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ON THE CONVERGENCE OF THE SIGNAL RELIABILITY OF ITERATIVE AND SEQUENTIAL SYSTEMS

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Abstract

Signal reliability is an accurate measure of digital systems reliability. Unfortunately, its evaluation requires a large number of calculations, especially in the case of sequential systems for which the signal reliability has to be recalculated at every clock pulse. It is shown in this paper that, under certain conditions, the signal reliability of a synchronous sequential system converges to a steady state value within a few clock pulses, thus making the calculation of the signal reliability computationally feasible.

Keywords

Signal reliability, iterative system, sequential system, fault probability, reliability transfer matrix, Markov chain.

1. Introduction

Signal reliability as a measure of digital systems reliability was introduced by Amarel and Brzozowski [1]. Recently, algorithms for the evaluation of this measure have been presented [2–4] and the possible applications of signal reliability have been discussed [2–6]. However, the methods presented in [2] and [4] are restricted to combinational systems and the algorithm in [3] which can be applied to sequential systems is computationally inefficient since it implies recalculation of the system signal reliability at every clock pulse.

In this paper we employ the method in [4] to show that in many cases the signal reliability of iterative systems and of synchronous sequential systems (with some restrictions on the nature of the faults) converges to a steady state value. The existence of such a steady state makes the analysis of sequential system reliability computationally feasible.

2. Preliminaries

The signal reliability of a system depends upon the nature of the possible faults and their probabilities of occurrence. We assume that the possible faults are multiple lead failures which may be either permanent faults or intermittent ones. We denote by S_X the probability of a fault at line X. This probability is in general time dependent and the exact expression for $S_X(t)$ depends upon the model selected for the

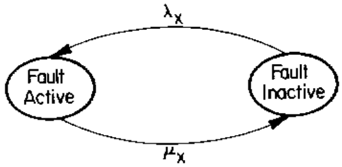

Fig. 1. The continuous-parameter Markov model for intermittent faults.

fault. For permanent faults the most commonly used fault probability function is $S_x(t) = 1 - \exp(-\lambda_x t)$ where λ_x is the failure rate at line X. For intermittent faults we select the continuous-parameter Markov model [7] shown in Figure 1. Here $S_x(t)$ is the probability that the intermittent fault is active at time t and is given by

$$S_x(t) = S_x(0) \exp[-(\lambda_x + \mu_x)t] + \frac{\lambda_x}{\lambda_x + \mu_x} \{1 - \exp[-(\lambda_x + \mu_x)t]\} \qquad (2.1)$$

where $\lambda_x(\mu_x)$ is the transition rate from the fault inactive (active) state to the fault active (inactive) state of the intermittent fault at line X.

The reliability of the system is clearly time-dependent. However, to simplify notation, we shall omit t as an argument of the reliability and failure probability functions, and these functions are understood to be time-dependent.

The presence of faults in the system may cause incorrect logic signals on some lines. Consequently, the signal on any line X in the system may assume one out of four values, namely: correct 0, correct 1, incorrect 0, and incorrect 1. These values are designated by 0, 1, 2 and 3, respectively. Thus, the signal on line X is a four-valued logic signal and the probabilities of its four possible values are

$$\Pr\{X = 0\} = \Pr\{X \text{ is correctly a } 0\} \triangleq \mathrm{R}_0(X)$$
$$\Pr\{X = 1\} = \Pr\{X \text{ is correctly a } 1\} \triangleq \mathrm{R}_1(X)$$
$$\Pr\{X = 2\} = \Pr\{X \text{ is incorrectly a } 0\} \triangleq \mathrm{R}_2(X)$$
$$\Pr\{X = 3\} = \Pr\{X \text{ is incorrectly a } 1\} \triangleq \mathrm{R}_3(X)$$

These four probabilities form a reliability vector $\mathrm{R}(X) = [\mathrm{R}_0(X), \mathrm{R}_1(X), \mathrm{R}_2(X), \mathrm{R}_3(X)]$ whose elements satisfy

$$\mathrm{R}_0(X) + \mathrm{R}_1(X) + \mathrm{R}_2(X) + \mathrm{R}_3(X) = 1.$$

The signal reliability of line X, denoted $\mathrm{SR}(X)$, is the probability that the signal on line X is correct, hence,

$$\mathrm{SR}(X) = \mathrm{R}_0(X) + \mathrm{R}_1(X)$$

The signal reliability of a system, whose output is Z, is $\mathrm{SR}(Z)$. This reliability is calculated from the input lines' reliability vectors using a reliability transfer function whose evaluation is based on the reliability model devised in [3]. In this model, the occurrence of faults is introduced through special elements called fault occurrence networks (FON). Such an element is inserted into each line of the system. Faults may occur in these elements only, and the rest of the system is considered fault-free.

For a given system M we calculate a reliability transfer function in a form of a matrix relating the output reliability vector to the input reliability vectors. This reliability transfer matrix is derived in the following way. Let $X_1, X_2, \ldots, X_n$ and Z be the n input lines and the output line of the system, respectively. Let $\mathbf{X}$ denote the input vector $X_1, X_2, \ldots, X_n$ and $i = (i_1, i_2, \ldots, i_n)$ denote a specific four-valued vector assumed by $\mathbf{X}$. Each element of $\mathrm{R}(Z)$ can be expressed as follows

$$R_j(Z) = \Pr\{Z = j\} = \sum_{\substack{\text{all four-valued} \\ \text{vectors } i}} \Pr\{Z = j \mid \mathbf{X} = i\} \cdot \Pr\{\mathbf{X} = i\}; \quad j = 0, 1, 2, 3$$

The sum is over all 4^n four-valued vectors of length n; $i = (i_1, i_2, \ldots, i_n)$; $i_k = 0, 1, 2, 3$. To simplify notation, i will be used to denote a four-valued vector and its decimal value interchangeably. Hence,

$$R_j(Z) = \sum_{i=0}^{4^n-1} \Pr\{Z = j \mid \mathbf{X} = i\} \cdot \Pr\{\mathbf{X} = i\} \tag{2.2}$$

Let $\Pr\{\mathbf{X} = i\}$ be denoted by $V_i(\mathbf{X})$, and let t_{ij} denote the conditional probability $\Pr\{Z = j \mid \mathbf{X} = i\}$. Using this notation, we obtain

$$\mathrm{R}_j(Z) = \sum_{i=0}^{4^n-1} t_{ij} V_i(\mathbf{X}) \tag{2.3}$$

The conditional probabilities t_{ij}; $i = 0, 1, \ldots, 4^n - 1$; $j = 0, 1, 2, 3$, form a stochastic matrix $T = \{t_{ij}\}$ of order $4^n \times 4$. The terms $V_i(\mathbf{X})$; $i = 0, \ldots, 4^n - 1$ form a vector $V(\mathbf{X})$ of length 4^n. Thus, equation (2.3) takes on the following matrix form

$$\mathrm{R}(Z) = V(\mathbf{X}) \cdot T \tag{2.4}$$

The matrix T is called the reliability transfer matrix, abbreviated RTM. The size 4^{n+1} of the RTM increases rapidly with n the number of inputs. However, it can be shown [4] that only a reduced matrix of size $2^{n+1} = \sqrt{4^{n+1}}$ is actually needed. The reduced matrix contains only the first two columns of T and only the binary-indexed rows of T where the ith row of T is a binary-indexed row if the four-valued vector $i = (i_1, i_2, \ldots, i_n)$ is a binary vector, i.e. $i_k \in \{0, 1\}$ for $k = 1, 2, \ldots, n$. For the sake of brevity we omit the justification for the row and column reduction and we use the unreduced matrix T.

The definition of an RTM is not restriced to single output systems and an RTM for multiple-output systems can be defined in a similar way [4].

To evaluate the RTM of a given system, it is decomposed into subsystems and an appropriate RTM is calculated for each subsystem. The RTM of the overall system is then calculated using the RTMs of the subsystems. The smallest subsystems considered are the basic elements of the model. In the follwing we first derive the RTMs of some basic elements and then we show how the RTM of a system is calculated from the RTMs of its components.

FON

Let X, Z be the input and output lines of a FON, respectively. The elements of the RTM T_{FON} depend upon the type of faults assumed to occur at line X. If the possible faults are stuck-at-zero (s-a-0) and stuck-at-1 (s-a-1) with probabilities q_{0_x} and q_{1_x}, respectively, satisfying $S_x = q_{0_x} + q_{1_x}$, then the elements of the RTM are

$$\begin{aligned} t_{00} &= \Pr\{Z = 0 \mid X = 0\} = \Pr\{Z \text{ is correctly a } 0 \mid X \text{ is correctly a } 0\} \\ &= \Pr\{\text{No s-a-1 fault occurred}\} = 1 - q_{1_x} \\ t_{01} &= \Pr\{Z = 1 \mid X = 0\} = \Pr\{Z \text{ is correctly a } 1 \mid X \text{ is correctly a } 0\} = 0 \end{aligned}$$

In a similar manner, $t_{02} = 0$.

$$\begin{aligned} t_{03} &= \Pr\{Z = 3 \mid X = 0\} = \Pr\{Z \text{ is incorrectly a } 1 \mid X \text{ is correctly a } 0\} \\ &= \Pr\{\text{A s-a-1 fault occurred}\} = q_{1_x} \end{aligned}$$

Similarly, all other elements of T_{FON} are calculated, yielding

$$T_{\mathrm{FON}} = \begin{bmatrix} 1 - q_{1_x} & 0 & 0 & q_{1_x} \\ 0 & 1 - q_{0_x} & q_{0_x} & 0 \\ 0 & q_{1_x} & 1 - q_{1_x} & 0 \\ q_{0_x} & 0 & 0 & 1 - q_{0_x} \end{bmatrix} \tag{2.5}$$

If the possible fault is an "inverted signal" fault (i.e. $Z = X'$) with probability S_x, the resulting RTM is

$$T_{\mathrm{FON}} = \begin{bmatrix} 1 - S_x & S_x & 0 & 0 \\ S_x & 1 - S_x & 0 & 0 \\ 0 & 0 & 1 - S_x & S_x \\ 0 & 0 & S_x & 1 - S_x \end{bmatrix}$$

In both cases the lead failures are not necessarily permanent.

Similarly, a matrix T_{FON} can be derived for any other kind of lead failure. Furthermore, the type of fault and its probability have not necessarily to be the same for the various leads in the system. Some of the leads may even be fault-free wires (i.e. $S_x = 0$) yielding

$$T_{\mathrm{FON}} = I$$

NOT gate

Let X, Z be the input and output lines of a fault-free NOT gate. Then,

$$\begin{aligned} t_{00} &= \Pr\{Z = 0 \mid X = 0\} = \Pr\{Z \text{ is correctly a } 0 \mid X \text{ is correctly a } 0\} = 0 \\ t_{01} &= \Pr\{Z = 1 \mid X = 0\} = \Pr\{Z \text{ is correctly a } 1 \mid X \text{ is correctly a } 0\} = 1 \\ t_{02} &= \Pr\{Z = 2 \mid X = 0\} = \Pr\{Z \text{ is incorrectly a } 0 \mid X \text{ is correctly a } 0\} = 0 \\ t_{03} &= \Pr\{Z = 3 \mid X = 0\} = \Pr\{Z \text{ is incorrectly a } 1 \mid X \text{ is correctly a } 0\} = 0 \end{aligned}$$

Besides t_{01}, the other non-zero elements of T_{NOT} are

$$t_{10} = \Pr\{Z = 0 \mid X = 1\} = \Pr\{Z \text{ is correctly a } 0 \mid X \text{ is correctly a } 1\} = 1$$
$$t_{23} = \Pr\{Z = 3 \mid X = 2\} = \Pr\{Z \text{ is incorrectly a } 1 \mid X \text{ is incorrectly a } 0\} = 1$$
$$t_{32} = \Pr\{Z = 2 \mid X = 3\} = \Pr\{Z \text{ is incorrectly a } 0 \mid X \text{ is incorrectly a } 1\} = 1$$

Thus,

$$T_{\mathrm{NOT}} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

The RTM of any other fault-free gate is computed in a similar fashion.

JK *flip-flop*

Let j, K, Y_P and Y_N denote the two input lines, the present state and the next state of a fault-free *JK* flip-flop, respectively. The RTM T_{JKFF} is a $4^3 \times 4$ matrix with the elements

$$T_{(i,j,k),l} = \Pr\{Y_N = l \mid Y_P = i, J = j \text{ and } K = k\}$$

For example,

$$t_{000,0} = \Pr\{Y_N = 0 \mid Y_P = 0, J = 0 \text{ and } K = 0\} = 1$$
$$t_{003,0} = \Pr\{Y_N = 0 \mid Y_P = 0, J = 0 \text{ and } K = 3\} = 1$$
$$t_{023,1} = \Pr\{Y_N = 1 \mid Y_P = 0, J = 2 \text{ and } K = 3\} = 0$$
$$t_{023,2} = \Pr\{Y_N = 2 \mid Y_P = 0, J = 2 \text{ and } K = 3\} = 1$$

In a similar way we may calculate the other elements of T_{JKFF} or the RTM of any other flip-flop.

The RTM of a system

The RTM of a system is calculated from the RTMs of its components which are either the basic elements of the reliability model [3] or subsystems whose RTMs are known. This calculation is performed using the following theorem.

Theorem 2.1 [4]: Let $A^{(0)}$, $A^{(1)}$ and $A^{(2)}$ be the RTMs of the three subsystems M_0, M_1 and M_2 in Figure 2, respectively. The RTM of the system is

$$T = (A^{(1)} \circledast A^{(2)}) \cdot A^{(0)}$$

where the star product $\circledast$ is a generalization of the Kronecker matrix product and is defined in [4].

Theorem 2.1 can be generalized to the case where l subsystems $M_1, \ldots, M_l$ feed the subsystem M_0. Let $A^{(i)}$ denote the RTM of $M^{(i)}$, $i = 0, 1, \ldots, l$. Then

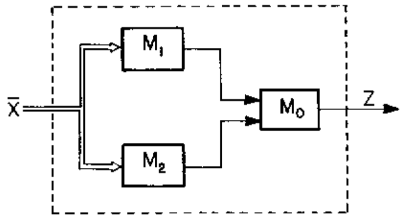

Fig. 2. A system constructed of three subsystems.

Theorem 2.2 [4]: The RTM of a system constructed of l subsystems feeding M_0 is

$$T = (A^{(1)} \circledast A^{(2)} \circledast \ldots \circledast A^{(l)}) \cdot A^{(0)}$$

It is shown in [4] that theorem 2.2 can be applied to an arbitrary system after proper partitioning of the system into subsystems. This partitioning is applied recursively until a level is reached where the RTMs of the subsystems are known.

This method of calculating the RTM of a given system using the RTMs of its subsystems is especially attractive in the following cases:

1. The system is constructed of standard LSI modules. In this case, if a standard module is used more than once throughout the system, its RTM has to be calculated just once.
2. The system consists of several indentical subsystems, e.g. cellular arrays and NMR systems.

The second case is illustrated in the following example.

Consider a TMR configuration of a Full-Adder. The RTM of the TMR system, denoted by T_{TMR}, is calculated using the RTM of a single Full-Adder, T_{FA}, and the RTM of the Majority-Voter, T_V, as follows

$$T_{\mathrm{TMR}} = (T_{\mathrm{FA}} \circledast T_{\mathrm{FA}} \circledast T_{\mathrm{FA}}) \cdot T_V$$

The extension of this equation to NMR system is straightforward.

In the next section we consider iterative arrays. The final results obtained in that section are extended later to the synchronous sequential systems in section 4.

3. Iterative systems

In this section we investigate the signal reliability of a cell in an iterative system and the conditions under which this reliability converges toward a steady state value. When such a convergence takes place a considerable reduction in the amount of computation needed is achieved.

Let $\mathbf{X} = X_1, X_2, \ldots, X_n$, $\mathbf{Y} = Y_1, Y_2, \ldots, Y_m$ and $\mathbf{Z} = Z_1, Z_2, \ldots, Z_l$ be the vectors of n primary input lines, m carry lines between the cells and l primary output lines, respectively, of a typical cell in an iterative system. Let $T_Y(T_Z)$ be the RTM of the carry output (primary output) of the cell. The logical operation of the fault-free cell can be described by a deterministic state diagram D_S with 2^m states $\{S_0, S_1, \ldots, S_{2^m-1}\}$ where S_i is the state corresponding to $\mathbf{Y} = i$ and $i = (i_1, i_2, \ldots, i_m)$; $i_k = 0, 1$. When faults are introduced into the cell we replace the two-valued signals by four-valued signals and consequently, we obtain an expanded state diagram D_U with 4^m states designated $\{U_0, U_1, \ldots, U_{4^m-1}\}$ where U_i is the state corresponding to $\mathbf{Y} = i$ and $i = (i_1, i_2, \ldots, i_m)$; $i_k = 0, 1, 2, 3$. Clearly, the expanded diagram D_U contains D_S as a proper subgraph. The additional transitions in $D_U - D_S$ are due to faults present in the cell. An edge from U_i to U_j in D_U is labelled with the transition probability P_{ij} given by the following expression

$$P_{ij} = \Pr\{U_j \mid U_i\} = \Pr\{\mathbf{Y}_N = j \mid \mathbf{Y}_P = i\} \tag{3.1}$$

where $\mathbf{Y}_P$ is the present state vector (i.e. the carry in) and $\mathbf{Y}_N$ is the next state vector (i.e. the carry out). An example of the expansion of D_S to D_U is shown in Figures 3 and 4.

We rewrite the transition probability P_{ij} as follows

$$P_{ij} = \sum_{k=0}^{4^n-1} \Pr\{\mathbf{Y}_N = j \mid \mathbf{Y}_P = i, \mathbf{X} = k\} \cdot \Pr\{\mathbf{X} = k \mid \mathbf{Y}_P = i\} \tag{3.2}$$

Since the primary inputs are independent of the carry signals the second probability in (3.2) equals $\Pr\{\mathbf{X} = k\}$. The first probability in (3.2) is, by definition, the element

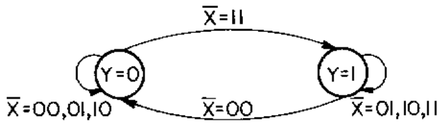

Fig. 3. The state diagram D_S of a Full-Adder.

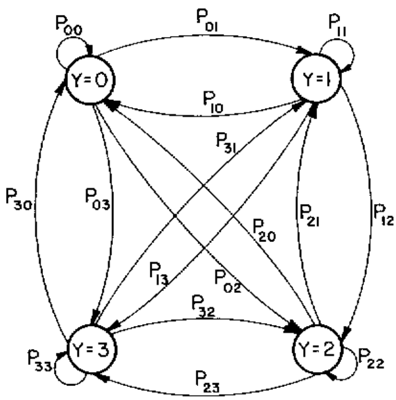

Fig. 4. The finite Markov chain D_U of a Full-Adder.

$t_{(i,k),j}$ of the RTM T_Y where (i, k) is the concatenation of i, the present state vector, and k, the primary input vector.

Hence,

$$P_{ij} = \sum_{k=0}^{4^n-1} t_{(i,k),j} \Pr\{\mathbf{X} = k\} \tag{3.3}$$

Thus, if the primary input sequence has a stationary (time invariant) probability then D_U is a probablistic state diagram that describes a finite Markov chain. The properties of this Markov chain depend upon the transition matrix $P = \{P_{ij}\}$ which from (3.3) depends on the RTM T_Y which in turn depends upon the possible faults in the cell and their probabilities of occurrence. To investigate the properties of the Markov chain we define the following function

$$\phi(X) = \begin{cases} 0 & \text{if } X = 0 \text{ or } 3 \\ 1 & \text{if } X = 1 \text{ or } 2 \end{cases}$$

i.e. $\phi(X)$ is the correct value of X. We extend the definition of ϕ to vectors $\mathbf{X}$ by applying ϕ to each element separately. When applied to the states of D_U the function ϕ defines a relation E as follows, two states U_i and U_j satisfy

$$(U_i, U_j) \in E \quad \text{iff} \quad \phi(U_i) = \phi(U_j) \tag{3.4}$$

The relation E is clearly an equivalence relation and each equivalence class contains 2^n states, one of which is a correct state designated U_{i*} satisfying $U_{i*} = \phi(U_{i*})$.

The function ϕ is employed in the proof of the following lemma showing that the Markov chain D_U is irreducible if D_S is strongly connected and the failures in all the input and output carry leads have non-zero probabilities. If some of these faults have zero probabilities, it is possible to obtain a reducible Markov chain D_U even if D_S is strongly connected. However, the above condition is satisfied in most practical cases.

Lemma 3.1: If the faults in all the input and output carry leads have non-zero probabilities then the finite Markov chain D_U is irreducible iff the cell state diagram D_S is strongly connected.

Proof: A finite Markov chain is irreducible, i.e. its state diagram is strongly connected, if for any two states U_i and U_j there is a directed path from U_i to U_j. We first prove that there is an edge from U_i to U_j in D_U iff there is an edge from U_{i*} to U_{j*}. To prove the only if part suppose that an edge from U_i to U_j exists, meaning that a cell with an input carry U_i may produce an output carry U_j. If no faults exist then the input carry to this cell is U_{i*} and the resulting output is some correct state U_k. In the presence of faults the actual output carry may be either U_k or some incorrect output carry which is ϕ-equivalent to U_k (under relation E). Since U_j is such an incorrect output carry we have $\phi(U_j) = \phi(U_k) = U_{j*}$. To prove the if part consider a cell with a correct input carry U_{i*} and a correct output carry

U_{j*}. Given that the faults in all input carry leads have non-zero probabilities, the actual input carry to this cell may be any state U_i which is ϕ-equivalent to U_{i*}. Similarly, the non-zero probabilities of the faults in all the output carry leads imply that the actual output carry may be any state U_j which is ϕ-equivalent to U_{j*}.

As a consequence, there is a path from U_i to U_j in D_U iff there is a path of the same length from U_{i*} to U_{j*} in D_S and the lemma follows. Q.E.D.

Lemma 3.2.: The periodicity of D_U equals that of D_S.

Proof: D_S is a proper subgraph of D_U, therefore, all the circuits in D_S are contained in D_U. Following the steps in the proof of lemma 3.1 there is a circuit of length k in D_U only if there is a circuit of the same length in D_S. Hence, the periodicity of D_U equals that of D_S. Q.E.D.

Corollary 3.3: D_U is aperiodic iff D_S is aperiodic.

Proof: Immediate from lemma 3.2.

As a result of these lemmas and corollary, the Markov chain D_U is irreducible and aperiodic if the state diagram D_S of the typical cell in the iterative system is strongly connected, aperiodic and the faults in all the input and output carry leads have non-zero probabilities. Consequently, the probabilities of the states in D_U converge to steady state values that satisfy the following set of equations,

$$\Pr\{U_i\} = \sum_{j=0}^{4^m-1} P_{ij} \cdot \Pr\{U_j\}; \quad i = 0, 1, \ldots, 4^m - 2 \tag{3.5}$$

and

$$\sum_{i=0}^{4^m-1} \Pr\{U_i\} = 1 \tag{3.6}$$

The signal reliability of the output carry is the probability that the output state is correct, therefore, it is the sum of the correct states' probabilities. Hence, under the above conditions, the signal reliability SR(Y) of the output carry of a cell in an iterative system converges to a steady state value.

A similar convergence is achieved by the primary output's signal reliability SR(Z) since a stationary probability of the primary input X is assumed.

Example: The signal reliability of a 40-bit ripple-carry parallel adder has been calculated using an APL program. The typical cell in this iterative system is a Full-Adder whose state diagram D_S is strongly connected and aperiodic as shown in Figure 3. It has been assumed that the possible faults are multiple, stuck-at-type permanent faults with probabilities $q_0 = q_1 = 0.01$ for any line in the cell. The state diagram of the resulting Markov chain D_U is shown in Figure 4 where the transition probabilities are given by equation (3.3). The primary input signals that appear in (3.3) are assumed to be either correct 0 or correct 1 with equal probabilities, i.e. $R(X) = (0.5, 0.5, 0, 0)$. The numerical results have shown that convergence of the signal reliability is achieved within 13 stages, i.e. stages 13–40 have the same signal reliabilities, namely, output carry reliability of $SR(Y) = 0.93841$ and primary output reliability of $SR(Z) = 0.82374$. The main conclusions drawn from this example are that the signal reliability of a parallel adder with a reasonable number of stages is independent of the number of stages and that a larger number of stages does not imply a lower reliability. The latter conclusion is in contrast to the conclusion drawn when the functional reliability measure is employed. When evaluating the functional reliability of a system, the reliability of the basic cell of the system (a Full-Adder stage in this case) is raised to the power of k where k is the number of these cells in the system. Consequently, the functional reliability of a parallel adder is a decreasing function of the number of stages.

4. Synchronous sequential systems

To analyse the signal reliability of synchronous sequential systems we model them as infinite iterative systems. This enables us to extend the results of the previous section to apply to synchronous systems. The modelling is illustrated in Figure 5. Part (a) shows a sequential system with a single JK flip-flop. Part (b) shows a typical cell of the iterative system model. The flip-flop equivalent in (b) is a combinational circuit whose RTM is identical to the RTM T_{JKFF} (developed in section 2). It calculates the next state Y_N from the present state Y_P and the excitation signals J and K.

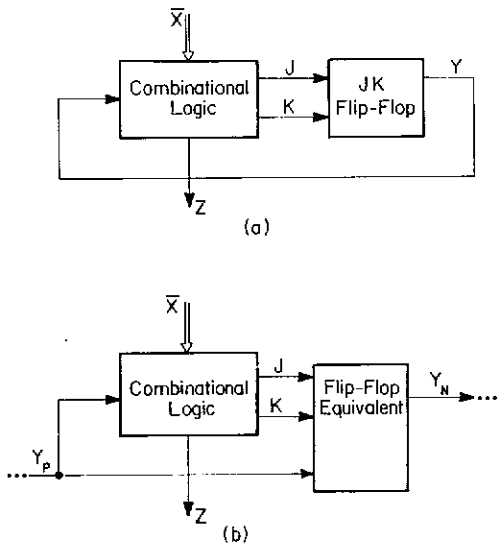

Fig. 5. Modelling a synchronous sequential system by an iterative system.

In the analysis of iterative systems the same RTM is used for all cells. This is possible since the sets of faults in all cells and their probabilities are assumed to be identical. However, if the occurrence of some faults in cell i depends on the occurrence of faults in cell j, the sets of faults probabilities are not necessarily identical. For example, if the occurrence of a fault in cell i implies the occurrence of the same fault in cell j and the fault in cell i indeed occurs, the probability of this fault in cell j is 1. Dependence of fault occurrences in separate cells is more likely to happen in an iterative system model of a given sequential system since the cells are not distinct circuits but images of the same circuit in various time instants. A permanent fault in the sequential system is represented by a fault in each cell of the iterative system and clearly those faults are dependent. However, an intermittent fault in the sequential system may, under certain conditions, be represented by a set of independent faults in the cells of the iterative system. The main condition is that the fault duration does not exceed the clock period. Then, the occurrence of a fault in cell i does not imply its occurrence in cell $i+1$. The condition above is effectively met when the probability that the fault duration exceeds the clock period is negligible. The duration of an intermittent fault characterized by the continuous-parameter Markov model [7] is exponentially distributed with mean $1/\mu_x$ and variance $1/\mu_x^2$. Hence, this condition is satisfied if

$$\mu_x \gg f_c \tag{4.1}$$

for every line X in the cell where f_c is the clock frequency.

Consequently, to evaluate the signal reliability of a sequential system with intermittent faults satisfying (4.1) we may represent the sequential system by an iterative system and apply the results of the previous section. The state probabilities of the underlying Markov chain converge to steady-state values if the state diagram of the sequential system is strongly connected, aperiodic and all its transition probabilities (and hence, the fault probabilities) are constant. The first two conditions for convergence are satisfied in many practical sequential systems that have a reset input. The third condition is met since the fault probability given by (2.1) is in practice a weak function of time and it takes weeks or months for the fault probability to change significantly. The convergence of the state probabilities on the other hand is a matter of a few clock cycles.

In the following example we illustrate the convergence of the signal reliability of a sequential system.

Example: A serial adder containing a single JK flip-flop is subject to occurrences of multiple intermittent faults in all of its leads. The probability that a single intermittent fault is active is selected to be 0.01 and the reliabilities of the input signals are assumed to be $R(X) = (0.5, 0.5, 0, 0)$. This sequential system is clearly strongly connected and aperiodic (see Figure 3) and it is assumed that condition (4.1) is satisfied too. To calculate the signal reliability of the serial adder it has been modelled by an iterative system. The results have shown that convergence is achieved within 10 clock cycles yielding an output signal reliability of $SR(Z) = 0.8912$ and a

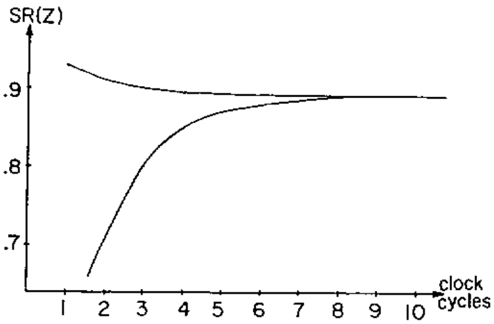

Fig. 6. The signal reliability of a serial adder.

carry signal reliability of $SR(Y) = 0.9620$. The values of the signal reliability in the first 10 clock cycles depend upon the reliability of the flip-flop's initial state. These values have been calculated first for $R(Y_0) = (1, 0, 0, 0,)$ (i.e. the flip-flop is in the reset state with probability 1) and then for $R(Y_0) = (0.5, 0, 0, 0.5)$. The results are summarized in Figure 6 illustrating the speed of convergence.

5. Conclusions

The signal reliabilities of iterative and sequential systems have been analysed in this paper. It has been shown that, under certain conditions, the signal reliabilities of these systems converge to steady state values, thus avoiding the need for prohibitively large number of calculations. Furthermore, this convergence provides a new insight when the signal reliability measure is compared to the more pessimistic functional reliability measure as illustrated in the example in section 3.

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Résumé

La fiabilité du signal est une mesure précise de la fiabilité des systèmes digitaux. Malheureusement, son évaluation nécessite une grande quantité de calcul, spécialement dans le cas des systèmes séquentiels, pour lesquels la fiabilité du signal doit être recalculée à chaque impulsion d'horloge. On montre dans cet article que dans certaines conditions, la fiabilité du signal converge vers une valeur permanente en l'espace de quelques impulsions d'horloge, ce qui rend possible le calcul de cette fiabilité.

Zusammenfassung

Signal-Zuverlässigkeit ist ein genaues Mass der Zuverlässigkeit digitaler Systeme. Leider erfordert ihre Festellung einen hohen Rechenaufwand, insbesondere im Fall sequentieller Systeme, für welche die Signal-Zuverlässigkeit in jedem Zeittakt neu berechnet werden muss. In dieser Arbeit wird gezeigt, dass unter bestimmten Bedingungen die Signal-Zuverlässigkeit eines synchronen sequentiellen Systems innerhalb weniger Takte gegen einen konstanten Zustandswert konvergiert, und dadurch die Berechnung der Signal-Zuverlässigkeit effizient durchführbar macht.