ECE344 Midterm Exam
November 18, 2005

1. **Problem.** Consider a sample of silicon at $T = 300$ K. Assume that the electron concentration varies linearly from $n(0)$ at $x = 0$ to 5×10^{14} cm^{-3} at $x = 0.01$ cm. The diffusion current density is measured to be $j_n = 0.19$ A/cm^2. Knowing the electron diffusion constant, $D_n = 25$ cm^2/s, determine the electron concentration $n(0)$ at x =0.

   Solution. The electron diffusion current density is given by Eq. (89) of the Notes:

$$j_{n,diff} \; = \; eD_n \, \frac{dn}{dx} \; . \tag{1}$$

   In our case:

$$e \; 25 \; \frac{dn}{dx} \; = \; e \; 25 \; \frac{5 \times 10^{14} \; - \; n(0)}{0.01} \; = \; 0.19 \; . \tag{2}$$

   Solving for $n(0)$:

$$n(0) \; = \; 5 \times 10^{14} \; - \; \frac{0.19 \times 0.01}{1.6 \times 10^{-19} \times 25} \; = \; 2.5 \times 10^{13} \mathrm{cm}^{-3} \; . \tag{3}$$

2. **Problem.** A p type sample of GaAs is doped with $N_A = 10^{16}$ cm^{-3} and N_D = 0. Assume for the recombination lifetime of both electrons and holes $\tau_n = \tau_p = 2 \times 10^{-7}$ s. Assume also that the sample is under illumination resulting in a constant and spatially uniform generation rate of electron-hole pairs $G = 2 \times 10^{21}$ cm^{-3}/s. Finally, assume $T = 300$ K.

   (a) Calculate the steady-state electron density.

   (b) Assuming for the electron and hole mobilities $\mu_n = 8{,}500$ cm^2/Vs and $\mu_p = 400$ cm^2/Vs, respectively, calculate the change in conductivity due to the illumination.

   Solution. The steady-state electron density can be obtained from Eq. (85) of the Notes noting that the sample is p-type, so in absence of illumination $n \approx 0$, so that $\delta n \approx n$:

$$\frac{dn}{dt} \; = \; G \; - \; \frac{n}{\tau_n} \; . \tag{4}$$

   (a) At steady-state $dn/dt = 0$, so:

$$n \; = \; G \, \tau_n \; = \; 2 \times 10^{21} \times 2 \times 10^{-7} \; = \; 4 \times 10^{14} \; \mathrm{cm}^{-3} \tag{5}$$

(b) The conductivity in the dark can be evaluated recalling that $n \approx 0$ and using Eq. (1) above:

$$\sigma_{dark} \;=\; e\mu_p p \;=\; e\;400\;10^{16}\;. \tag{6}$$

The conductivity under illumination can be evaluated noticing that $\delta p = n = 4 \times 10^{14}$ cm^{-3}:

$$\sigma_{light} \;=\; e\mu_p(p + \delta p) + e\mu_n n \;=\; e\,400\,(10^{16}{+}4{\times}10^{14}) + e\,8500 \times 4{\times}10^{14}\;. \tag{7}$$

Therefore the change in conductivity will be:

$$\Delta\sigma \;=\sigma_{light}\; - \sigma_{dark}\; =$$

$$e\mu_p\delta p + e\mu_n n \;=\; e\,400\,4{\times}10^{14} + e\,8500 \times 4{\times}10^{14} \;=\; e\,8900 \times 4{\times}10^{14} \;=\; 0.57/\Omega\text{cm}\;. \tag{8}$$

3. **Problem.** A Si p-n junction has $N_D = 10^{18}$ cm^{-3} and $N_A = 10^{16}$ cm^{-3}. The junction area is 100 μm^2. Calculate the junction (depletion) capacitance in the absence of any applied bias.

   Solution. The depletion capacitance is given by Eq. (184) of the Lecture Notes:

$$C_{depl} \;=\; \frac{\epsilon_s A}{W}\;. \tag{9}$$

The width of the depletion region – given by Eq. (183) of the notes – requires the knowledge of the built-in potential:

$$V_{bi} \;=\; \frac{k_B T}{e}\ln\left(\frac{N_A N_D}{n_i^2}\right) \;=\; 0.0258\ln(5\times10^{13}) \;=\; 0.814\text{ V}\;. \tag{10}$$

Getting back to the evaluation of W, expressing everything in cm we have:

$$W \;=\; \left(\frac{2\times11.7\times8.85\times10^{-14}\times0.814}{1.6\times10^{-19}} \times \frac{1.01\times10^{18}}{10^{34}}\right)^{1/2} \;=\; 0.325\,\mu\text{m}\;. \tag{11}$$

So, finally:

$$C_{depl} \;=\; \frac{11.7\times8.85\times10^{-14}\times10^{-6}}{3.25\times10^{-5}} \;=\; 3.19\times10^{-14}\text{ F} \;=\; 31.9\text{ fF}\;. \tag{12}$$

4. **Problem.** The current trends in VLSI technology also demand reduced applied biases (*i.e.*, a smaller V_a). Without doing any calculation, do you think that the strictest breakdown limitations for *p*-*n* junctions will be due to impact ionization or to Zener tunneling?

   Solution. Zener tunneling will dominate, since the energy threshold for impact-ionization is $\sim E_G$, while for Zener tunneling is $E_G - eV_{bi}$, where V_{bi} is the built-in potential of the drain/body junction in a MOSFET.

5. **Problem.** **(a)** Consider an $\mathrm{SiO_2}$ gate insulator of thickness t_{ox}. Using Eqns. (231) and (232) of the Lecture Notes, estimate the oxide thickness t_{ox} at which the direct-tunneling current at $F_{ox} = 0$ (Eq. (231)) equals the Fowler-Nordheim tunneling current (Eq. (232)) at $F_{ox} = 10^7$ V/cm. [Use $m_{ox} = 0.5 m_0$, where m_0 is the free electron mass, and $e\phi'_B = 3.2$ eV].

   (b) Based on this result and considering that the current VLSI technology requires ever thinner $\mathrm{SiO2_2}$ films, now approaching 1 nm, which of the two components of the tunneling current (direct or FN) is more important today? For your convenience, below are the equations mentioned above: The direct tunneling current is proportional to:

$$P_d \approx \exp\left\{ - \frac{4(2em_{ox})^{1/2}}{3\hbar F_{ox}} \phi_B'^{3/2} [1 - (1 - t_{ox} F_{ox}/\phi'_B)]^{3/2} \right\} \sim \exp\left\{ - \frac{2(2m_{ox} e\phi'_B)^{1/2}}{\hbar} t_{ox} \right\}$$

$$= e^{-2\overline{\kappa}\ t_{ox}} \ , \qquad (231)$$

while the FN tunneling current is proportional to:

$$P_{FN} \approx \exp\left\{ - \frac{4(2em_{ox})^{1/2}}{3\hbar F_{ox}} \phi_B'^{3/2} \right\} == e^{-(4/3)\overline{\kappa}\ z_t} \ , \qquad (232)$$

Solution. **(a)** The direct tunneling current, Eq. (231) will be equal to the FN tunneling current, Eq. (232) when:

$$e^{-2\bar{\kappa} t_{ox}} = e^{-(4/3)\bar{\kappa} z_t} \ , \qquad (233)$$

which implies:

$$t_{ox} = \frac{2}{3} \frac{\phi'_B}{F_{ox}} \approx 2.1 \text{ nm} \ . \qquad (234)$$

(b) Clearly, since the thickness of the gate oxide is now much smaller than the result of Eq. (234), direct tunneling is becoming the most important component of the gate tunneling current.

6. **Bonus Problem**. The limit of *low injection* is usually defined to be the value of the forward bias required to have a concentration of the minority carriers at the edge of the depletion region in the low-doped side of the junction equal to 1/10 of the concentration of majority carriers in that region. Determine the value of forward bias at which the low-injection limit is reached for a p^+-n Ge diode at $T = 300$ K with $N_D = 10^{16}$ cm^{-3}, $N_A = 10^{18}$ cm^{-3}, Use for the intrinsic carrier concentration of Ge at 300 k the value of 2.5×10^{13} cm^{-3}.

   Solution. By definition we must have:

$$p(x_n) \;=\; \frac{1}{10} n(x_n)\ , \tag{235}$$

since the low-doped side of the junction is the n-type side. Since $p(x_n) = p_{n0} \exp[eV_a/(k_B T)]$ and $n(x_n) = x_{n0} = N_D$, we must have at the onset of strong injection:

$$p_{n0}\ \exp\left(\frac{V_a}{0.0258}\right) \;=\; \frac{1}{10}\ N_D\ , \tag{236}$$

where, as usual, V_a is measured in V. Recalling that $p_{n0} = n_i^2/N_D$, solving for V_a:

$$V_a \;=\; \frac{k_B T}{e}\ \ln\left(\frac{N_D}{10 \times p_{n0}}\right) \;=\; 0.0258 \times \ln(1.6 \times 10^4) \;=\; 0.25\ \text{V}\ . \tag{237}$$