(c) For a low frequency gain increase of 3.16, and the pole at 3.16 rad/sec, the zero needs to be at 10 in order to maintain the crossover at $\omega_c = 31.6$ rad/sec. So the lag compensator is

$$D_2(s) = 3.16 \frac{s}{10} + 1 \frac{s}{3.16} + 1$$

and

$$D_1(s)D_2(s) = 100 \frac{s}{100} + 1 \frac{s}{10} + 1 \frac{s}{3.16} + 1$$

The Bode plots of the system before and after adding the lag compensation are

(d) By using the `margin` routine from Matlab, we see that

$$PM = 49^\circ \ (\omega_c = 3.16 \ deg/sec)$$

58. Golden Nugget Airlines had great success with their free bar near the tail of the airplane. (See Problem 5.41) However, when they purchased a much
larger airplane to handle the passenger demand, they discovered that there was some flexibility in the fuselage that caused a lot of unpleasant yawing motion at the rear of the airplane when in turbulence and was causing the revelers to spill their drinks. The approximate transfer function for the dutch roll mode (See Section 9.3.1) is

$$\frac{r(s)}{\delta_r(s)} = \frac{8.75(4s^2 + 0.4s + 1)}{(s/0.01 + 1)(s^2 + 0.24s + 1)}$$

where $r$ is the airplane’s yaw rate and $\delta_r$ is the rudder angle. In performing a Finite Element Analysis (FEA) of the fuselage structure and adding those dynamics to the dutch roll motion, they found that the transfer function needed additional terms that reflected the fuselage lateral bending that occurred due to excitation from the rudder and turbulence. The revised transfer function is

$$\frac{r(s)}{\delta_r(s)} = \frac{8.75(4s^2 + 0.4s + 1)}{(s/0.01 + 1)(s^2 + 0.24s + 1)} \cdot \frac{1}{\left(\frac{s}{\omega_b} + 2\zeta\frac{s}{\omega_b} + 1\right)}$$

where $\omega_b$ is the frequency of the bending mode (= 10 rad/sec) and $\zeta$ is the bending mode damping ratio (= 0.02). Most swept wing airplanes have a “yaw damper” which essentially feeds back yaw rate measured by a rate gyro to the rudder with a simple proportional control law. For the new Golden Nugget airplane, the proportional feedback gain, $K = 1$, where

$$\delta_r(s) = -Kr(s). \quad (3)$$

(a) Make a Bode plot of the open-loop system, determine the PM and GM for the nominal design, and plot the step response and Bode magnitude of the closed-loop system. What is the frequency of the lightly damped mode that is causing the difficulty?

(b) Investigate remedies to quiet down the oscillations, but maintain the same low frequency gain in order not to affect the quality of the dutch roll damping provided by the yaw rate feedback. Specifically, investigate one at a time:

i. increasing the damping of the bending mode from $\zeta = 0.02$ to $\zeta = 0.04$. (Would require adding energy absorbing material in the fuselage structure)

ii. increasing the frequency of the bending mode from $\omega_b = 10$ rad/sec to $\omega_b = 20$ rad/sec. (Would require stronger and heavier structural elements)

iii. adding a low pass filter in the feedback, that is, replace $K$ in Eq. (3) with $KD(s)$ where

$$D(s) = \frac{1}{s/\tau_p + 1}. \quad (4)$$

Pick $\tau_p$ so that the objectionable features of the bending mode are reduced while maintaining the PM $\geq 60^\circ$. 

iv. adding a notch filter as described in Section 5.5.3. Pick the frequency of the notch zero to be at $\omega_n$ with a damping of $\zeta = 0.04$ and pick the denominator poles to be $(s/100 + 1)^2$ keeping the DC gain of the filter $= 1$.

(c) Investigate the sensitivity of the two compensated designs above (iii and iv) by determining the effect of a reduction in the bending mode frequency of -10%. Specifically, re-examine the two designs by tabulating the GM, PM, closed loop bending mode damping ratio and resonant peak amplitude, and qualitatively describe the differences in the step response.

(d) What do you recommend to Golden Nugget to help their customers quit spilling their drinks? (Telling them to get back in their seats is not an acceptable answer for this problem! Make the recommendation in terms of improvements to the yaw damper.)

Solution:

(a) The Bode plot of the open-loop system is:

$$PM = 97.6^\circ \ (\omega = 0.0833 \text{ rad/sec})$$

$$GM = 1.28 \ (\omega = 10.0 \text{ rad/sec})$$

The low $GM$ is caused by the resonance being close to instability.
From the Bode plot of the closed-loop system, the frequency of the lightly damped mode is:

$$\omega \approx 10 \text{ rad/sec}$$
and this is borne out by the step response that shows a lightly damped oscillation at 1.6 Hz or 10 rad/sec.

i. The Bode plot of the system with the bending mode damping increased from $\zeta = 0.02$ to $\zeta = 0.04$ is:

\[ PM = 97.6^\circ \ (\omega = 0.0833 \text{ rad/sec}) \]
\[ GM = 7.31 \ (\omega = 10.0 \text{ rad/sec}) \]

and we see that the $GM$ has increased considerably because the resonant peak is well below magnitude 1; thus the system will be much better behaved.

ii. The Bode plot of this system ($\omega_b = 10 \text{ rad/sec} \implies \omega_b = 20\)
and again, the GM is much improved and the resonant peak is significantly reduced from magnitude 1.

iii. By picking up $\tau_p = 1$, the Bode plot of the system with the low
pass filter is:

\[ P_M = 92.9^\circ \ (\omega = 0.0831 \text{ rad/sec}) \]
\[ G_M = 34.97 \text{ dB} \ (\omega = 8.62 \text{ rad/sec}) \]

which are healthy margins and the resonant peak is, again, well below magnitude 1.
iv. The Bode plot of the system with the given notch filter is:

\[
P_M = 97.6^\circ \ (\omega = 0.0833 \text{ rad/sec})
\]
\[
G_M = 55.3 \ (\omega = 99.7 \text{ rad/sec})
\]

which are the healthiest margins of all the designs since the notch filter has essentially canceled the bending mode resonant peak.

(b) Generally, the notch filter is very sensitive to where to place the notch zeros in order to reduce the lightly damped resonant peak. So if you want to use the notch filter, you must have a good estimation of the location of the bending mode poles and the poles must remain at that location for all aircraft conditions. On the other hand, the low pass filter is relatively robust to where to place its break point. Evaluation of the margins with the bending mode frequency lowered by 10% will show a drastic reduction in the margins for the notch filter and very little reduction for the low pass filter.

<table>
<thead>
<tr>
<th></th>
<th>Low Pass Filter</th>
<th>Notch Filter</th>
</tr>
</thead>
<tbody>
<tr>
<td>(G_M)</td>
<td>34.97 ((\omega = 8.62) rad/sec)</td>
<td>55.3 ((\omega = 99.7) rad/sec)</td>
</tr>
<tr>
<td>(P_M)</td>
<td>92.9(^\circ) ((\omega = 0.0831) rad/sec)</td>
<td>97.6(^\circ) ((\omega = 0.0833) rad/sec)</td>
</tr>
<tr>
<td>Closed-loop bending mode damping ratio</td>
<td>(\simeq 0.02)</td>
<td>(\simeq 0.04)</td>
</tr>
<tr>
<td>Resonant peak</td>
<td>0.087</td>
<td>0.068</td>
</tr>
</tbody>
</table>
The magnitude plots of the closed-loop systems are:

![Bode Diagrams](image1)

The closed-loop step responses are:

![Unit Step Response](image2)

(c) While increasing the natural damping of the system would be the best solution, it might be difficult and expensive to carry out. Likewise, increasing the frequency typically is expensive and makes the
structure heavier, not a good idea in an aircraft. Of the remaining two options, it is a better design to use a low pass filter because of its reduced sensivity to mismatches in the bending mode frequency. Therefore, the best recommendation would be to use the low pass filter.

Problems and Solutions for Section 6.8

59. A feedback control system is shown in Fig. 6.106. The closed-loop system is specified to have an overshoot of less than 30% to a step input.

(a) Determine the corresponding PM specification in the frequency domain and the corresponding closed-loop resonant peak value \( M_r \). (See Fig. 6.37)

(b) From Bode plots of the system, determine the maximum value of \( K \) that satisfies the PM specification.

(c) Plot the data from the Bode plots (adjusted by the \( K \) obtained in part (b)) on a copy of the Nichols chart in Fig. 6.73 and determine the resonant peak magnitude \( M_r \). Compare that with the approximate value obtained in part (a).

(d) Use the Nichols chart to determine the resonant peak frequency \( \omega_r \) and the closed-loop bandwidth.

Solution:

(a) From Fig. 6.37:

\[
M_p \leq 0.3 \implies PM \geq 40^\circ \implies M_r \leq 1.5
\]

resonant peak: \( M_r \leq 1.5 \)

(b) A sketch of the asymptotes of the open loop Bode shows that a PM of \( \approx 40^\circ \) is obtained when \( K = 8 \). A Matlab plot of the Bode can be used to refine this and yields

\[
K = 7.81
\]

for \( PM = 40^\circ \).