Analysis of Ladder Circuits (Alternate Series,)

Series element currents
Shunt element voltages

\[ I_{M-1} = V_L Y_M \]
\[ U_M = V_L \]

\[ F_k = \text{series impedance or shunt admittance of branch } k \]

\[ U_k = \text{series branch current or shunt branch voltage of branch } k \]

\[ U_M = \text{series branch current or shunt branch voltage of last branch on the right} \]

\[ U_o = \text{current or voltage associated with the driving generator on the left} \]

\[ U_o = V_g \]
\[ I_g \]

\[ U_o = I_g \]
\[ A_{M+1} := 1 = \frac{U_M}{U_M} \]

\[ A_M = F_M A_{M+1} = \frac{U_{M-1}}{U_M} \]

\[ A_{M-1} = F_{M-1} A_M + A_{M-1} = \frac{U_{M-2}}{U_M} \]

\[ \vdots \]

\[ A_k = F_k A_{k+1} + A_{k+2} = \frac{U_{k-1}}{U_M} \]

\[ \vdots \]

\[ A_1 = F_1 A_2 + A_3 = \frac{U_0}{U_M} = \frac{V_g}{V_L} \]

\[ \therefore \frac{V_L}{V_g} = \frac{1}{A_1} \]

\[ (F_{in})_k = \text{impedance looking right through branch } k \text{ if branch } k \text{ is in series, or admittance looking right through branch } k \text{ if branch } k \text{ is in shunt} \]

\[ = A_k / A_{k+1} \]
In the figure above if \( M = 4 \)

\[
A_5 = 1 = \frac{U_4}{U_4}, \quad U_4 = V_L; \quad Y_4 = \frac{I_3}{V_L}
\]

\[
A_4 = Y_4 A_5 = Y_4 = \frac{I_3}{U_4} = \frac{I_3}{V_L}
\]

\[
A_3 = Z_3 A_4 + A_5 = Z_3 Y_4 + 1 = \frac{V_2}{V_L}
\]

\[
A_2 = Y_2 A_3 + A_4 = Y_2 Z_3 Y_4 + Y_2 + Y_4 = \frac{I_1}{V_L}
\]

\[
A_1 = Z_1 A_2 + A_3
\]

\[
= Z_1 Y_2 Z_3 Y_4 + Z_1 Y_2 + Z_1 Y_4 + Z_3 Y_4 + 1
\]

\[
= \frac{V_g}{V_L} \quad \Rightarrow \quad \frac{V_L}{V_g} = \frac{1}{(Z_1 Y_2 Z_3 Y_4 + Z_1 Y_2 + Z_1 Y_4 + Z_3 Y_4 + 1)}
\]

\[
Z_{in} = \frac{A_1}{A_2} = \frac{Z_1 + \frac{Z_3 Y_4 + 1}{Y_2 Z_3 Y_4 + Y_2 + Y_4}}{Z_1 + \frac{1}{Y_2 + \frac{1}{Z_3 + \frac{1}{Y_4}}}}
\]