1. **Problem.**
   Using Eq. (158) (page 96) of the Notes and using the values for the $k \cdot p$ parameters $A$, $B$, and $C$ listed there for Si and Ge, calculate the effective masses for the heavy- and light-hole bands along the [100], [110] and [111] directions.

   **Solution.**
   a. Along the [100] direction we have $k = k(1, 0, 0)$. Inserting into Eq. (158):

   $$E_{100}(k) = \frac{\hbar^2 k^2}{2m} (A \pm B).$$  \hspace{0.5cm} (1)

   Note that I have reversed the sign of the energy, measuring kinetic energies. Thus, all masses will be negative electron masses, corresponding to positive hole masses. Since the effective mass along a given direction is defined as (see the equation after Eq. (180) on page 104 of the Notes)

   $$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k^2},$$  \hspace{0.5cm} (2)

   we have from Eq. (1):

   $$m^* = \frac{m}{A \pm B}. \hspace{0.5cm} \hspace{0.5cm} (3)$$

   With the parameters given in the Notes (from Ridley’s text, p. 32), we have:

   $$m_{hh} = 0.34, \hspace{0.5cm} m_{lh} = 0.19 \hspace{0.5cm} \text{(Si)}, \hspace{0.5cm} \hspace{0.5cm} (4)$$

   and

   $$m_{hh} = 0.21, \hspace{0.5cm} m_{lh} = 0.05 \hspace{0.5cm} \text{(Ge)}. \hspace{0.5cm} (5)$$
b. Along the [110] direction we have \( \mathbf{k} = (k/\sqrt{2})(1, 1, 0) \). Inserting into Eq. (120):

\[
E_{110}(k) = \frac{\hbar^2 k^2}{2m} \left[ A \pm \left( B^2 + C^2/4 \right)^{1/2} \right].
\] (6)

Then:

\[
m^* = \frac{m}{A \pm (B^2 + C^2/4)^{1/2}}.
\] (7)

so that:

\[
m_{hh} = 0.60, \quad m_{lh} = 0.16 \quad \text{(Si),}
\] (8)

and

\[
m_{hh} = 0.37, \quad m_{lh} = 0.04 \quad \text{(Ge).}
\] (9)

c. Finally, along the [111] direction we have \( \mathbf{k} = (k/\sqrt{3})(1, 1, 1) \) and

\[
E_{111}(k) = \frac{\hbar^2 k^2}{2m} \left[ A \pm \left( B^2 + C^2/3 \right)^{1/2} \right].
\] (10)

Then:

\[
m^* = \frac{m}{A \pm (B^2 + C^2/3)^{1/2}}.
\] (11)

so that:

\[
m_{hh} = 0.72, \quad m_{lh} = 0.15 \quad \text{(Si),}
\] (12)

and

\[
m_{hh} = 0.48, \quad m_{lh} = 0.04 \quad \text{(Ge).}
\] (13)

2. **Problem.**

Using the approximate \( \mathbf{k} \cdot \mathbf{p} \) nonparabolic correction, Eq. (159), assuming \( \gamma(k) = \hbar^2 k^2/(2m^*) \), derive the
expression for the group velocity as a function of \( k \) to first-order in the nonparabolicity parameter \( \alpha \).

**Solution.**

From the definition of the group velocity (page 105 of the notes)

\[
v = \frac{1}{\hbar} \frac{dE}{dk} = \frac{\hbar k}{m^*} \left[ 1 + \alpha \frac{\hbar^2 k^2}{m^*} \right] \approx \frac{\hbar k}{m^*} \left[ 1 + 2 \alpha E(k) \right],
\]

the last step being valid to first order in \( \alpha \).

3. **Problem.**

In order to derive the ‘envelope equation’, Eq. (179), we have assumed that the potential varies slowly enough to render inter-band coupling negligible. An important exception to this assumptions is given by electrons tunneling from the top of the valence band to the bottom of the conduction band when we apply to the semiconductor an electric field sufficiently strong. This is called ‘Zener’ (or ‘band-to-band’) tunneling and this process represents an important cause of breakdown in heavily-doped junctions.

a. Using the WKB approximation outlined in the first Homework assignment, assuming a constant effective mass for the valence band, conduction band, and also inside the gap (yes, it has a meaning, ask me if you would like to know), calculate the tunneling probability

\[
T(E) = \exp \left\{ -2 \int_0^d \kappa(x) \, dx \right\},
\]

where \( \kappa(x) = \left\{ 2m^* \left[ V(x) - E \right] \right\}^{1/2}/\hbar \) is the imaginary component of the electron wavevector in the gap. Assume a field \( E = 10^5 \) V/cm, a gap of 1.42 eV (appropriate for GaAs) and an effective mass of 0.063 \( m_0 \), where \( m_0 = 9.1 \times 10^{-31} \) Kg is the free electron mass. Use the help provided by the figure below.

b. Repeat the calculation for InAs \( (m^* = 0.031m_0, E_{gap} = 0.36 \) eV).  

c. If you have time, plot \( T(E) \) for GaAs and InAs as a function of \( E \) in the range \( 5 \times 10^4 - 2 \times 10^6 \) V/cm.
Solution.
Setting the zero of the energy scale at the top of the valence band at \( x = 0 \) in the figure, \( V(x) = E_{gap} - e E x \), and \( d = E_{gap}/(e E) \), so that \( \kappa(x) = [2m^*(E_{gap} - e E x)]^{1/2}/\hbar \). Therefore:

\[
T(\mathcal{E}) = \exp \left\{ -\frac{2(2m^*)^{1/2}}{\hbar} \int_0^{E_{gap}/(e E)} [E_{gap} - e E x]^{1/2} \, dx \right\} .
\] (16)
Let’s change the integration variable to $y = E_{gap} - e E x$:

$$
T(\mathcal{E}) = \exp \left\{ - \frac{2(2m^*)^{1/2}}{\hbar e \mathcal{E}} \int_0^{E_{gap}} y^{1/2} \, dy \right\} = \exp \left\{ - \frac{4(2m^*)^{1/2} E_{gap}^{3/2}}{3e \hbar \mathcal{E}} \right\}. \quad (17)
$$

Now it’s just a matter of plotting the function $T$. Note how the smaller gap of InAs renders the tunneling probability exponentially larger.
InAs

GaAs \times 10^6