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Reliability Analysis of *N*-Modular Redundancy Systems with Intermittent and Permanent Faults

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***Abstract*—It is well known that static redundancy techniques are very efficient against intermittent (transient) faults which constitute a large portion of logic faults in digital systems. However, very little theoretical work has been done in evaluating the reliability of modular redundancy systems which are subject to intermittent malfunction occurrences. In this paper we present a statistical model for intermittent faults and use it to analyze the reliability of NMR systems in mixed intermittent and permanent fault environments.**

***Index Terms*—Digital system, intermittent fault, modular redundancy, permanent fault, reliability.**

I. Introduction

ALTHOUGH our current knowledge on the conditions causing intermittent faults and their behavior is limited, it is believed that intermittent faults constitute a large portion of the logic faults that occur in digital systems [1]–[5]. Because of the complexity involved in the diagnosis of intermittent faults, these faults are a major cause of digital system downtime. In order to increase the reliability and availability of a system which is subject to intermittent malfunctions we can use static or dynamic redundancy techniques to incorporate fault tolerance into the system [6]–[8].

Recently the problem of intermittent fault recovery in dynamic redundancy systems was studied by Merryman and Avizienis [1] and by Ng and Avizienis [2]. In this paper we study N-modular static redundancy in mixed intermittent and permanent fault environments. We first introduce a Markov model for intermittent faults and use it to analyze the reliability of a nonredundant module. We next define the reliability of an N-modular redundant system which is subject to occurrences of intermittent faults, and we show how to calculate this reliability. Finally, this reliability is compared to the classical expression for the reliability of NMR systems and it is shown that the predicted mission time (i.e., the time the system will operate at or above a given reliability) is considerably larger than the time predicted using the classical results as can be expected.

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II. Modeling Intermittent Faults

An intermittent fault is a physical event (a defect in a component) that manifests itself intermittently in an unpredictable manner [2], [6]–[8]. Thus, an intermittent fault when existing in the system may be *active* at one instant of time causing a malfunction of the system or may be *inactive* at another instant allowing the system to operate correctly. Hence, unlike permanent faults, for intermittent faults we distinguish between the *existence* of a defective component and the *activity* of the logical fault caused by it.

Defects causing intermittent faults come from a variety of sources, e.g., environmental causes, design deficiencies and partially defective components. Each of these sources generates faults in a different stochastic process. Unfortunately, there is no published information on the statistics of these processes and consequently, accurate modeling of intermittent faults is not possible. We adopt here the viewpoint [1]–[5] that most intermittent faults can be characterized by a first-order Markov model which has the advantage of being mathematically tractable and which appears to be consistent with observations on the behavior of intermittent faults [2], [3], [8]. A discrete-parameter two-state Markov model was introduced by Breuer [4] and a continuous-parameter two-state Markov model was presented by Su *et al.* [5]. A similar continuous model was used in [1], [2], and [11]. This model can be described by the state diagram in Fig. 1 where FA is the *Fault Active* state in which the intermittent fault is active and FN is the *Fault Not Active* state in which the existing intermittent fault is inactive. The arcs in the state diagram are marked with the transition rates λ and μ which are assumed to be time independent.

Such a two-state Markov model is adequate for devising testing procedures where the testing time is very short compared to the lifetime of the system. However, when evaluating the reliability, we have to distinguish between defects introduced in the manufacturing process and defects introduced only after a period of use in the field. Thus, a slightly more general Markov model that contains three states will be used, as depicted in Fig. 2. In this state diagram the additional state NE is the fault *Not Existing* state in which the corresponding component is defect-free and no logical fault can be caused by it. The rate of fault occurrences (transitions from the initial state (NE) to the FN state) is denoted by ν.

The Markov model shown in Fig. 2 can easily be reduced

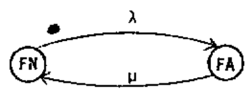

Fig. 1. A two-state Markov model.

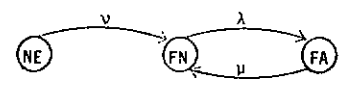

Fig. 2. A three-state Markov model.

to the previous model in Fig. 1 by letting ν approach ∞. Clearly, a permanent fault is also a special case of our model for which $\mu \to 0$ and $\lambda \to \infty$ (or $\nu \to \infty$). Consequently, the reliability expressions for these two special cases can be derived from our final results.

III. Reliability of a Module with Intermittent Faults

A module is a digital system composed of several nonredundant components (e.g., integrated circuit packages) C_i with reliability $R_i(t)$; $i = 1, 2, \cdots, n$ where n is the number of components in the module. Let f_i be the fault that may occur in component C_i and let ν_i, λ_i, and μ_i be the parameters associated with f_i. The reliability of the component C_i, denoted by $R_i(t)$, is defined as the probability that this component operates correctly during the time interval $[0, t]$, i.e.,

$$R_i(t) = \Pr\{\text{the fault } f_i \text{ is inactive in } [0, t]\}. \tag{3.1}$$

The reliability of the module, denoted by $R_M(t)$, is defined as the probability that it operates correctly in $[0, t]$. It is usually assumed that all n components must operate correctly in order for the module to operate correctly and it is further assumed that the occurrences of faults in different components are statistically independent. Hence,

$$R_M(t) = \Pr\{\text{no fault is active in } [0, t]\} = \prod_{i=1}^{n} R_i(t). \tag{3.2}$$

The component reliability is given in the following lemma.

Lemma 3.1:

$$R_i(t) = \frac{\lambda_i}{\lambda_i - \nu_i}\left(e^{-\nu_i t} - \frac{\nu_i}{\lambda_i} e^{-\lambda_i t}\right). \tag{3.3}$$

Proof: $R_i(t) = 1 - \Pr\{f_i \text{ is active at least once in } [0, t]\}$. Let x denote the time instant at which the fault f_i occurs (i.e., the transition from the NE state in Fig. 2 to the FN state takes place), we have

$$\Pr\{f_i \text{ is active at least once in } [0, t]\}$$
$$= \int_0^t \Pr\{f_i \text{ is active at least once in } [x, t]/f_i \text{ occurred at time } x\} \cdot \Pr\{f_i \text{ occurred at time } x\}$$
$$= \int_0^t (1 - e^{-\lambda_i(t-x)})\nu_i e^{-\nu_i x}\, dx$$
$$= 1 - \frac{\lambda_i}{\lambda_i - \nu_i}\left(e^{-\nu_i t} - \frac{\nu_i}{\lambda_i} e^{-\lambda_i t}\right).$$

The lemma now follows. Q.E.D.

Substituting the result of Lemma 3.1 into (3.2) yields

$$R_M(t) = \prod_{i=1}^{n}\left[\frac{\lambda_i}{\lambda_i - \nu_i}\left(e^{-\nu_i t} - \frac{\nu_i}{\lambda_i} e^{-\lambda_i t}\right)\right]. \tag{3.4}$$

If the fault f_i is a permanent fault for which $\mu_i = 0$ and $\lambda_i \to \infty$ we have $R_i(t) = e^{-\nu_i t}$. Similarly, if all the faults are permanent the reliability of the module is

$$R_M(t) = \prod_{i=1}^{n} R_i(t) = e^{-\sum_{i=1}^{n} \nu_i t}. \tag{3.5}$$

According to this well-known expression the failure rate of a nonredundant module which is subject to permanent malfunctions, is given by the sum of the permanent faults' failure rates. Such a simplification is not possible in the case of intermittent faults and we still have to consider each intermittent fault separately.

IV. Reliability of an N-Modular Redundancy System

The well-known triple modular redundant (TMR) system with permanent faults has been thoroughly analyzed. The classical equation for the reliability of a TMR system, denoted by $R_{TMR}(t)$, is

$$R_{TMR}(t) = \Pr\{\text{no two modules are faulty in } [0, t]\}$$
$$= 3R_M^2(t) - 2R_M^3(t) \tag{4.1}$$

where a module is called faulty if at least one permanent fault is present in it. A more general and less pessimistic reliability expression for a TMR system has been developed by Bouricius *et al.* [9] but still only permanent faults were considered. To incorporate intermittent faults in the TMR reliability model we have first to change the definition in (4.1) because of the following reason. For permanent faults in static redundancy systems where no repair or replacement are taking place, a module that becomes faulty at time instant t will remain faulty thereafter. However, if the possible faults are intermittent, it may happen that a fault is active at time instant t_1 causing the module to be faulty, and inactive at a later time instant t_2 ($t_2 > t_1$) allowing the module to become fault-free. This is clearly true if the module is combinational. In the case of a sequential module, unless it is in the output logic, the intermittent fault may place the sequential machine in an erroneous state [10]. Hence, the intermittent fault may have a permanent effect. It has been shown by Wakerly [10] that the correct state of a sequential module can be restored after an intermittent fault occurrence if and only if the machine has a synchronizing sequence. Thus, if the machine has no synchronizing sequence, all the intermittent faults having a permanent effect will be considered as permanent faults (i.e., $\mu_i = 0$). If the machine possesses synchronizing sequences and these sequences occur frequently during the normal operation of the module, the faults can still be considered as intermittent.

We define therefore the reliability of a TMR system with intermittent faults in the following way:

$$R_{TMR}(t) = \Pr\{\text{no two modules are faulty at the same time in } [0, t]\} \tag{4.2}$$

where a module is called faulty if at least one fault is active in it. Note that we have to exclude intermittent faults with high correlation among parallel modules (e.g., intermittent faults caused by a design deficiency and which occur under rare data conditions). For such faults NMR configurations are inefficient.

The conventional approach to derive an expression for the reliability in (4.2) is to draw the complete state diagram of the Markov model for the TMR system and then calculate the system's reliability from this multistate model. The computational complexity of this approach depends mainly upon the number of states in the system's diagram. Hence, reliability analysis of systems with intermittent faults that do not have the same parameters or faults with a more complicated model is limited due to the vast increase in the number of states in the system's diagram. To overcome this limitation we derive an expression for the reliability in a different way which can be used for other models of intermittent faults as well. First we define the following two probabilities to be used later when calculating the reliability in (4.2),

$$Q_i(t) = \Pr\{\text{the fault } f_i \text{ is inactive at time } t\} \tag{4.3}$$

$$Q_M(t) = \Pr\{\text{the module is not faulty at time } t\}. \tag{4.4}$$

Since the faults are assumed to be independent we have the relation

$$Q_M(t) = \prod_{i=1}^{n} Q_i(t). \tag{4.5}$$

The probability $Q_i(t)$ is given in the following lemma.

Lemma 4.1:

$$Q_i(t) = \frac{\mu_i}{\lambda_i + \mu_i} + \frac{\lambda_i}{\lambda_i + \mu_i - \nu_i}\left(e^{-\nu_i t} - \frac{\nu_i}{\lambda_i + \mu_i} e^{-(\lambda_i + \mu_i)t}\right). \tag{4.6}$$

Proof: Let x be the time instant at which the fault f_i occurs (i.e., goes from NE to FN state), thus

$$\begin{aligned} Q_i(t) = \int_0^t & \Pr\{f_i \text{ is inactive at time } t/f_i \text{ occurred at time } x\} \\ & \cdot \Pr\{f_i \text{ occurred at time } x\} \\ & + \Pr\{f_i \text{ is inactive at time } t/f_i \text{ did not occur in } [0, t]\} \\ & \cdot \Pr\{f_i \text{ did not occur in } [0, t]\} \end{aligned}$$

$$\begin{aligned} &= \int_0^t \left[\frac{\mu_i}{\lambda_i + \mu_i} + \frac{\lambda_i}{\lambda_i + \mu_i} e^{-(\lambda_i + \mu_i)(t - x)}\right] \cdot e^{-\nu_i x} \nu_i \, dx + e^{-\nu_i t} \\ &= \frac{\mu_i}{\lambda_i + \mu_i} + \frac{\lambda_i}{\lambda_i + \mu_i - \nu_i}\left(e^{-\nu_i t} - \frac{\nu_i}{\lambda_i + \mu_i} e^{-(\lambda_i + \mu_i)t}\right). \end{aligned}$$

Q.E.D.

Clearly, if the fault f_i is a permanent fault we should obtain

$$Q_i(t) = R_i(t) = e^{-\nu_i t} \tag{4.7}$$

as can be verified from (4.6) when $\mu_i \to 0$ and $\lambda_i \to \infty$. If all the possible faults in the module are permanent we have

$$Q_M(t) = R_M(t) = e^{-\sum_{i=1}^{n} \nu_i t}. \tag{4.8}$$

The probabilities $Q_i(t)$ and $Q_M(t)$ are useful when deriving equations for the reliability of a TMR system. We first derive an equation for the simple case where all the intermittent faults have the same parameters. Next, this equation is extended to NMR systems and finally, we extend the result to the general case where the faults have different parameters.

If all the faults have equal parameters, i.e., $\nu_1 = \nu_2 = \cdots = \nu_n \triangleq \nu$; $\lambda_1 = \lambda_2 = \cdots = \lambda_n \triangleq \lambda$ and $\mu_1 = \mu_2 = \cdots = \mu_n \triangleq \mu$, we obtain

$$Q_M(t) = [Q(t)]^n \tag{4.9}$$

where

$$Q(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu - \nu}\left(e^{-\nu t} - \frac{\nu}{\lambda + \mu} e^{-(\lambda + \mu)t}\right). \tag{4.10}$$

The reliability of the TMR system in this case is given in the following theorem.

Theorem 4.1:

$$\ln R_{TMR}(t) = -6n\lambda \int_0^t \frac{1 - Q_M(t)}{3 - 2Q_M(t)} \cdot \frac{Q(t) - e^{-\nu t}}{Q(t)} \, dt. \tag{4.11}$$

Proof: We define

$$\begin{aligned} \bar{R}_{TMR}(t) &= 1 - R_{TMR}(t) \\ &= \Pr\{\text{at least two modules are faulty at the same time in } [0, t]\}. \end{aligned} \tag{4.12}$$

Let x denote the time instant at which two modules are faulty simultaneously for the first time in $[0, t]$, i.e., one module is already faulty at time x and another module becomes faulty in $[x, x + dx]$. Let these two modules be designated M_1 and M_2, respectively, and since there are $\binom{3}{2} = 6$ ways to select them we obtain

$$\bar{R}_{TMR}(t) = 6 \int_0^t \Pr\{A \cap B \cap C\} \tag{4.13}$$

where A, B, and C are events defined as follows:

$A = M_1$ is faulty at time x;
$B = M_2$ becomes faulty in $[x, x + dx]$;
$C =$ No two modules are faulty at the same time in $[0, x]$.

To calculate the integral in (4.13) we use the relation

$$\Pr\{A \cap B \cap C\} = \Pr\{B/A \cap C\} \cdot \Pr\{A/C\} \cdot \Pr\{C\}. \tag{4.14}$$

The third term in (4.14) is simply $\Pr\{C\} = \bar{R}_{TMR}(x)$ by the definition in (4.2). Since the underlying Markov process is memoryless we may write for the second term in (4.14)

$$\begin{aligned}\Pr\{A/C\} &= \Pr\{M_1 \text{ is faulty at time } x/\text{no two modules are faulty at time } x\}\\ &= \frac{\Pr\{\text{only } M_1 \text{ is faulty at time } x\}}{\Pr\{\text{no two modules are faulty at time } x\}}\\ &= \frac{[1 - Q_M(x)]Q_M^2(x)}{Q_M^3(x) + 3[1 - Q_M(x)]Q_M^2(x)}\\ &= \frac{1 - Q_M(x)}{3 - 2Q_M(x)}. \end{aligned} \tag{4.15}$$

Similarly, the first term in (4.14) is

$$\Pr\{B/A \cap C\} = \Pr\{M_2 \text{ becomes faulty in } [x, x + dx]/M_2 \text{ is not faulty at time } x\}.$$

Applying the principle of total probability we obtain

$$\begin{aligned}\Pr\{B/A \cap C\} &= \sum_{k=0}^{n} \Pr\{A \text{ fault in } M_2 \text{ becomes active in } [x, x + dx]/k \text{ inactive faults exist in } M_2 \text{ at time } x\}\\ &\quad \cdot \Pr\{k \text{ inactive faults exist in } M_2 \text{ at time } x/M_2 \text{ is not faulty at time } x\}\\ &= \sum_{k=0}^{n} \lambda k\, dx\\ &\quad \cdot \Pr\{k \text{ inactive faults exist in } M_2 \text{ at time } x/M_2 \text{ is not faulty at time } x\}. \end{aligned} \tag{4.16}$$

To calculate the conditional probability in (4.16) we employ Bayes' formula

$$\begin{aligned}&\Pr\{k \text{ inactive faults exist in } M_2 \text{ at time } x/M_2 \text{ is not faulty at time } x\}\\ &= \frac{\Pr\{M_2 \text{ is not faulty at time } x/k \text{ inactive faults exist in } M_2 \text{ at time } x\}}{\Pr\{M_2 \text{ is not faulty at time } x\}}\\ &\quad \cdot \Pr\{k \text{ inactive faults exist in } M_2 \text{ at time } x\}\\ &= \frac{P^k \cdot \binom{n}{k} (1 - e^{-\nu x})^k (e^{-\nu x})^{n-k}}{Q_M(x)} \end{aligned} \tag{4.17}$$

where $P = \Pr\{$the fault f is inactive at time x/the fault f exists at time $x\}$. Following the steps in the proof of Lemma 4.1 we obtain

$$P = \frac{Q(x) - e^{-\nu x}}{1 - e^{-\nu x}}. \tag{4.18}$$

Substituting (4.18) into (4.17) and then into (4.16) yields

$$\Pr\{B/A \cap C\} = \sum_{k=0}^{n} \lambda k\, dx \frac{1}{Q_M(x)} \binom{n}{k} (e^{-\nu x})^{n-k} \cdot [Q(x) - e^{-\nu x}]^k. \tag{4.19}$$

After some combinatorial manipulations we obtain

$$\Pr\{B/A \cap C\} = \frac{Q(x) - e^{-\nu x}}{Q(x)} \cdot n\lambda\, dx. \tag{4.20}$$

Now substitute (4.15) and (4.20) into (4.13) using relation (4.14)

$$\bar{R}_{TMR}(t) = 6 \int_0^t R_{TMR}(x) \cdot \frac{1 - Q_M(x)}{3 - 2Q_M(x)} \cdot \frac{Q(x) - e^{-\nu x}}{Q(x)} n\lambda\, dx. \tag{4.21}$$

After differentiating (4.21), dividing by $R_{TMR}(t)$ and integrating again the theorem follows. Q.E.D.

Corollary 4.1: If all n faults are permanent, i.e., $\lambda \to \infty$ and $\mu \to 0$, then (4.11) reduces to the known equation in (4.1).

Proof: To calculate the limit of $R_{TMR}(t)$ we use (4.7), (4.8) and the following equation

$$\lim_{\lambda \to \infty, \mu \to 0} \lambda(Q(t) - e^{-\nu t}) = \nu e^{-\nu t}. \tag{4.22}$$

Substituting in (4.11) yields

$$\begin{aligned}\lim_{\lambda \to \infty, \mu \to 0} \ln R_{TMR}(t) &= -6n \int_0^t \frac{1 - e^{-n\nu t}}{3 - 2e^{-n\nu t}} \cdot \nu\, dt\\ &= -2n\nu t + \ln(3 - 2e^{-n\nu t}). \end{aligned}$$

Hence,

$$\begin{aligned}R_{TMR}(t) &= e^{-2n\nu t}(3 - 2e^{-n\nu t})\\ &= 3R_M^2(t) - 2R_M^3(t). \end{aligned}$$

Q.E.D.

Equation (4.11) is extended now to NMR systems where $N = 2m + 1$, i.e., the reliability $R_{NMR}(t)$ is the probability that at most m modules are faulty at the same time in $[0, t]$.

Theorem 4.2:

$$\ln R_{NMR}(t) = -N\binom{N-1}{m} n\lambda \cdot \int_0^t \frac{[1 - Q_M(t)]^m}{\sum_{k=0}^{m}\binom{N}{k}[1 - Q_M(t)]^k[Q_M(t)]^{m-k}} \cdot \frac{Q(t) - e^{-\nu t}}{Q(t)}\, dt. \tag{4.23}$$

Proof: The proof is similar to that of Theorem 4.1 and is therefore omitted.

In the next theorem we generalize (4.23) to the case where the n faults have different parameters. To specify the faults present in a module at a given time instant t we define a binary vector $B(t) = [b_1(t), b_2(t), \cdots, b_n(t)]$ as follows:

$$b_i(t) = \begin{cases} 1 & \text{if the fault } f_i \text{ exists at time } t \\ 0 & \text{otherwise.} \end{cases} \tag{4.24}$$

For convenience we will omit t as an argument of the binary vector B.

Theorem 4.3:

$$\ln R_{NMR}(t) = -N\binom{N-1}{m} \cdot \int_0^t \frac{[1 - Q_M(t)]^m}{\sum_{k=0}^{m}\binom{N}{k}[1 - Q_M(t)]^k[Q_M(t)]^{m-k}} \cdot P(t)\, dt \tag{4.25}$$

where

$$P(t) = \frac{1}{Q_M(t)} \sum_{B=0}^{2^n-1} \left(\sum_{i=1}^{n} \lambda_i b_i\right) \prod_{i=1}^{n} [Q_i(t) - e^{-\nu_i t}]^{b_i}[e^{-\nu_i t}]^{\bar{b}_i} \tag{4.26}$$

and $\bar{b}_i = 1 - b_i$.

Proof: Following the steps in the proof of Theorem 4.1 we obtain

$$\bar{R}_{NMR}(t) = N\binom{N-1}{m}\int_0^t \Pr\{B/A \cap C\} \Pr\{A/C\}\{\Pr C\} \tag{4.27}$$

where A, B, and C are the events

$A = M_1, M_2, \cdots, M_m$ are faulty at time x.
$B = M_{m+1}$ becomes faulty in $[x, x + dx]$.
$C =$ No $(m + 1)$ modules are faulty at the same time in $[0, x]$.

The three terms in the integral (4.27) are $\Pr\{C\} = R_{TMR}(x)$,

$$\Pr\{A/C\} = \frac{[1 - Q_M(x)]^m}{\sum_{k=0}^{m}\binom{N}{k}[1 - Q_M(x)]^k[Q_M(x)]^{m-k}}$$

and the first term $\Pr\{B/A \cap C\}$ which can easily be shown to be equal to $P(x)\,dx$ where $P(x)$ is defined in (4.26).

Q.E.D.

Corollary 4.3: If all n faults are permanent, i.e., $\lambda_i \to \infty$ and $\mu_i \to 0$, then (4.25) reduces to the known equation for NMR systems

$$R_{NMR}(t) = \sum_{k=0}^{m}\binom{N}{k}[1 - R_M(t)]^k[R_M(t)]^{N-k}. \tag{4.28}$$

Proof: The proof is similar to that of Corollary 4.1 and is therefore omitted.

V. Examples

In this section we calculate the reliability of an NMR system in the following two examples:

1) A TMR system for which all ten possible faults in a module are intermittent faults with the same parameters: $\nu = 0.01$, $\lambda = 1$, and $\mu = 100$.

2) A TMR system and a 5MR system for which six out of the ten possible faults in a module are intermittent faults with the same parameters as in (1); the other four are permanent faults with $\nu = 0.01$, $\lambda = 100$, and $\mu = 0$.

In these two examples we compare numerically the reliability expression (4.25) for N-modular redundancy systems with intermittent faults to the classical expression (4.28) for the reliability of NMR systems with permanent faults. To carry out such a comparison we associate with every intermittent fault (with parameters ν_i, λ_i, and μ_i) an "equivalent" permanent fault (with a failure rate α_i) where "equivalence" means that it takes the same average time for both faults to become active for the first time and cause a malfunction of the module. For a permanent fault this average lifetime is $1/\alpha_i$; for an intermittent fault f_i the average lifetime of the component c_i, whose reliability is given in (3.3), is

$$\int_0^\infty R_i(t)\, dt = \frac{\nu_i + \lambda_i}{\nu_i \lambda_i}. \tag{5.1}$$

Consequently,

$$\alpha_i = \frac{\nu_i \lambda_i}{\nu_i + \lambda_i}. \tag{5.2}$$

In the examples we also compare the reliability of a module with intermittent faults [given in (3.4)] to the reliability of a module with permanent faults, i.e.,

$$R_M(t) = e^{-\sum_{i=1}^{n} \alpha_i t}. \tag{5.3}$$

The results of examples 1) and 2) are plotted in Figs. 3 and 4, respectively. In these graphs the time has been normalized and one time unit is equal to the average lifetime of a module whose reliability is given in (5.3),

$$\int_0^\infty R_M(t)\, dt = \frac{1}{\sum_{i=1}^{n} \alpha_i}. \tag{5.4}$$

The results in Figs. 3 and 4 can be used to calculate the mission time improvement factor MTIF [6], [9]. For exam-

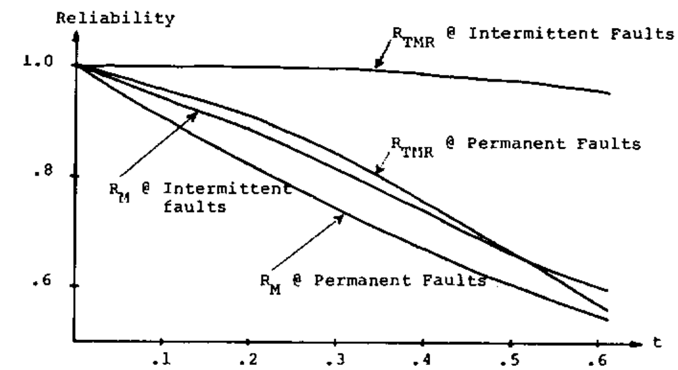

Fig. 3. The reliability of the TMR system in example 1).

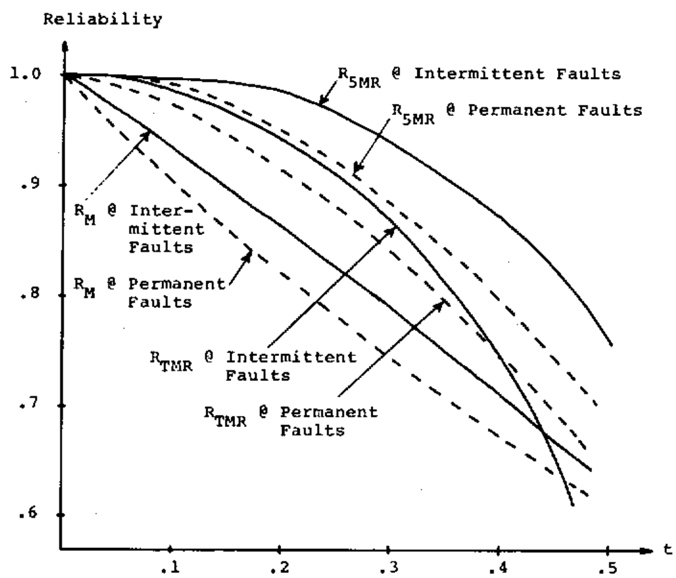

Fig. 4. The reliability of the NMR systems in example 2).

ple, in Fig. 3 the predicted mission time for $R = 0.9$, when (4.28) is used, is 0.81 while the predicted mission time when using the classical expression (4.25) is 0.22. Hence, for $R = 0.9$ we obtain MTIF = 3.68. The improvement in mission time (which is clearly expected) depends upon the parameters μ and λ of the intermittent faults. The larger the ratio μ/λ, the more intermittent are the faults and the greater is the improvement in mission time. A similar analysis of the results in Fig. 4 reveals that the presence of four permanent faults reduces the improvement in mission time. For the TMR system we obtain MTIF = 1.18 (for $R = 0.9$ as before) and for the 5MR system we obtain MTIF = 1.28. These improvements in mission time are intuitively obvious since intermittent faults are active for shorter periods of time compared to their "equivalent" permanent faults. However, this is true only for static NMR systems while for hybrid NMR systems the latency of the intermittent faults can cause a smaller mission time [11]. Even for static redundancy the reliability curve of an NMR system with intermittent faults may go below the reliability curve of an NMR system with the "equivalent" permanent faults as can be seen in Fig. 4. The crossover between the two curves of $R_{TMR}(t)$ occurs when $R = 0.75$. Similar crossovers occur for $R_{5MR}(t)$ in Fig. 4 and $R_{TMR}(t)$ in Fig. 3 but for lower reliabilities and hence, are not shown. The reason for these crossovers is believed to lie in the fact that the reliability $R_I(t)$ of a component with an intermittent fault [given by (3.3)] gets, for very large t, lower than the reliability of the same component with a permanent fault that has the same life time.

VI. Conclusions

N-modular redundancy systems with intermittent faults have been studied in this paper. A new definition of the reliability of such systems is presented and expressions for its evaluation are developed. The method used to calculate the system's reliability is computationally simple and can be employed for various models of intermittent faults. In this study a continuous-parameter three-state Markov model is used to characterize intermittent faults. Special cases of this model are permanent faults and intermittent faults which can be characterized by a two-state model.

Finally, the new reliability expression is compared to the classical expression and it is shown that for static NMR systems there is an improvement in the predicted mission time.

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