UNIVERSITY OF MASSACHUSETTS

Department of Electrical and Computer Engineering

Fault Tolerant Computing ECE655

Homework 3

1. Given a number X and its residue modulo-3, $C(X) = |X|_3$; how will the residue change when X is shifted by one bit position to the left if the shifted-out bit is 0? Repeat this for the case where the shifted-out bit is 1. Verify your rule for $X = 01101$ shifted five times to the left.

2. You have a RAID1 system where failures occur at individual disks at a constant rate λ per disk. The repair time of disks is exponentially distributed with rate μ. Suppose we are in an earthquake-prone area, where building-destroying earthquakes occur according to a Poisson process with rate λ_e. If the building is destroyed, so too is the entire RAID system. Derive an expression for the probability of data loss for such a system as a function of time. Assuming that the mean time between such earthquakes is 50 years, plot the probability of data loss as a function of time using the parameters $1/\lambda = 500,000$ *hours* and $1/\mu = 1$ *hour*.

3. For the example shown in Figure **??** the four nodes have an availability 1 while the links have the availabilities indicated in the figure. Use Heuristic 2 to assign votes to the four nodes, write down the possible values for w and the corresponding minimal values of r, and calculate the availability for each possible value of (r, w). Assume that read operations are twice as frequent as write operations.

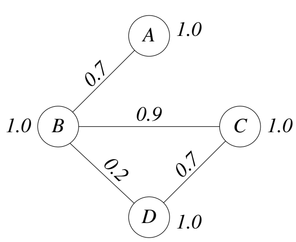

Figure 1: An example network (numbers indicate availabilities).

4. Show how checksums can be used to detect and correct errors in a scalar by matrix multiplication for the following example. Assume a 3×3 matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Show the corresponding column weighted matrix A_C and assume that during the mulitplication of A_C by the scalar 2 a single error has occured resulting in the following output

$$2 \cdot A = \begin{bmatrix} 2 & 4 & 6 \\ 8 & 10 & 12 \\ 14 & 17 & 18 \end{bmatrix}$$

5. Consider an 8×8 butterfly network. Suppose that each processor generates a new request every cycle. This request is independent of whether or not its previous request was satisfied, and is directed to memory module 0 with probability 1/2 and to memory module i with probability 1/14, for $i \in \{1, 2, \cdots, 7\}$. Obtain the expected bandwidth of this network.

6. We showed how to obtain the probability, for a multistage network, that a given processor is unable to connect to *any* memory. In our analysis, only link failures were considered. Extend the analysis to include switchbox failures, which occur with probability q_s. Assume that link and switchbox failures are all mutually independent of one another.

7. Derive an approximate expression for the reliability of a square $(4, 4)$ interstitial redundancy array with 16 primary nodes and 9 spares. Denote the reliability of a node by R and assume that the links are fault-free.

8. A 3×3 crossbar has been augmented by adding a row and a column, and input demultiplexers and output multiplexers. Assume that a switch can fail with probability q_s and when it fails all the incident links are disconnected. Also assume that all links are fault-free but multiplexers and demultiplexers can fail with probability q_m. Write expressions for the reliability of the original 3×3 crossbar and for the fault-tolerant crossbar. (For the purposes of this question, the reliability of the fault-tolerant crossbar is the probability that there is a functioning 3×3 crossbar embedded within the 4×4 system). Will the fault-tolerant crossbar have always a higher reliability than the original 3×3 crossbar?