UNIVERSITY OF MASSACHUSETTS
Department of Electrical and Computer Engineering

Fault Tolerant Computing ECE655

Homework 2

1. A duplex system consists of two active units and a comparator. Assume that each unit has a failure rate of λ and a repair rate of μ. The outputs of the two active units are compared, and when a mismatch is detected, a procedure to locate the faulty unit is performed. The probability that upon a failure, the faulty unit is correctly identified and the fault-free unit (and consequently, the system) continues to run properly is the coverage factor c. Note that when a coverage failure occurs, the entire system fails and both units have to be repaired (at a rate μ each). When the repair of one unit is complete, the system becomes operational and the repair of the second unit continues, allowing the system to return to its original state.
(a) Show the Markov model for this duplex system.
(b) Derive an expression for the long-term availability of the system assuming that $\mu = 2\lambda$.

2. A duplex system consists of a switching circuit and two computing units: an active unit with a failure rate of λ_1 and a standby idle unit that has a lower failure rate $\lambda_2 < \lambda_1$ while idle. The switching circuit frequently tests the active unit, and when a fault is detected, the faulty unit is switched out, and the second unit is switched in and becomes fully operational with a failure rate λ_1. The probability that upon a failure, the fault is correctly detected and the fault-free idle unit resumes the computation successfully is denoted by c (the coverage factor). Note that when a coverage failure occurs, the entire system fails.
(a) Show the Markov model for this duplex system (hint: three states are sufficient).
(b) Write the differential equations for the Markov model and derive an expression for the reliability of the system.

3. To an n-bit word with a single parity bit (for a total of $(n + 1)$ bits), a second parity bit for the $(n+1)$-bit word has been added. How would the error detection capabilities change?

4. Show that the Hamming distance of an M-of-N code is 2.

5. Compare two parity codes for data words consisting of 64 data bits: (1) a $(72, 64)$ Hamming code and (2) a single parity bit per byte. Both codes require eight check bits. Indicate the error correction and detection capabilities, the expected overhead, and list the types of multiple errors that are detectable by these two codes.

6. A communication channel has a probability of 10^{-3} that a bit transmitted on it is erroneous. The data rate is 12000 bps. Data packets contain 240 information bits, a 32-bit CRC for error detection, and 0, 8, or 16 bits for error correction coding (ECC). Assume that if eight ECC bits are added, all single-bit errors can be corrected, and if sixteen ECC bits are added all double-bit errors can be corrected.
   (a) Find the throughput in information bits per second of a scheme consisting of error detection with retransmission of bad packets (i.e., no error correction).
   (b) Find the throughput if eight ECC check bits are used, so that single-bit errors can be corrected. Uncorrectable packets must be retransmitted.
   (c) Finally find the throughput if sixteen ECC check bits are appended, so that two-bit errors can be corrected. As in (b), uncorrectable packets must be retransmitted. Would you recommend increasing the number of ECC check bits from 8 to 16?

7. Derive all codewords for the separable 5-bit cyclic code based on the generating polynomial $X + 1$ and compare the resulting codewords to those for the nonseparable code.

8. Given that $X^7 - 1 = (X + 1)g_1(X)g_2(X)$, where $g_1(X) = X^3 + X + 1$
   (a) Calculate $g_2(X)$.
   (b) Identify all the $(7, k)$ cyclic codes that can be generated based on the factors of $X^7 - 1$. How many different such cyclic codes exist?
   (c) Show all the codewords generated by $g_1(X)$ and their corresponding data words.