Design of matching network: Given $\Gamma_s$, find $l_1$ and $l_2$

Towards "load" \\

\[ Y_c = Y_0 \]

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Normalized Immittances.

\[ Y_B = 1 + j\tan\beta l_2 \]

\[ \Gamma_B = \frac{1 - Y_B}{1 + Y_B} = \frac{-j\tan\beta l_2}{2 + j\tan\beta l_2} \]

\[ V(z) = V^+ e^{-j\beta z} + V^- e^{j\beta z} \]

\[ \Gamma_A = \frac{V^-}{V^+} \]

\[ \Gamma_B = \frac{V^- e^{j\beta l_1}}{V^+ - j\beta l_1} = \Gamma_A e^{j2\beta l_1} \] towards load
\[
\Gamma_A = \Gamma_B e^{-j2 \beta l_1} = \Gamma_s.
\]

or
\[
\Gamma_s = \frac{-j \tan \beta l_2}{2 + j \tan \beta l_2} e^{-j2 \beta l_1}.
\]

\[
1 + j \tan \beta l_2 = \frac{1 - \Gamma_s e^{j2 \beta l_1}}{1 + \Gamma_s e^{j2 \beta l_1}}
\]

\[
j \tan \beta l_2 = \frac{-2 \Gamma_s e^{j2 \beta l_1}}{1 + \Gamma_s e^{j2 \beta l_1}}
\]

**Example:**
\[
\Gamma_s = 0.872 e^{j123\pi/180}
\]
\[
l_1 = 0.12 \lambda, \quad \beta l_1 = 0.24 \pi
\]
\[
\Gamma_s e^{j2 \beta l_1} = 0.872 e^{j1.1633\pi}
\]
\[
1 + \Gamma_s e^{j2 \beta l_1} = 0.2403 - j0.428 = 0.4909 e^{-j0.3371\pi}
\]
\[ \tan \beta l_2 = -\frac{2 \times 0.872 e^{j\frac{3\pi}{2}}}{0.4909} = j3.55 \]

\[ \beta l_2 = 1.2964 \Rightarrow l_2 = 0.206\lambda \checkmark \]

\[ l_1 = 0.12\lambda \]

\[ l_2 = 0.206\lambda \]

is the correct solution

\[ |\Gamma_s| = 0.872 \]

Point B (admittance)

Point B' (impedance)

\[ |\Gamma| = 1 \]
\[ l_1 = 0.077 + 0.044 \approx 0.12 \lambda \]

\[ l_2 = 0.204 \lambda \]