a) \[ T_e = \frac{P_e}{4R} = \frac{(0.01) \times 10^{-9.5}}{(1.38 \times 10^{-23})(75 \times 10^4)} = 305.5 K \] 

b) \[ F_e = 1 + (L-1) \frac{T_e}{T_0} = 1 + (1.413 - 1) \frac{300}{290} = 1.43, \quad F_a = 1 + \frac{T_e}{T_0} = 1.62 = 2.1 dB \]

c) \[ F_c = F_e + \frac{F_a - 1}{G_m} = 1.43 + \frac{1.62 - 1}{1.413} = 2.30 = 3.6 dB \]
\[ T_c = (F_c - 1) T_0 = (2.30 - 1)(290) = 378 K \]

d) \[ N_c = \frac{1}{k (T_e + T_c)} B G_e = \left( 1.38 \times 10^{-23} \right) (378 + 305.5)(75 \times 10^4) \left( \frac{15.9}{1.413} \right) \]
\[ = 7.9 \times 10^{-12} W = 7.9 \times 10^{-9} mW = -81.0 dBm \]
From (10.23) the noise figure of the cascade is \( F = IL + 1.5 \text{dB} \)

\[
F_{\text{Cas}} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_2 G_3} = 1.41 + (1.41)(1.58 - 1) + \frac{1.41}{10} (1.58 - 1)
\]

\[= 2.31 = 3.64 \text{dB}
\]

Also, \( P_{\text{in}} = -90 \text{dBm} \), then \( P_{\text{out}} = -90 \text{dBm} -1.5 \text{dB} + 10 \text{dB} + 20 \text{dB} = -61.5 \text{dBm} \)

The noise power output is then,

\[
P_n = G_{\text{cas}} K T_{\text{cas}} B = A (F_{\text{cas}} - 1) T_{\text{B}} G_{\text{cas}}
\]

\[
= (1.38 \times 10^{-23}) (2.31 - 1)(290)(10^8)(10^{28.5/10}) = 3.71 \times 10^{-10} \text{W}
\]

\[= -64.3 \text{ dBm}
\]

Thus,

\[
\frac{S_p}{N_p} = -64.5 + 64.3 = 2.8 \text{ dB}
\]

The best noise figure would be achieved with the arrangement shown below:

\[
G = 20 \text{dB} \quad F = 2 \text{dB} \quad G = 10 \text{dB} \quad F = 2 \text{dB} \quad IL = 1.5 \text{dB} \quad BW = 100 \text{MHz}
\]

Then,

\[
F_{\text{Cas}} = 1.58 + \frac{(1.58 - 1)}{10} + \frac{(1.41 - 1)}{1000} = 1.586 = 2.0 \text{ dB}
\]

(In practice, however, the initial filter may serve to prevent overload of the amplifier, and may not be allowed to be moved.)
a) **Resistive Divider**

When the input noise power at port 1 is $kTB$, and the divider is at temperature $T$, the system is in thermodynamic equilibrium. Thus, the output noise power at port 2 must be $kTB$. We can also express this as due to the attenuated input noise power and noise power added by the network (ref. at input). Thus,

$$\begin{align*}
P_2^- &= kTB = \frac{kTB}{4} + \frac{N_{added}}{4} \\
\therefore N_{added} &= 3kTB
\end{align*}$$

The equivalent noise temperature is then,

$$T_e = \frac{N_{added}}{kT} = 3T$$

And the noise figure is,

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{3T}{T_0}$$

At room temperature, $T = T_0$, so $F = 4 = 6\,\text{dB}$. (this result checks with that obtained using the available gain method)
In this case, if the input noise power is $kTB$, and the system is in thermodynamic equilibrium, the net output power at port 2 is $\frac{3}{2}kTB$, because of the mismatch of the output ports ($\frac{1}{4}$ of output power is reflected). Then we have,

$$P_2^- = \frac{3}{2}kTB = \frac{kTB}{2} + \frac{\text{Nadded}}{2}$$

(Nadded ref at input)

Thus:

$$\text{Nadded} = \frac{3}{2}kTB$$

$$\frac{T_e}{R_B} = \frac{\text{Nadded}}{R_B} = \frac{1}{2}$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{1}{2T_0}$$

If $T = T_0$, $F = \frac{3}{2} = 1.76$ dB.

(Result verified with HP-5855, calculations using available gain, and direct measurement)
c) QUADRATURE HYBRID

Using the same thermodynamic arguments as above, the output noise power is \( kTB \) (outputs are matched). Thus,

\[
P_z = \frac{kTB}{2} + \frac{N_{added}}{2}
\]

\[
N_{added} = kTB
\]

\[
T_e = \frac{N_{added}}{kT} = T
\]

\[
F = 1 + \frac{T_e}{T_o} = 1 + \frac{F}{T_o}
\]

If \( T = T_o \), we have \( F = 2 \) = 3 dB

10.8 From (10.33), \( T_e = \frac{(L-1)(L+1|\Gamma_s|^2)}{L\left(1-|\Gamma_s|^2\right)} \cdot T \)

Let \( x^2 = |\Gamma_s|^2 \); \( C = (L-1)T/L \). Then \( T_e = C \frac{L+x^2}{1-x^2} \)

\[
\frac{dT_e}{dx} = C \frac{(1-x^2)(2x) + (2x)(L+x^2)}{(1-x^2)^2} = \frac{2x(1+x)}{(1-x^2)^2} = 0
\]

Thus \( x = 0 \), so \( |\Gamma_s| = 0 \) minimizes \( T_e \)
\[ S_\pm = \frac{v_2}{2} \]
\[ v_1^A = \frac{v_2}{\sqrt{2L}} \]
\[ v_1^B = -j \frac{v_2}{\sqrt{2L}} \]
\[ v_2^A = \frac{v_2}{\sqrt{2L}} \]
\[ v_2^B = -j \frac{v_2}{\sqrt{2L}} \]
\[ v_0 = -j \frac{v_2^A}{\sqrt{2L}} + \frac{v_2^B}{\sqrt{2L}} = -j \frac{v_2}{\sqrt{2L}} \]
\[ S_0 = \frac{v_0^2}{2L} = \frac{\sqrt{2L}}{2L} = \frac{G_S s_0}{L} \]
\[ N_1^A = N_1^B = kT_0 B \]
\[ N_2^A = N_2^B = \frac{kT_0 B G}{1 + f - 1} = kT_0 B F \]
\[ N_0 = \frac{N_2^A}{2L} + \frac{N_2^B}{2L} + \frac{N_{\text{added}}}{2L} = \frac{kT_0 B G}{L} F + \frac{kT_0 B}{2L} \cdot \frac{(2L - 2)}{L} \]
\[ F_{\text{total}} = \frac{s_i}{s_0} \left[ \frac{G F}{L} + \left( 1 - \frac{1}{L} \right) \right] = LF + \frac{L}{G} (L - 1) \]

CHECK: If \( L = 1 \), \( F_{\text{total}} = F \)

**N_{\text{added}} for hybrid:**

\[ N_0 = \frac{kT_0 B}{2L} + \frac{kT_0 B}{L} + \frac{N_{\text{added}}}{2L} = kT_0 B \]

\[ : \quad N_{\text{added}} = 2kT_0 B (L - 1) \quad \text{(Ref. at input)} \]