On the Properties of Sensitized Paths

ISRAEL KOREN AND ZVI KOHAVI

Abstract—**In this note we investigate the conditions under which the paths which propagate a fault in a combinational network to the network output are not necessarily entirely sensitized.**

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I. Koren is with the Department of Electrical Engineering and Computer Science, University of California, Santa Barbara, CA 93106.

Z. Kohavi is with the Department of Electrical Engineering, Technion-Israel Institute of Technology, Haifa, Israel.

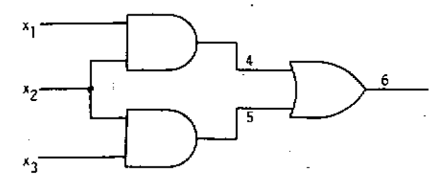

Fig. 1. A combinational network.

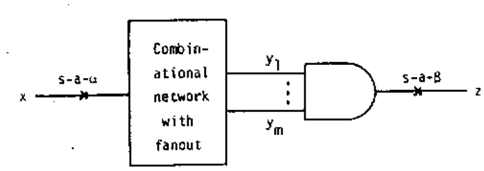

Fig. 2. A reconvergent fanout line.

Index Terms—**Backward tracing, combinational network, reconvergent fanout line, sensitized path, unsensitized propagating line.**

Fault detection by path sensitization is a well-known technique which has been studied extensively in the literature [1], [2]. A *sensitized path* is informally defined as a path in a digital network leading from the site of a fault to a network output and where the network inputs are specified in such a manner so as to generate the appropriate value at the site of the fault and to propagate the fault signal along this path to the output [1], [2]. In most cases the faults that occur along a sensitized path will also be detected at the network output.

In this correspondence we investigate the conditions under which a test will propagate a fault through paths in a network without sensitizing the entire paths [3]. We shall consider combinational networks with single stuck-at type faults. As an example consider the network in Fig. 1 [3]. The test $x_1 x_2 x_3 = 111$ detects the faults x_2 *s-a-*0 and x_6 *s-a-*0, but does not detect any of the faults on either line 4 or line 5, which are clearly part of the propagating paths from line 2 to line 6. We shall subsequently refer to such lines, which propagate faults to the output without being sensitized, as *unsensitized propagating lines.*

The detection of unsensitized propagating lines is very important in backward tracing when specifying the faults covered by a given test [4]. In the backward-tracing approach, once an unsensitized line is reached the subnetwork feeding this line can usually be ignored, except when the unsensitized lines are unsensitized propagating lines. In such a case further backward tracing within the unsensitized subnetwork must be performed. The use of backward tracing for specifying the faults covered by a given test is usually preferable to forward simulation techniques (parallel or deductive simulation). While in the forward simulation *all paths* in the network emanating from the primary input lines must be checked; in the backward tracing operation *only the sensitized* paths and the *unsensitized propagation lines* must be checked. Hence, the required computation time is considerably smaller.

As will be shown later, the existence of unsensitized propagating lines is closely related to the existence of reconvergent fanout lines within the network. Let x be a fanout line and let z denote the output of the reconvergence gate whose input lines are y_1, $y_2, \cdots, y_m$, as depicted in Fig. 2. This reconvergence gate and all other gates in the network are assumed to be conventional gates like AND, OR, NOT, NAND, NOR. Such a gate can be uniquely

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described by a binary vector $(\gamma_1, \gamma_2, \cdots, \gamma_m, \delta)$, where $\gamma_1, \cdots, \gamma_m$ is the only input combination for which the output z equals δ [4]. For example, a three-input NAND gate is described by (1, 1, 1, 0). We shall refer to such a vector as the *describing vector* for the gate.

Let T be a given test that covers the fault x *s-a-*α and initially suppose that the fault propagates to the output z through just a single path. Suppose also that the *same test* can be used to test z for *s-a-*β. In this case all the lines along the path are sensitized. Clearly, if the number of inversions along the path is even (odd), then $\alpha = \beta$ ($\alpha = \bar{\beta}$). If, however, for a given test T the fault propagates through two or more paths leading from x to z, we have two possible cases. If the inversion parity of the propagating paths is not the same, the fault will not propagate to z and thus will not be detected by the test [1]. On the other hand, if the inversion parity of the propagating paths is the same, the fault will propagate to z. Thus, equality of the inversion parities along the propagating paths is a necessary condition for the detection of x *s-a-*α.

Theorem 1: A sensitized subnetwork corresponding to a test T contains unsensitized propagating lines iff:

1) The test T covers a fault *s-a-*α at a fanout point x and this fault propagates to the output of a reconvergence gate through at least two paths, all having the same inversion parity; and

2) the test T covers a fault *s-a-*β at the output z of the reconvergence gate, such that $\beta = \delta$.

Proof: Suppose the two conditions above are satisfied. If T covers the fault z *s-a-*β and since $\beta = \delta$, the output z for the fault-free network is $z = \bar{\delta}$. Hence, by the definition of the describing vector, at least one of the input lines to the reconvergence gate must satisfy $y_j \neq \gamma_j$. Clearly, for each line y_j to be a propagating line, $y_j \neq \gamma_j$. (For example, if the reconvergence gate is a four-input AND gate whose describing vector is (1 1 1 1 1), the fault covered by test T is z *s-a-*1 and for the fault-free output we have $z = \bar{\delta} = 0$. Clearly, a fault can propagate from x to z only through the input lines satisfying $y_j = 0 \neq \gamma_j$.) Now, by condition 1, there are at least two input lines to the reconvergence gate for which $y_j \neq \gamma_j$ and a single fault in one of them will not propagate to the output z. Thus, it follows that neither of these lines is sensitized.

To prove that the above conditions are also necessary, suppose that the test T covers a fault *s-a-*β at the output line z of a gate in the network. There are two cases to be considered: 1) The output z for the fault-free network is $z = \delta$, i.e., $\beta = \bar{\delta}$. Hence, $y_1 \cdots y_m = \gamma_1 \cdots \gamma_m$, and consequently each of the input lines $y_1 \cdots y_m$ is also sensitized; and 2) $z = \bar{\delta}$ and $\beta = \delta$. In this case at least one of the inputs to the gate satisfies $y_j \neq \gamma_j$. Clearly, if only one line satisfies $y_j \neq \gamma_j$, this line is sensitized. If, on the other hand, there are two or more input lines for which $y_j \neq \gamma_j$, a single fault in one of them will not propagate to the output. Hence, these lines are not sensitized. However, these unsensitized lines may propagate a fault *s-a-*α from a preceding line x. In fact, such a fault will propagate from x to z iff it changes simultaneously all the y_j for which $y_j \neq \gamma_j$, but does not affect the remaining y's. Such a situation occurs only if all the y_j's for which $y_j \neq \gamma_j$ are on paths emanating from a fanout point x and all these propagating paths have the same inversion parity. Q.E.D.

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Revision of the Buffer Length Derivation for a Modified $E_k/D/1$ Systems by Maritsas and Hartley

J. S. HILL

Abstract—**The problem of loss of items arriving at a queue of limited length is relevant to the design of computers with real-time inputs and of similar equipments where only a limited input buffer size is possible. The length of buffer required to reduce the overflow loss to an acceptably low level is an important design parameter.**

This problem has been considered by Dor [1] for a random (Poisson) arrival rate and by Maritsas and Hartley [2] for an Erlang arrival rate. A discrepancy between the two models for an Erlang input of degree 1 led to a re-examination of the latter model.

This paper introduces a revised model for Erlang input. It gives the same results as Dor's model for degree 1, and substantially affects the results for degrees 2 and higher.

Index Terms—**Buffer length, Erlang input/constant removal rate, fractional loss, queue length distribution.**

Introduction

Items arriving at a queue of limited length are lost by overflow if the queue is already full when an item arrives. In many equipments, items arrive at random and are placed in a buffer of limited size from which they are removed in the order of arrival at regular sampling intervals. If the input buffer is too small, the fractional loss of input items may be unacceptable.

A model for this situation was developed by Dor [1] for random (Poisson) arrivals, and subsequently Maritsas and Hartley [2] published a model for Erlang input. These models will be referred to as the Dor model and the Maritsas model in this paper, which will use the notation of (and assume a knowledge of) reference [2].

The derivation of [2, eq. (8)] shows that it is in fact a transformation which, acting on an arbitrary set of probabilities $q(W)$ existing at a sampling instant, generates the set of probabilities $q'(W)$ at the next sampling instant. The equilibrium condition is the special case of $q'(W) = q(W)$.

The transformation equation is therefore

$$q'(W) = \begin{cases} \sum_{r=0}^{W} q(W+K-r)p(r) + \sum_{r=0}^{\min(W,K-1)} q(r)p(W-r), & 0 \le W \le KL+K-1; \\ 0, & W \ge KL+K. \end{cases} \tag{1}$$

Solutions for the fractional loss R_L of items were obtained by the author by successive application of (1) from an arbitrary initial set of $q(W)$, normalizing $q'(W)$ so that

$$\sum_{W=0}^{KL+K-1} q'(W) = 1$$

after each application, until equilibrium was reached, and then using equation [2, eq. (11)] to find R_L. The results obtained agreed with those of reference [2] for $K = 2, 3, 4$, and 5, but were some three times greater for $K = 1$ than those of reference [1]. This gave rise to suspicion that there was an error in one of the models, and the fact that the sum of all $q'(W)$ was always less than 1 before normalization indicated that some of the transition probabilities in the Maritsas model were incorrect.

To investigate this, a single application of (1) was performed for

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The author is with the Admiralty Surface Weapons Establishment, Portsmouth, Hampshire, England.

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