UNIVERSITY OF MASSACHUSETTS

Department of Electrical and Computer Engineering

Fault Tolerant Computing ECE655

Homework 1

1. The lifetime (measured in years) of a processor is exponentially distributed, with a mean lifetime of 2 years. You are told that a processor failed sometime in the interval $[4, 8]$ years. Given this information, what is the conditional probability that it failed before it was 5 years old?

2. Write the expression for the reliability $R_{system}(t)$ of the series/parallel system shown in Figure 1, assuming that each of the five modules has a reliability of $R(t)$.

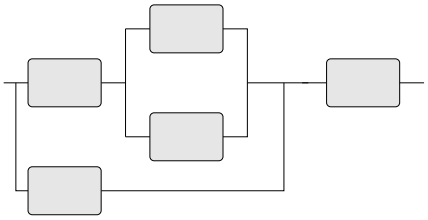

Figure 1: A 5-module series-parallel system.

3. The lifetime of each of the seven blocks in Figure 2 is exponentially distributed with parameter λ. Derive an expression for the reliability function of the system, $R_{system}(t)$, and plot it over the range $t = [0, 100]$ for $\lambda = 0.02$.

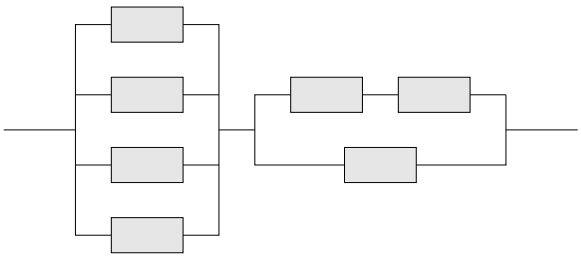

Figure 2: A 7-module series-parallel system.

4. Consider a triplex that produces a one-bit output. Failures that cause the output of a processor to be permanently stuck at 0 or stuck at 1 occur at constant rates λ_0 and λ_1, respectively. The voter never fails. At time t, you carry out a calculation the correct

output of which should be 0. What is the probability that the triplex will produce an incorrect result? (Assume that stuck-at faults are the only ones that a processor can suffer from, and that these are permanent faults; once a processor has its output stuck at some logic value, it remains stuck at that value forever).

5. Design a majority voter circuit out of two- and three-input logic gates. Assume that you are voting on one-bit inputs.

6. Derive an expression for the reliability of the voter you designed in the previous question. Assume that, for a given time t, the output of each gate is stuck-at-0 or stuck-at-1 with probability P_0 and P_1, respectively (and is fault-free with probability $1-P_0-P_1$). What is the probability that the output of your voter circuit is stuck-at-0 (stuck-at-1) given that the 3 inputs to the voter are fault-free and do change between 000 and 111?

7. Show that the MTTF of a parallel system of N modules, each of which suffers permanent failures at a rate λ, is $MTTF_p = \sum_{k=1}^{N} \frac{1}{k\lambda}$.

8. Consider a system consisting of two subsystems in series. For improved reliability, you can build subsystem i as a parallel system with k_i units, for $i = 1, 2$. Suppose permanent failures occur at a constant rate λ per unit.

   (a) Derive an expression for the reliability of this system.

   (b) Obtain an expression for the MTTF of this system with $k_1 = 2$ and $k_2 = 3$.

9. Write expressions for the upper and lower bounds and the exact reliability of the following non series/parallel system shown in Figure 3 (denote by $R_i(t)$ the reliability of module i). Assume that D is a bidirectional unit.

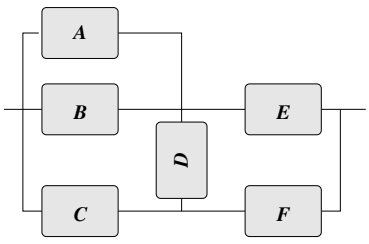

Figure 3: A 6-module non series/parallel system.