ECE 584 : Problem Set #1, F'2009

Due: Tuesday, Sept. 29
2:30 pm
(in class)

1.) Let $\bar{k} = \hat{x} k_x + \hat{y} k_y + \hat{z} k_z$

$\bar{r} = \hat{x} x + \hat{y} y + \hat{z} z$

a.) Show that $\nabla e^{-j\bar{k}\cdot\bar{r}} = -j\bar{k}\, e^{-j\bar{k}\cdot\bar{r}}$

by calculating all individual derivatives and reassembling the $\bar{k}$ vector.

b.) Assuming a plane wave solution to Maxwell's equations, $\bar{E}(x,y,z) = \bar{E}_o e^{-j\bar{k}\cdot\bar{r}}$, where $\bar{E}_o$ is a constant vector, and that there are no free charges $(\nabla\cdot\bar{E} = 0)$, show that the electric field vector is always perpendicular to the direction of propagation, $\hat{k}$

2.) Do problems 1.1, 1.4, 1.6 and 1.10

NOTE: It is ok to work in groups, but all work should ultimately be your own. That is, you should understand everything that you write down, because this will be important while taking the exams.

ECE 584 F'09 Ch#1 Problem Set

Problem #1

a.) $\nabla e^{-j\bar{k}\cdot\bar{r}} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) e^{-j(k_x x + k_y y + k_z z)}$

$$= -j\left(\hat{x}k_x + \hat{y}k_y + \hat{z}k_z\right) e^{-j\bar{k}\cdot\bar{r}}$$

$$= -j\bar{k}\, e^{-j\bar{k}\cdot\bar{r}}$$

b.) If $\bar{E} = \bar{E}_o e^{-j\bar{k}\cdot\bar{r}}$ then

$$\nabla\cdot\bar{E} = \nabla\cdot\left(\bar{E}_o e^{-j\bar{k}\cdot\bar{r}}\right) = \bar{E}_o\cdot\nabla e^{-j\bar{k}\cdot\bar{r}} + e^{-j\bar{k}\cdot\bar{r}}\,\nabla\cdot\bar{E}_o$$

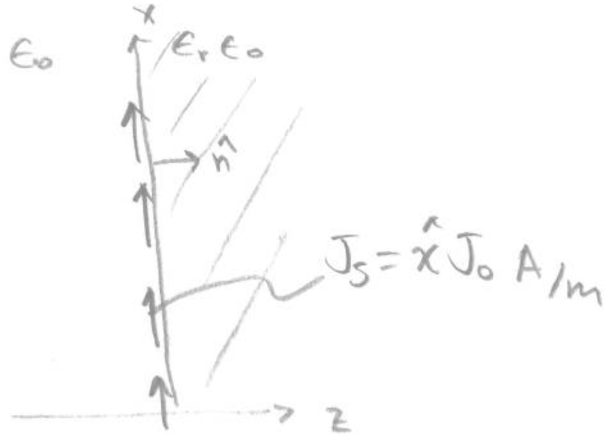
$$= \bar{E}_o\cdot\nabla e^{-j\bar{k}\cdot\bar{r}}$$ since $\bar{E}_o$ is constant in x, y & z.

Then $\nabla\cdot\bar{E} = \bar{E}_o(-j\bar{k})e^{-j\bar{k}\cdot\bar{r}}$

$$= -j(\bar{E}_o\cdot\bar{k})e^{-j\bar{k}\cdot\bar{r}}$$

if $\nabla\cdot\bar{E} = 0$, then $(\bar{E}_o\cdot\bar{k})$ must equal zero. Hence, no component of E can be in the direction of propagation.

Problem 1.1



Since there is no incoming field, and the current is uniform in $\hat{x}$ & $\hat{y}$, assume outgoing plane waves in each region.

$z < 0$	$z > 0$
$\bar{E}_1 = \hat{x} A e^{jk_0 z}$	$\bar{E}_2 = \hat{x} B e^{-jkz}$
$\bar{H}_1 = -\hat{y} \dfrac{A}{\eta_0} e^{jk_0 z}$	$\bar{H}_2 = +\hat{y} \dfrac{B}{\eta} e^{-jkz}$
$\eta_0 = \sqrt{\dfrac{\mu_0}{\epsilon_0}} \quad k = \omega\sqrt{\mu_0 \epsilon_0}$	$\eta = \sqrt{\dfrac{\mu}{\epsilon}} \quad k = \omega\sqrt{\mu\epsilon}$

The boundary condition at $z=0$ is

$$\hat{n} \times (\bar{H}_2 - \bar{H}_1) = J_S \qquad \text{(see 1.37)}$$

and

$$(\bar{E}_2 - \bar{E}_1) \times \hat{n} = 0 \qquad (1.36)$$

Hence $A = B$ from (1.36) and $(\hat{z} \times \hat{y})(H_2 - H_1) = \hat{x} J_0$

$$\Rightarrow (-\hat{x})\left[\frac{B}{\eta} + \frac{A}{\eta_0}\right] = \hat{x} J_0 \Rightarrow \frac{B}{\eta} + \frac{A}{\eta_0} = -J_0 \quad \text{from (1.37)}$$

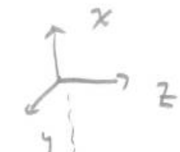
Which gives $A\left(\dfrac{1}{\eta} + \dfrac{1}{\eta_0}\right) = -J_0 \Rightarrow A\left(\dfrac{\eta_0 + \eta}{\eta\eta_0}\right) = -J_0$

OR $\boxed{A = B = -J_0 \dfrac{\eta\eta_0}{\eta + \eta_0}}$

Problem 1.4

$E_y = E_0 \cos(\omega t - kz)$ If $E_0 = 30\,V/m$, $f = 2.4\,GHz$ and $\epsilon_r = 2.55$

a.) what is the amplitude & direction of the magnetic field?



$$\eta \stackrel{\Delta}{=} \frac{E_y}{H_x} = \sqrt{\frac{\mu}{\epsilon}} = \frac{1}{\sqrt{\epsilon_r}}\eta_0 = \frac{377\Omega}{\sqrt{2.55}} = 236\Omega$$

since field is propagating in $+\hat{z}$-direction $\bar{S} = \bar{E} \times \bar{H}^*$ must be in the $+\hat{z}$-direction. Using the Right-hand rule & that E is in $+\hat{x}$-direction.

$$H_x = -\frac{30\,V/m}{236\,\Omega}\cos(\omega t - kz)$$

$$= -0.127\,\frac{A}{m}\cos(\omega t - kz)$$

b.) $$v_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3\times10^8\,m/s}{\sqrt{2.55}} = 1.88\times10^8\,m/s$$

c.) since $v_p = \frac{\omega}{k} \Rightarrow k = \frac{2\pi}{\lambda} = \frac{\omega}{v_p} = \frac{2\pi(2.4\times10^9\,Hz)}{1.88\times10^8\,m/s}$

$= 80.2$ radians/meter

then $\Delta\phi = \Delta z \cdot k = (1.7 - 0.5\,m)\,\frac{80.2\,rad}{m}$

$= (1.2\,m)\,80.2\,rad/m$

$= 96.2\,rad = 5514^\circ \mod 360^\circ$

$= \underline{114^\circ}$

Problem 1.6

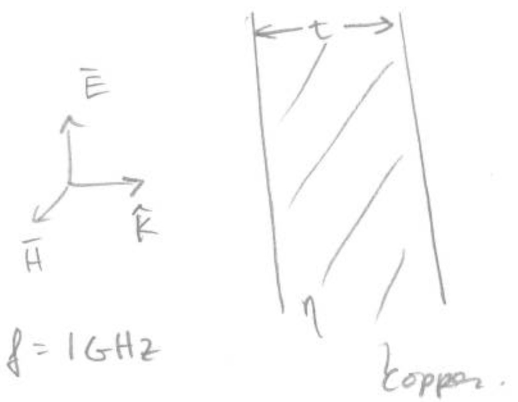
we have $\bar{E} = \bar{E}_0 e^{-j\bar{k}\cdot\bar{r}}$ and $\bar{H} = \frac{1}{\eta_0}\hat{n}\times\bar{E}$ (1.76)

Since $\bar{S} = \bar{E}\times\bar{H}^* \Rightarrow \bar{S} = \frac{1}{\eta_0}\left(\bar{E}_0\times\hat{n}\times\bar{E}_0^*\right)e^{-j\bar{k}\cdot\bar{r}}\,e^{+jk\cdot r}$

$$= \frac{1}{\eta_0}\left[(\bar{E}_0\cdot\bar{E}_0^*)\hat{n} - \underbrace{(\bar{E}_0\cdot\hat{n})}_{=0}\bar{E}_0^*\right] \quad \text{from (B.5) identity.}$$

$$= \frac{1}{\eta_0}|\bar{E}_0|^2\,\hat{n}$$

Problem 1.10



The losses of the copper is computed by

$$\alpha = \sqrt{\frac{\omega\mu\sigma}{2}} = 4.8\times10^5\ Np/m$$

where $\sigma = 5.8\times10^7$ S/m (Appendix F),

and $$\eta = \frac{(1+j)}{\sigma\,\delta_s} = 8.2\times10^{-3}(1+j)\ \Omega$$

a.) The transmission loss at the air copper interface is given by $1-|\Gamma|^2$ where $\Gamma = \frac{\eta_c - \eta_0}{\eta_c + \eta_0} = \frac{8.2\times10^{-3}(1+j)-377}{8.2\times10^{-3}(1+j)+377}$

$$\Gamma \approx \frac{8.2\times10^{-3}(1+j)-377}{377} = -0.999956 + j4.35\times10^{-5}$$

hence $1-|\Gamma|^2 = -40.6$ dB lost to the field entering the copper. Note that by symmetry, this is the same loss experienced at the copper - air interface

Problem 1.6 cont'd

The attenuation within the copper sheet is

$$-20 \log e^{-\alpha t}$$

thickness

loss

20 dB factor because this is power.

Hence, for a 150 dB loss:

$$-20 \log e^{-\alpha t} = 150\ \text{dB} - 40.6\ \text{dB} - 40.6\ \text{dB}$$

first interface

second interface.

$\Rightarrow$ $t = 0.017\ \text{mm}$