Sample Exam
UNIVERSITY OF MASSACHUSETTS
Department of Electrical and Computer Engineering

ECE 666 (DS 730-A)

Computer Arithmetic - Mid-term I

Open notes - 2 hours.
Solve all five problems.
Write your answers in the provided space.
**Show your work** and clearly mark your final answer.

Student name: ________________________________

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Total ______ (100 points)

1. (20 points) Show the exact steps in the non-restoring division with the (negative) dividend $X = 1011001$ in two’s complement representation and the divisor $D = 0110$.

2. (a) (10 points) Show the representation of the following operands in the IEEE short format (use hexadecimal notation), perform the multiplication and show the final result in all four rounding schemes (round-to-nearest-even, round toward zero (truncate), round toward infinity, and round toward $-\infty$).

   $$(1 + 2^{-23}) \times (1 + 2^{-22})$$

   [Note: $(1 + 2^{-23})$ is the number $1.00000000000000000000000001$].

   **Solution:** 3F80 0003 for all rounding schemes except round-toward $+\infty$ for which the result is 3F80 0004.

   (b) (10 points) Show the result of the following subtraction of numbers in the IEEE short format in all four rounding schemes. The operands are already given in the hexadecimal notation.

   $3F80 \ 0000 - \ 3EFF \ FFFF$

   **Solution:** 3F00 0000 for all rounding schemes except round-toward $+\infty$ for which the result is 3F00 0001.

3. (20 points) Problem 4.10 from the textbook.

4. (20 points) Prove that the optimal way to implement a two-level combinatorial shifter for $k$ bits, where $k = m^2$, is for the first level to shift by multiples of $m$, and the second level to shift from 0 to $m$. Assume that the speed is proportional to the number of destinations for each line in the two levels. Can you generalize this result for any value of $k$?

5. (a) (10 points) Problem 4.12 from the textbook.

   (b) (10 points) Problem 4.13 from the textbook.