ENGIN 112 – Midterm Exam 1

Fall 2009

Prof. Ciesielski and Prof. Kelly

Name: Solutions

ID Number: ____________________

	Maximum	Achieved
Question 1	15	
Question 2	15	
Question 3	20	
Question 4	20	
Question 5	30	
Question 6	20	
Total	120	

This exam is closed book, closed notes. No calculators or other electronic devices allowed. Be concise, but show your work. All your answers must fit in the provided space; no additional worksheets will be accepted.

Write legibly! Unreadable answers will not be graded.

Table with Boolean postulates and theorems is provided.

Time: 120 minutes.

Question 1 (15 points):

Answer the following questions regarding number representations.

a) What is the decimal value of the binary number $(11011101)_2$? (4 points)

$$1\times2^0 + 1\times2^2 + 1\times2^3 + 1\times2^4 + 1\times2^6 + 1\times2^7$$
$$= 1+4+8+16+64+128 = 221$$

Answer: $(221)_{10}$

b) What is the representation of the decimal number $(21)_{10}$ as 6-bit binary number? (4 points)

$21 \div 2 = 10$, rem $= 1 \Rightarrow a_0 = 1$
$10 \div 2 = 5$, rem $= 0 \Rightarrow a_1 = 0$
$5 \div 2 = 2$, rem $= 1 \Rightarrow a_2 = 1$
$2 \div 2 = 1$, rem $= 0 \Rightarrow a_3 = 0$
$1 \div 2 = 0$, rem $= 1 \Rightarrow a_4 = 1$, $a_5 = 0$

Answer: $(010101)_2$

c) What is the binary representation of $(0.625)_{10}$? (3 points)

$.625\times2 = 1 + .25 = a_{-1} = 1$
$.25\times2 = 0 + .5 \Rightarrow a_{-2} = 0$
$.5\times2 = 1 + 0 \Rightarrow a_{-3} = 1$

Answer: $(0.101)_2$

d) What is the 2's complement representation of $(-21)_{10}$ when using 8-bit signed numbers? (4 points)

Use 8-bit 2's complement (21)

binary(21) : 00010101
1's complement : 11101010
+1 : 11101011

Answer: 11101011

Question 2 (15 points):

a) What is the hexadecimal representation of $(189)_{10}$? (5 points)

$189 \div 16 = 11$, rem $= 13 = D \Rightarrow a_0 = D$
$11 \div 16 = 0$, rem $= 11 = B \Rightarrow a_1 = B$

Answer: $(BD)_{16}$

b) Without performing conversion, determine the number of bits needed to represent the number $(323)_{10}$ as a binary number, assuming the following representations (5 points):

- Unsigned integer: $2^8 = 256$, $2^9 = 512 \Rightarrow \boxed{9 \text{ bits}}$

- Signed two's complement form:
  max value is $2^{n-1} - 1$ – so need $2^{n-1} > 323$
  $\Rightarrow n = \boxed{10 \text{ bits}}$

c) Three 8-bit values, $(DE)_{16}$, $(CA)_{16}$, and $(FE)_{16}$, are encoded using an error detecting code with even parity (7 data bits + 1 error detection bit (appended on the right)). Which of the three 8-bit values are error-free and which contain an error? (5 points)

Answer (check one box per line, but show how you derived the result in the space below):

	error-free	contains error
$(DE)_{16}$	✓	
$(CA)_{16}$	✓	
$(FE)_{16}$		✓

DE = 1101 1110 → ok (6 1's)
CA = 11001010 → ok (4 1's)
FE = 11111110 → error (7 1's)

Question 3 (20 points):

Answer the following questions regarding binary arithmetic.

a) What is the binary result of the addition of unsigned binary numbers $(01100111)_2$ and $(10001110)_2$? (7 points)

```
  01100111
  10001110
  --------
  11110101
```

$(11110101)_2$

What is the corresponding decimal value of the result?

Answer: $(245)_{10}$

b) The numbers 01100111 and 10001110 are now represented as 8-bit numbers using signed 2's complement representation. What is their sum and the decimal value of the result? Clearly state if the number is positive or negative (7 points)

$\boxed{\text{Sum} = 11110101}$ (as in part (a))

First bit = 1 ⇒ $\boxed{\text{negative number}}$

```
           11110101
1's compl. : 00001010
+1         : 00001011
```

⇒ represents $-(00001011) = (-11)_{10}$

Answer: $(-11)_{10}$

c) What is the binary result of the multiplication of unsigned binary numbers $(10111010)_2$ and $(111)_2$? (6 points)

```
      10111010
           111
  ------------
      10111010
     10111010
  ------------
    1000101110
    10111010
  ------------
   10100010110
```

Answer: $(10100010110)_2$

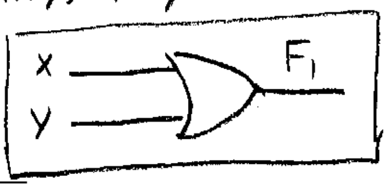
Question 4 (20 points):

Answer the following questions regarding manipulations of expressions in Boolean algebra. Note that for this question minimization with Karnaugh maps is not an acceptable substitute for algebraic manipulation.

a) Simplify function $F_1(x,y)=xy' + x'y + xy$ as sum of product form and draw the resulting logic circuit diagram. (5 points)

$XY'+XY = X(Y'+Y)=X$

$X+X'Y = (X+X')(X+Y)=X+Y$



Answer: $F_1 =$ $X+Y$

b) Represent the same expression as sum of minterms and product of maxterms (5 points).

$X+Y = XY+XY'+XY+X'Y = X'Y+XY'+XY = m_1+m_2+m_3$

Answer: $F_1 = \Sigma\ (1,2,3)$

Answer: $F_1 = \prod\ (0)$

c) What is the simplified representation of the function

$F_2(x,y,z)=z+z(y'x+y')+(z+x')(y+x)$?

Express your solution as a sum of products. (5 points)

$Z+Z(Y'X+Y') = Z$; $Z+Z(Y+X) = Z$

$X'(Y+X) = X'Y$

Answer: $F_2 =$ $X'Y+Z$

d) Given function $F_3(x,y,z) = xz'+z'y'+x'y'$, what is the product of maxterms representation of the complement function F_3' ? (5 points)

$F_3 = \underbrace{XYZ'}_{m_6}+\underbrace{XY'Z'}_{m_4}+\cancel{XY'Z'}+\cancel{X'Y'Z'}+\underbrace{X'Y'Z}_{m_1}+\underbrace{X'Y'Z'}_{m_0}$

$\Rightarrow F_3' = M_0\, M_1\, M_4\, M_6$

Answer: $F_3' =$ $\Pi(0,1,4,6) = (X+Y+Z)(X+Y+Z')(X'+Y+Z)(X'+Y'+Z)$

Question 5 (30 points):

Answer the following questions regarding minimizing Boolean functions with Karnaugh maps.

a) Show the Karnaugh map for function $F_4(x,y,z) = \sum(1,4,5,6)$. List all prime implicants. Which prime implicants are essential? Write down the algebraic expression for the minimized function. (10 points)

x \ yz	00	01	11	10
0		1		
1	1	1		1

Prime implicants: $\Sigma(1,5) = Y'Z$, $\Sigma(4,6) = XZ'$, $\Sigma(4,5) = XY'$

Essential prime implicants: $Y'Z$, XZ'

Simplified $F_4 =$ $XZ' + Y'Z$

b) Show the Karnaugh map for incompletely specified Boolean function: $F_5(A,B,C,D) = \sum(0,1,8,9,10,11,13,15)$ with don't care set $d_5(A,B,C,D) = \sum(2,5,6,7,14)$. Write down the algebraic expression for the minimized function. (10 points)

AB \ CD	00	01	11	10
00	1	1		X
01		X	X	X
11		1	1	X
10	1	1	1	1

Simplified $F_5 =$ $AD + AB' + B'C'$

(Several other answers are also correct – eg: $AC + B'C' + C'D$; $AC + B'C' + AD$; $AB' + BD + B'C'$; $AD + C'D + B'D'$; etc.)

c) Show the Karnaugh map for $F_6(A,B,C) = \sum(0,1,2)$. Write down the algebraic expression for the minimized function. Convert the algebraic expression such that it only uses a minimal number of 2-input NAND functions. Draw the circuit diagram that implements the function with NAND gates (AND-invert variety). Assume that only the original form of the variables are available as inputs (i.e., you need to generate the complements of A, B, and C if necessary). (10 points)

A \ BC	00	01	11	10
0	1	1		1
1				

$$F_6 = A'B' + A'C'$$

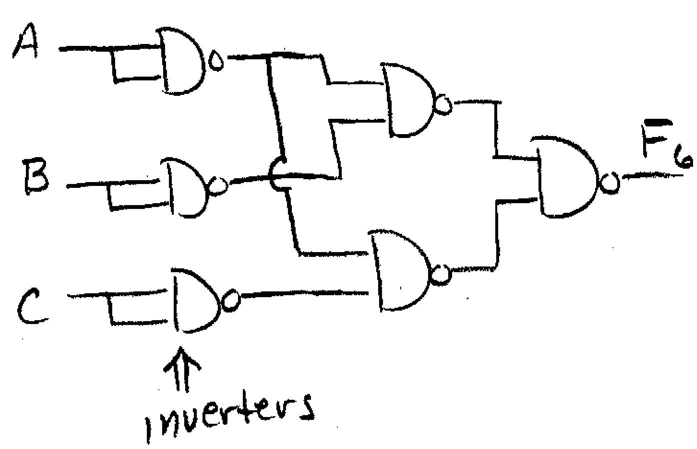
$$= \{(A'B' + A'C')'\}'$$

$$= \{(A'B')' \cdot (A'C')'\}'$$

$$= (A' \text{ NAND } B') \text{ NAND } (A' \text{ NAND } C')$$

Minimized F_6= $A'B' + A'C'$

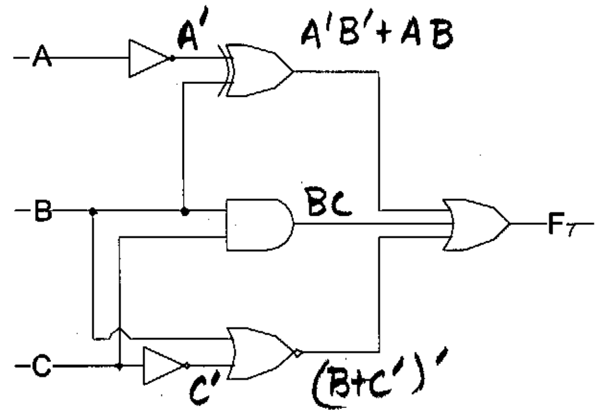
Circuit diagram:



Question 6 (20 points):

Answer the following questions regarding minimizing circuits.

a) What is the algebraic expression for F_7 that corresponds to the following circuit (exactly as shown, no minimization, using only NOT, AND, OR)? (10 points)



Answer: $F_7 = (AB + A'B') + BC + (B+C')'$

b) Modify the algebraic expression to a sum of minterms. (5 points)

$(B+C')' = B'C$ – So: $F_7 = AB + A'B' + BC + B'C$

$= \underbrace{ABC}_{m_7} + \underbrace{ABC'}_{m_6} + \underbrace{A'B'C}_{m_1} + \underbrace{A'B'C'}_{m_0} + \cancel{ABC} + \underbrace{A'BC}_{m_3} + \underbrace{AB'C}_{m_5} + \cancel{A'B'C}$

Answer $F_7 = A'B'C' + A'B'C + A'BC + AB'C + ABC' + ABC$

$= \Sigma(0, 1, 3, 5, 6, 7)$

c) What is the product of maxterm representation of this function? (5 points)

$F_7 = M_2 M_4 = (A + B' + C)(A' + B + C)$

Answer $F_7 = (A + B' + C)(A' + B + C)$

$= \Pi(2, 4)$