a malfunction of the system or may be *inactive* at another instant allowing the system to operate correctly. Thus, unlike permanent faults, for intermittent faults we distinguish between the *existence* of an intermittent fault and its being *active*. The system is said to be in the Fault Active state (FA) if a fault existing in the system is active and it is said to be in the Fault Not Active state (FN) if the fault is existing but inactive.

We will restrict our attention to well-behaved and signal-independent intermittent faults [3]. A fault is said to be well-behaved if during the application of a test pattern, the system behaves as if either it is fault-free or a permanent fault exists. A fault is said to be signal-independent if its being active is independent of the signal values present in the system.

Two probabilistic models for describing the behavior of intermittent faults were presented in the literature. The first model is a discrete parameter Markov model of the first order which was introduced by Breuer [3]. The second one is a zero-order Markov model introduced by Kamal and Page [6] and used later by Savir [4] and by Koren and Kohavi [5].

A more realistic model for intermittent faults is proposed here. This model is a continuous-parameter Markov model which is convenient to use and removes some of the restrictions faced when using the discrete models. Using appropriate approximations, the relations obtained from the continuous model are easily reduced to those obtained from the previous discrete models.

While using the discrete models, only repetitive testing is considered, i.e., detection tests are applied repeatedly until a decision criterion is satisfied. The continuous model enables us to consider another testing strategy, besides repetitive testing the continuous testing, in which a test is applied continuously until a given confidence level is achieved. Both testing strategies are analyzed in Section III.

In the last section optimal detection experiments for both testing strategies are presented for the case where the system may have one out of several possible intermittent faults.

II. THE CONTINUOUS PARAMETER MODEL

The continuous-parameter Markov model for intermittent faults presented next is a generalization of the discrete-parameter model introduced by Breuer [3]. In the discrete parameter model shown in Fig. 1, r and s are the one time-step transition probabilities, e.g., s is the probability of going to the FA state at time t_{q+1} given that the system was in FN state at time t_q , r is the probability of remaining in FA state. Clearly, the estimated values for the one-step transition probabilities depend upon the time-step selected; and they have to be reestimated if this time-step is changed. Since the time-step is not an attribute of the fault but merely depends upon the clock rate of the testing device, it is desirable to have a model whose parameters are independent of the time-step.

In order to devise such a model, consider the transition probabilities between the two states for an infinitesimal time-step Δt . Clearly, these probabilities should increase as the time-step increases. Hence, as a first approximation, we select transition probabilities which depend linearly on the time-step Δt , i.e., the probability for going from FN state at time t to FA state at time $t + \Delta t$ is $\lambda \Delta t$, and the probability for going from FA state at time t to FN state at time $t + \Delta t$ is $\mu \Delta t$, as depicted in Fig. 2. This model is a well-known continuous-parameter Markov model [7] and expressions for the transition probabilities have been derived. Let 0 and 1 denote the states FN and FA, respectively and let $P_{i,j}(t)$ denote the probability for going from state i at time t_0 to state j at time $t_0 + t$. The equations for these probabilities are [7]

$$P_{0,1}(t) = \frac{\lambda}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}) \quad (1)$$

$$P_{0,0}(t) = 1 - P_{0,1}(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu} e^{-(\lambda + \mu)t} \quad (2)$$

$$P_{1,0}(t) = \frac{\mu}{\lambda + \mu} (1 - e^{-(\lambda + \mu)t}) \quad (3)$$

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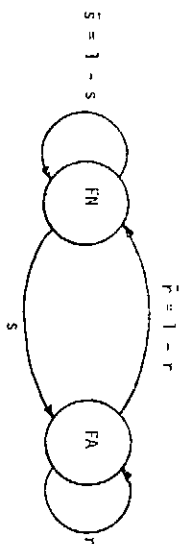


Fig. 1. The discrete-parameter Markov model.

$$P_{1,1}(t) = 1 - P_{1,0}(t) = \frac{\lambda}{\lambda + \mu} + \frac{\mu}{\lambda + \mu} e^{-(\lambda + \mu)t}. \quad (4)$$

An important property of this model is that the time period during which the system stays in state FA (FN) is exponentially distributed with mean $1/\mu(1/\lambda)$.

Equations (1)-(4) can be compared to the corresponding equations for the discrete-parameter Markov model. Let the transition probabilities s and $\bar{r}$ for the latter model be defined for a time-step Δt which is sufficiently small so that a meaningful comparison can be made. In this case $s = \lambda \Delta t$ and $\bar{r} = \mu \Delta t$. Let $n = t/\Delta t$ denote the number of time-steps in the time period t . The n -step transition probability from state 0 to state 1 for the discrete-parameter Markov model is [7]

$$P_{0,1}(n) = \frac{s}{s + \bar{r}} [1 - (1 - \bar{r} - s)^n]. \quad (5)$$

From (1) we have

$$\begin{aligned} P_{0,1}(t) = n \Delta t &= \frac{\lambda}{\lambda + \mu} (1 - \exp [-(\lambda + \mu)n \Delta t]) \\ &= \frac{s}{s + \bar{r}} [1 - \exp [-(\bar{r} + s)n]] \end{aligned}$$

If the exponent $e^{-(\bar{r} + s)}$ is approximated by $(1 - \bar{r} - s)$, then $P_{0,1}(t)$ becomes the same as $P_{0,1}(n)$ for the discrete model. In a similar way the other three probabilities can be compared. Consequently, the results obtained by using the discrete model can be regarded as approximation to those derived by using the continuous model.

Similarly, (1)-(4) can be compared to the probabilities for the zero-order Markov model [6] as follows. From (1)-(4) the steady-state transition probabilities for $t \rightarrow \infty$ are

$$P_{0,1}(\infty) = P_{1,1}(\infty) = \frac{\lambda}{\lambda + \mu} \quad (6)$$

$$P_{0,0}(\infty) = P_{1,0}(\infty) = \frac{\mu}{\lambda + \mu} = 1 - \frac{\lambda}{\lambda + \mu}. \quad (7)$$

These equations indicate that the probability of the system being in a particular state is now independent of the previous state. Thus, for a very large time-step, the continuous Markov model reduces to the discrete zero-order Markov model for which $\lambda/(\lambda + \mu)$ is the probability of the fault being active.

III. TESTING STRATEGIES

Faults in digital systems are detected by applying test patterns to the system's input lines. If the possible fault in the system is permanent, then a single application of a test pattern can determine the existence of the fault. If the possible fault is intermittent, it may occur that the fault exists but is not active when the test is applied. Hence, a wrong conclusion might be drawn regarding the existence of the fault. One way to minimize such wrong conclusions is to apply the test pattern repeatedly until either the fault is detected or the number of test repetitions exceeds a precalculated number k . The number k is determined so that the probability of a wrong conclusion is smaller than some prespecified ϵ . This testing strategy is called *repetitive testing*. Another testing strategy is to

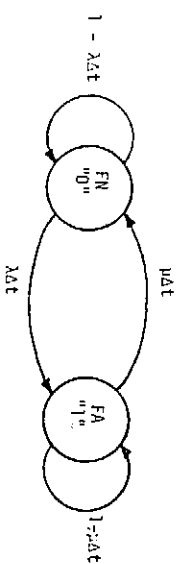


Fig. 2. The continuous-parameter Markov model.

apply the test pattern continuously until either the fault is detected or the testing time exceeds a precalculated time which is determined similarly to the number k . This testing strategy is called *continuous testing* and is clearly applicable only when there is no synchronizing clock in the system. Both testing strategies are analyzed below.

Continuous Testing

To increase our confidence in the results of the testing procedure, we have to minimize the probability that an intermittent fault exists and is not detected. The test pattern is applied to the system continuously from time t_0 to time $t_0 + s$ and the testing is terminated prior to $t_0 + s$ if the fault is detected. The maximum testing time s is determined so that the above probability is smaller than some prespecified ϵ , namely,

$$\Pr \{ \text{The fault exists and is not detected during the interval } [t_0, t_0 + s] \} \leq \epsilon. \quad (8)$$

This probability equals $\Pr \{A \cap B \cap C\}$ where A , B , and C are the events

A = The fault exists.

B = The fault is inactive at time t_0 .

C = The fault remains inactive from time t_0 to $t_0 + s$. (9)

To calculate the probability in (8) we use the relation

$$\Pr \{A \cap B \cap C\} = \Pr \{C/A \cap B\} \cdot \Pr \{B/A\} \cdot \Pr \{A\}. \quad (10)$$

For our model the probability $\Pr \{C/A \cap B\}$ equals $e^{-\lambda s}$ [7]. To determine the probability $\Pr \{B/A\}$ assume that the fault existed for a long time prior to t_0 so that the steady-state probability from (7) can be used. Substituting these two probabilities into (8) and denoting $\Pr \{A\}$, the *a priori* probability, by p yields the following inequality

$$e^{-\lambda s} \frac{\mu}{\lambda + \mu} \cdot p \leq \epsilon. \quad (11)$$

Consequently, the maximum testing time required is

$$s = \frac{1}{\lambda} \ln \frac{\mu p}{\epsilon(\lambda + \mu)}. \quad (12)$$

Notice that such an expression for continuous testing cannot be derived using discrete parameter models.

Repetitive Testing

Repetitive testing can be used for detecting intermittent faults in both combinational and synchronous sequential networks. In the case of combinational networks, only one test pattern is required to detect the fault and this test pattern is applied repeatedly by a clocked tester. In the case of synchronous sequential networks, a sequence of v test patterns ($v \geq 1$) is required to detect the fault and propagate it to the output lines. One test pattern samples the fault while the other patterns in the sequence either initialize the network or propagate the fault to the output. Let t_i be the time

period between successive samples of the network. Clearly, $t_1 = v \times$ clock period.

For repetitive testing we have to determine k , the maximum number of test repetitions, as the minimum number satisfying inequality (8). Using relation (10) and the fact that the network is sampled k times with sampling period t_1 , we obtain the following inequality

$$[P_{0,0}(t_1)]^{k-1} \frac{\mu}{\mu + \lambda} \cdot p \leq \epsilon. \quad (13)$$

The first term is the probability $\Pr \{C_i/A \cap B\}$ where C_i is the event the fault remains inactive at the time instants $t_0 + t_1, t_0 + 2t_1, \dots, t_0 + (k-1)t_1$. Substituting $P_{0,0}(t_1)$ from (2) and solving for the number of test repetitions we obtain

$$k \geq \frac{\ln \left[\frac{\epsilon}{p} \frac{\mu + \lambda}{\mu} \right]}{\ln \left[\frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda + \mu)t_1} \right]} + 1. \quad (14)$$

The resulting testing time is $k \cdot t_1$.

Example: A digital system has an intermittent fault with *a priori* probability $p = 0.1$ and parameters $\mu = 10^2/\text{ms}$, $\lambda = 1/\text{ms}$. We want to calculate the testing time needed for $\epsilon = 10^{-6}$ (i.e., the probability that a fault exists but is undetected should be less than 10^{-6}). If the unit can be tested continuously, then from (12) we have

$$s = \frac{1}{\lambda} \ln \frac{\mu p}{\epsilon(\lambda + \mu)} = 11.49 \text{ ms.}$$

If repetitive testing is to be used, then assuming that the unit is a combinational network and the clock period is 10^{-2} ms, we have $t_1 = 10^{-2}$ ms. Substituting in (14) we obtain $k \geq 1822.6$, thus, $k = 1823$ and the testing time is $k \cdot t_1 = 18.23$ ms. Assuming that the unit-under-test is a synchronous sequential network and the testing sequence consists of $v = 10$ test patterns, the sampling time is $t_1 = 10^{-1}$ ms. By (14) the number of repetitions is $k \geq 1157.1$, hence, $k = 1158$ and the testing time is $k \cdot t_1 = 115.8$ ms.

IV. OPTIMAL FAULT DETECTION EXPERIMENTS

A digital system may have, in general, several possible intermittent faults, $f_1, f_2, f_3, \dots, f_m$. Each of these intermittent faults, when active, is equivalent to a permanent fault which may be a single or multiple stuck-at type fault. We assume that at most one intermittent fault exists in the system at any instant of time and that for these n possible faults we are given a fault matrix $R = (r_{ij})$. The fault matrix consists of n rows corresponding to n different faults and m columns corresponding to m possible test patterns $T_1, T_2, T_3, \dots, T_m$ (or test sequences in the case of synchronous sequential networks). The elements of matrix R are defined as follows:

$$r_{ij} = \begin{cases} 1, & \text{if } f_i \text{, when active, is detected by } T_j, \\ 0, & \text{otherwise.} \end{cases}$$

With every fault f_i , we associate an *a priori* probability of existence in the system, denoted by p_i , and the parameters λ_i and μ_i . Consequently, for every fault f_i we have a set of transition probabilities $P_{0,0}^{(i)}(t)$, $P_{1,1}^{(i)}(t)$, and $P_{1,0}^{(i)}(t)$ given by (1)-(4).

A detection experiment is an experiment in which the test patterns $T_1, T_2, \dots, T_m$ are applied sequentially to the system, so that T_j is applied repeatedly k_j times or continuously for the time s_j and the probability of a wrong conclusion (fault exists but is not detected) is smaller than some prespecified ϵ . An optimal detection experiment is a detection experiment minimizing the total

testing time. In the following paragraphs, optimal detection experiments for continuous testing and repetitive testing are designed.

Continuous Testing

To design an optimal experiment for continuous testing we have to determine the testing periods $s_1, s_2, \dots, s_m$ so that inequality (8) is satisfied and the total testing time $\sum_{j=1}^m s_j$ is minimized. Since only one fault, at most, is assumed to exist in the system, we can rewrite inequality (8) in the following way:

$$\sum_{i=1}^n \{\Pr \text{ The fault } f_i \text{ exists and}$$

was not detected by the experiment}\} \leq \epsilon

or

$$\sum_{i=1}^n \Pr \{\text{The fault } f_i \text{ was not detected by}$$

$$T_1, T_2, \dots, T_m / f_i \text{ exists}\} \times \Pr \{f_i \text{ exists}\} \leq \epsilon. \quad (15)$$

The first probability in (15) can be written as a product of conditional probabilities while the second probability is the *a priori* probability p_i , thus yielding

$$\sum_{i=1}^n p_i \prod_{j=1}^m \Pr \{f_i \text{ was not detected by } T_j / f_i \text{ was not}$$

detected by the previously applied tests

$$T_1, T_2, \dots, T_{j-1} \cap f_i \text{ exists}\} \leq \epsilon. \quad (16)$$

The conditional probabilities in (16) are evaluated as follows:

$\Pr \{f_i \text{ was not detected by } T_j / f_i \text{ was not detected by}$

$$T_1, T_2, \dots, T_{j-1} \cap f_i \text{ exists}\} = [P^{(i)} \exp(-\lambda_i s_j)]^{r_{ij}} \quad (17)$$

where $P^{(i)}$ is the probability that the fault f_i was inactive when T_j was first applied, given that f_i exists. To prove (17) we note that if T_j does not test for f_i (i.e., $r_{ij} = 0$) the probability in (17) is equal to 1. If T_j tests for f_i (i.e., $r_{ij} = 1$) then the same reasoning as for the probability in (11) results in the expression $P^{(i)} e^{-\lambda_i s_j}$.

Substituting (17) in (16), we obtain

$$\sum_{i=1}^n p_i \prod_{j=1}^m [P^{(i)} \exp(-\lambda_i s_j)]^{r_{ij}} \leq \epsilon. \quad (18)$$

$P^{(i)}$ depends upon the time t^* elapsed from the application of the most recent test which tests for f_i until the first application of T_j . More explicitly, $P^{(i)} = P_{0,0}^{(i)}(t^*)$. If T_{j-1} tests for f_i then $t^* = 0$ and $P^{(i)} = P_{0,0}^{(i)}(0) = 1$; if T_j is the first test that tests for f_i then $t^* \rightarrow \infty$ and $P^{(i)} = P_{0,0}^{(i)}(\infty) = \mu_i / (\mu_i + \lambda_i)$. Since $P_{0,0}^{(i)}(t)$ is monotonic, we have

$$\frac{\mu_i}{\mu_i + \lambda_i} \leq P^{(i)} \leq 1. \quad (19)$$

In practice $\mu_i \gg \lambda_i$, otherwise the intermittent fault can be considered as a permanent fault during the relatively short testing period. Hence, the upper and lower bounds in (19) are nearly equal and we may, for simplicity, replace (18) by the following inequality

$$\sum_{i=1}^n p_i \prod_{j=1}^m \exp(-\lambda_i s_j r_{ij}) \leq \epsilon \quad (20)$$

or

$$\sum_{i=1}^n p_i \exp\left(-\lambda_i \sum_{j=1}^m s_j r_{ij}\right) \leq \epsilon. \quad (21)$$

Inequality (21), the conditions $s_j \geq 0$ ($j = 1, 2, \dots, m$) and the total testing time to be minimized $\sum_{j=1}^m s_j$ form a nonlinear programming problem.

If the computation time is to be reduced, the above problem can be reformulated by dividing ϵ into n parts thus enabling the decomposition of inequality (21) into n inequalities.

$$p_i \exp \left(-\lambda_i \sum_{j=1}^n s_j r_{ij} \right) \leq \frac{\epsilon}{n}; \quad i = 1, 2, \dots, n \quad (22)$$

or

$$\sum_{j=1}^n s_j r_{ij} \geq \frac{1}{\lambda_i} \ln \left(\frac{n p_i}{\epsilon} \right); \quad i = 1, 2, \dots, n. \quad (23)$$

Hence, the nonlinear programming problem reduces to a linear programming problem whose solution is not necessarily minimal but for which the required computation time is considerably shorter.

Repetitive Testing

To design an optimal detection experiment for repetitive testing, we have to determine the numbers of applications $k_1, k_2, \dots, k_m$ of the tests $T_1, T_2, \dots, T_m$, respectively, so that inequality (16) is satisfied and the total testing time $\sum_{j=1}^m k_j t_j^{(0)}$ is minimized, where $t_j^{(0)}$ is the time between two successive samplings of the network when T_j is applied. Reasoning on the same lines as for continuous testing results in

$$\sum_{i=1}^n p_i \prod_{j=1}^m (P_{0,0}^{(0)}(t_j^{(0)})^{k_j-1})^{r_{ij}} \leq \epsilon. \quad (24)$$

Using the upper bound of $P_j^{(0)}$ from (19), as for continuous testing, we replace (24) by the following inequality

$$\sum_{i=1}^n p_i \prod_{j=1}^m [P_{0,0}^{(0)}(t_j^{(0)})^{k_j-1}]^{r_{ij}} \leq \epsilon. \quad (25)$$

Defining $U_{ij} = -\ln [P_{0,0}^{(0)}(t_j^{(0)})]$ and substituting in (25) results in

$$\sum_{i=1}^n p_i \exp \left[-\sum_{j=1}^m U_{ij} (k_j - 1) r_{ij} \right] \leq \epsilon. \quad (26)$$

Inequality (26), the condition: all k_j 's are nonnegative integers and the total testing time to be minimized form an integer programming problem whose solution is the optimal detection experiment. If the computation time is to be reduced, the problem can be reformulated by decomposing inequality (26) into n inequalities, yielding

$$\sum_{j=1}^m U_{ij} (k_j - 1) r_{ij} \geq \ln \left(\frac{n p_i}{\epsilon} \right); \quad i = 1, 2, \dots, n. \quad (27)$$

Example: A unit may have one out of three intermittent faults f_1, f_2 , and f_3 with *a priori* probabilities $p_1 = p_2 = p_3 = 0.1$; and parameters $\mu_1 = \mu_2 = \mu_3 = 10^2/\text{ms}$ and $\lambda_1 = \lambda_2 = \lambda_3 = 1/\text{ms}$. The fault matrix for this unit is

$$R = \begin{matrix} & \begin{matrix} f_1 \\ f_2 \\ f_3 \end{matrix} & \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \\ \begin{matrix} T_1 \\ T_2 \\ T_3 \end{matrix} & & \end{matrix}$$

If the unit can be tested continuously, from (23) we have the following set of inequalities, for $\epsilon = 10^{-6}$:

$$\begin{aligned} s_1 + s_3 &\geq 12.6, \\ s_2 + s_3 &\geq 12.6, \\ s_1 + s_2 &\geq 12.6. \end{aligned}$$

The total testing time $\sum_{j=1}^3 s_j$ is minimal if $s_1 = s_2 = s_3 = 6.3$ ms. Hence, $\sum_{j=1}^3 s_j = 18.9$ ms. If, instead of using the three tests T_1, T_2 , and T_3 , the minimal detection set for permanent faults consisting of only T_1 and T_2 is used, we obtain the inequalities $s_1 \geq 12.6$ and $s_2 \geq 12.6$. Consequently, the total testing time is $s_1 + s_2 = 25.2$ ms.

If the unit-under-test is a synchronous sequential circuit with T_1, T_2 , and T_3 consisting of 10, 17, and 5 test patterns, respectively, a clock period of 10^{-2} ms, then from (27) we have

$$\begin{aligned} 9.950 \times 10^{-3} k_1 + 9.886 \times 10^{-3} k_3 &\geq 12.6, \\ 9.950 \times 10^{-3} k_2 + 9.886 \times 10^{-3} k_3 &\geq 12.6, \\ 9.950 \times 10^{-3} k_1 + 9.950 \times 10^{-3} k_2 &\geq 12.6, \end{aligned}$$

and the objective function to be minimized is

$$10 \times 10^{-3} k_1 + 17 \times 10^{-3} k_2 + 5 \times 10^{-3} k_3.$$

The optimal solution is $k_1 = 1270, k_2 = 0$, and $k_3 = 1278$. The total testing time required is $\sum_{j=1}^m t_j^{(0)} k_j = 190.90$ ms.

Using the minimal detection set for permanent faults, $\{T_1, T_2\}$ results in the following solution $k_1 = 1269, k_2 = 1269$ and the total testing time is 342.63 ms. From these results we can see that for both testing strategies, the optimal detection experiment for intermittent faults may be different from the optimal experiment for permanent faults as was observed by Savir [4].

V. CONCLUSIONS

A continuous-parameter Markov model for intermittent faults has been proposed. This model, which is more realistic than earlier discrete models, enables us to consider continuous testing of combinational systems in addition to the repetitive testing strategy. Both testing strategies have been analyzed and optimal detection experiments for them have been developed.

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