Sample Exam

UNIVERSITY OF MASSACHUSETTS
Department of Electrical and Computer Engineering

ECE 666 (DS 730-A)

Computer Arithmetic - Take-home Final Exam

Solve all four problems. You have two and a half days.

1. (Exercise 8.9) Given an operand A that is a normalized fraction ($0.5 \leq A < 1$), what function $z = f(A)$ will the following procedure calculate:
(i) $x_{i+1} = x_i \cdot r_i^2$ with $x_0 = A$.
(ii) $z_{i+1} = z_i \cdot r_i$ with $z_0 = A$.
with $r_i = 1 + (1 - x_i)/2$.
How many iterations are needed in this calculation and what operations are executed in each iteration? How would the multiplying coefficient be calculated?
Estimate the error in the final result. Can you suggest ways to speed up the calculation?
Given $A = 0.11100001$, calculate the 8-bit result $z = f(A)$ using the above procedure and compare it to the correct result.

2. (Exercise 10.6) Show that the maximum error of the approximation $\log_2(1 + x) \approx x$ is 0.08639. Suppose that the approximation $\log_2(1 + x) \approx x + c_i$ is used instead, where the interval [0,1] is divided into four subintervals, as in equation (10.14), and c_i is a constant employed for the ith subinterval ($i = 1, 2, 3, 4$). Find the best values for the c_i's so as to minimize the error, and calculate the resulting maximum error.

3. (Exercise 11.2) Given the set of moduli (7,5,3), find: (a) the range M, (b) the coefficients for the Chinese Remainder Theorem and the value represented by (2,3,2), (c) the corresponding mixed-radix representation, (d) the representation of 20 in the residue system and in the mixed-radix system.

4. Read the attached two papers "Redundant CORDIC Methods with a Constant Scale Factor for Sine and Cosine Computation," by N. Takagi *et al.*, *IEEE Trans. on Computers, vol. 40, pp. 989-994, Sept. 1991,* and "The CORDIC Algorithm: New Results for Fast VLSI Implementation," by J. Duprat and J.M. Muller, *IEEE Trans. on Computers, vol. 42, pp. 168-178, Feb. 1993.* Answer the following questions:
a. Explain the significance of equation (1) on p. 168 of the 2nd paper.
b. What is the problem that the two algorithms (SINCOS1 and Branching-CORDIC) attempt to solve and how do they solve it?
c. Explain the simplifications suggested at the end of Section III (on p. 991) of the 1st paper. Why are these possible?
d. Estimate the execution times of the proposed algorithms (SINCOS1 and Branching-CORDIC) in the two papers and compare them to that of a conventional CORDIC.
e. Show the calculation of sine and cosine of $\pi/6$ using the algorithm Branching-CORDIC proposed in the 2nd paper with the same precision used by the SINCOS1 algorithm in Figure 1 in the 1st paper. Compare the results obtained by the two algorithms to the correct value. Are the errors acceptable?