ECE344 Practice Problems for Midterm Exam 2
November 15, 2005

1. **Problem.** An n-type silicon sample has a resistivity of 5 Ω.cm at T = 300 K. Assume that the electron mobility at this temperature is 1,500 cm^2/Vs and that the mobility varies as $T^{-3/2}$.

   (a) What is the donor impurity concentration (assuming that N_A=0)?

   (b) What would the resistivity be at T = 200 K? **(c)** And at T = 400 K?

   Solution. The equation we must use is Eq. (67) of the Lecture Notes: The resistivity ρ – which is the inverse of the conductivity σ – is expressed in terms of the mobility and carrier density as follows:

   $$\rho \;=\; \frac{1}{\sigma} \;=\; \frac{1}{e\mu_n n} \;. \tag{1}$$

   (a) From this equation the electron density (and so the donor concentration) can be calculated:

   $$n \;=\; N_D \;=\; \frac{1}{e\mu_n\rho} \;=\; \frac{1}{1.6\times10^{-19}\times1.5\times10^3\times5} \;=\; 8.33\times10^{14}\ \text{cm}^{-3}\;. \tag{2}$$

   (b) Since the mobility scales with temperature as $T^{-3/2}$, we have $\rho \propto T^{3/2}$, so

   $$\rho(T) \;=\; \rho(300)\left(\frac{T}{300}\right)^{3/2} \;. \tag{3}$$

   Thus:

   $$\rho(200) \;=\; \rho(300)\left(\frac{200}{300}\right)^{3/2} \;=\; 5\;\times\;0.543\ \Omega\text{.cm} =\; 2.72\ \Omega\text{.cm}\;. \tag{4}$$

   (c) Similarly,

   $$\rho(400) \;=\; \rho(300)\left(\frac{400}{300}\right)^{3/2} \;=\; 5\;\times\;1.54\ \Omega\text{.cm} =\; 7.68\ \Omega\text{.cm}\;. \tag{5}$$

2. **Problem.** A Ge p^+-n diode at T = 300 K has the following parameters: $N_D = 10^{16}$ cm^{-3} and $N_A = 10^{18}$ cm^{-3}, D_n = 100 cm^2/s, D_p = 49 cm^2/s, $\tau_n = \tau_p$ = 5 μs and the junction area A is 10^{-4} cm^2. Determine the diode current for **(a)** a forward bias of 0.2 V and **(b)** a reverse bias of -0.2 V.

Solution. The current flowing through the diode will be given by the current density (see Eq. (150) of the Notes) multiplied by the area of the junction:

$$I \;=\; AJ_{total} \;=\; A\left(\frac{eD_p}{L_p}p_{n0} \;+\; \frac{eD_n}{L_n}n_{p0}\right)\;\left[\exp\left(\frac{eV_a}{k_BT}\right)-1\right] \;, \qquad (6)$$

where the diffusion lengths are $L_p = \sqrt{D_p\tau_p} = 1.57\times10^{-2}$ cm and $L_n = \sqrt{D_n\tau_n} = 2.24\times10^{-2}$ cm. Recall also that $p_{n0} = n_i^2/N_D$ and $n_{p0} = n_i^2/N_A$. Using $n_i = 2.5\times10^{13}$ cm^{-3} (see figures on page 95 of the text), we have $p_{n0} = 6.25\times10^{10}$ cm^{-3} and $n_{p0} = 6.25\times10^{8}$ cm^{-3}. So, finally:

$$I \;=\; A\left(\frac{1.6\times10^{-19}\times49\times6.25\times10^{10}}{1.57\times10^{-2}} \;+\; \frac{1.6\times10^{-19}\times100\times6.25\times10^{8}}{2.24\times10^{-2}}\right)$$

$$\times\;\left[\exp\left(\frac{V_a}{0.0258}\right)-1\right]\;(\text{Amp})\;, \qquad (7)$$

where V_a is in V. Thus:

$$I \;=\; (3.14\times10^{-7} \;+\; 4.46\times10^{-9})\left[\exp\left(\frac{V_a}{0.0258}\right)-1\right] \;=$$

$$=\; 3.18\times10^{-9}\;\left[\exp\left(\frac{V_a}{0.0258}\right)-1\right]\;(\text{Amp})\;. \qquad (8)$$

From this equation we get:

$$I(V_a = 0.2V) \;=\; 7.41\;\mu\text{A}\;, \qquad (9)$$

and

$$I(V_a = -0.2V) \;=\; -3.18\;\text{nA}\;. \qquad (10)$$

3. **Problem.** Derive Eq. (165) of the Lecture Notes, Part 2.

Solution. We require:

$$J(x=l_n) \;=\; J \;=\; J\left\{\frac{1}{M_n} \;+\; \int_{-l_p}^{l_n} dx\;\alpha_p\;\exp\left[-\int_{-lp}^{x} dx'(\alpha_n-\alpha_p)\right]\right\}\;\exp\left[\int_{-lp}^{l_n} dx'(\alpha_n-\alpha_p)\right]\;. \qquad (11)$$

Now let's multiply both sides of this equation by:

$$\frac{1}{J}\;\exp\left[-\int_{-lp}^{l_n} dx'(\alpha_n-\alpha_p)\right]\;,$$

so that Eq. (11) becomes:

$$\exp\left[-\int_{-lp}^{l_n} dx'(\alpha_n - \alpha_p)\right] = \frac{1}{M_n} + \int_{-l_p}^{l_n} dx\ \alpha_p\ \exp\left[-\int_{-lp}^{x} dx'(\alpha_n - \alpha_p)\right] , \tag{12}$$

or

$$\frac{1}{M_n} = \exp\left[-\int_{-lp}^{l_n} dx'(\alpha_n - \alpha_p)\right] - \int_{-l_p}^{l_n} dx\ \alpha_p\ \exp\left[-\int_{-lp}^{x} dx'(\alpha_n - \alpha_p)\right] . \tag{13}$$

Now let's add and substract α_n in the integrand of the last term:

$$\frac{1}{M_n} = \exp\left[-\int_{-lp}^{l_n} dx'(\alpha_n - \alpha_p)\right] + \int_{-l_p}^{l_n} dx\ (\alpha_n - \alpha_p)\ \exp\left[-\int_{-lp}^{x} dx'(\alpha_n - \alpha_p)\right] -$$
$$- \int_{-l_p}^{l_n} dx\ \alpha_n\ \exp\left[-\int_{-lp}^{x} dx'(\alpha_n - \alpha_p)\right] . \tag{14}$$

The second term on the right-hand side can be integrated immediately, since in general:

$$\int_a^b dx\ f(x)\ \exp\left(-\int_a^x dx' f(x')\right) = -\exp\left(-\int_a^x dx' f(x')\right)\Big|_a^b = 1 - \exp\left(-\int_a^b dx' f(x')\right) .$$

So, Eq. (14) becomes:

$$\frac{1}{M_n} = \exp\left[-\int_{-lp}^{l_n} dx'(\alpha_n - \alpha_p)\right] + 1 - \exp\left[-\int_{-lp}^{l_n} dx'(\alpha_n - \alpha_p)\right] -$$
$$- \int_{-l_p}^{l_n} dx\ \alpha_n\ \exp\left[-\int_{-lp}^{x} dx'(\alpha_n - \alpha_p)\right] = 1 - \int_{-l_p}^{l_n} dx\ \alpha_n\ \exp\left[-\int_{-lp}^{x} dx'(\alpha_n - \alpha_p)\right] , \tag{15}$$

so that

$$1 - \frac{1}{M_n} = \int_{-l_p}^{l_n} dx\ \alpha_n\ \exp\left[-\int_{-lp}^{x} dx'(\alpha_n - \alpha_p)\right], \tag{16}$$

which is Eq. (165) of the Lecture Notes, Part 2.

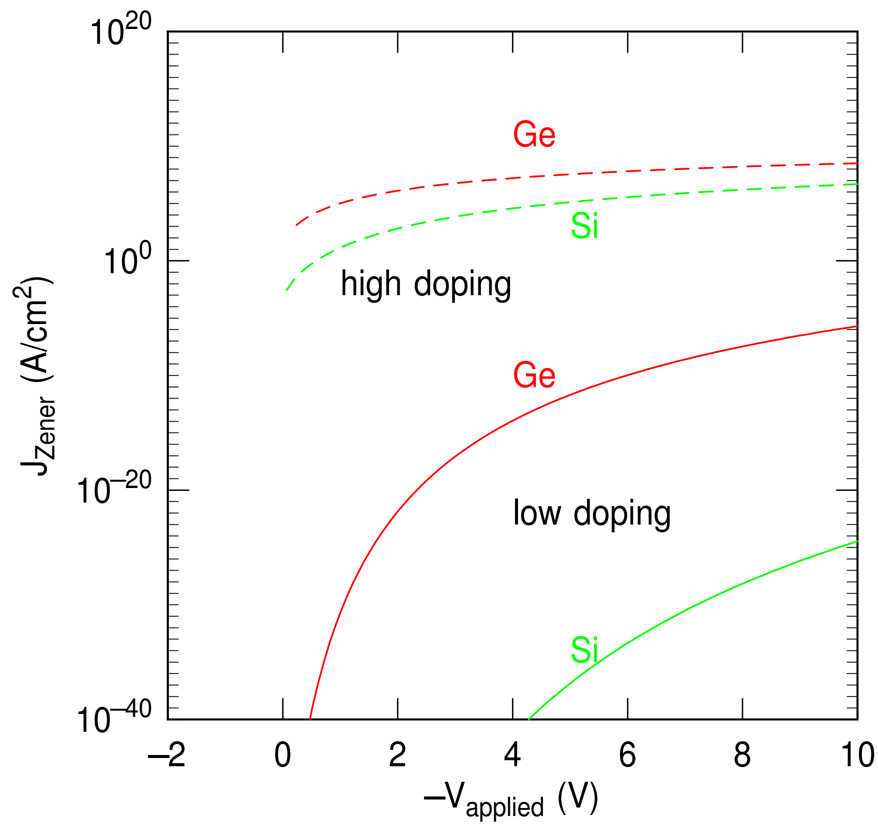
Note that the result above is completely general. We have not made use of the assumption $\alpha_n = \alpha_p$, nor of any arbitrary assumption of the type

$$\exp\left[\int_{-l_p}^{x} dx(\alpha_n - \alpha_p)\right] = \exp\left[(\alpha_n - \alpha_p)(x + l_p)\right] ,$$

which is only valid if α_n and α_p do not depend on x, and, finally, it is valid also far from breakdown, so it does not require any condition of the sort $\int_{-l_p}^{l_n} dx\alpha = 1$.

4. **Problem.** **(a).** Plot the Zener tunneling current J_{Zener} given by Eq. (172) of the Lecture Notes as a function of applied bias V_a using $m^* = 0.32\ m_{el} = 0.32 \times 9.1\ 10^{-31}$ kg and $E_G = 1.1$ eV (the gap of Si). Assume for the field F the approximate value $F \approx F_{max}$, where the maximum field in the junction, F_{max}, is given by Eq. (131). Assume $N_A = 10^{17}\text{cm}^{-3}$ and $N_D = 10^{19}\text{cm}^{-3}$ to estimate the width l_p and l_n of the depletion regions.
   (b). Repeat the calculation, but now using $E_G = 0.64$ eV (the gap of Ge) and $m^* = 0.22\ m_{el}$.
   (c). Repeat the calculations in **(a)** and **(b)** but now assuming higher doping, $N_A = 10^{19}\text{cm}^{-3}$ and $N_D = 10^{21}\text{cm}^{-3}$. What happens? Why?
   (d). Considering that the current VLSI technolgy requires increasingly higher doping, can you foresee any problems with the possible use of Ge (instead of Si) when the width of the depletion regions shrinks?
   Solution. Higher doping enhances tunneling. Ge exhibits a much larger Zener tunneling current than Si because of its smaller band-gap. This drawback will become more severe as the doping concentrations increase.
   The figure below shows the result: Note that the current is nonzero only under reverse bias such that $V_a < -(E_G - V_{bi})$ or under forward bias $V_a > E_G + V_{bi}$. For a bias of smaller magnitude, $|V_a|$, tunneling cannot occurr because there are no final states available. The plot show only the current in reverse bias, since this matters in devices. (Under forward bias the diode current is very large anyway and tunneling is not a concern.)
   NOTE: The most important dependence of the Zener tunneling current on the applied reverse bias V_a is via the depletion width l_p appearing in the expression for the field $F_{max} = eN_A l_p/\epsilon_s$. Do not forget this crucial dependence!

5. **Problem.** Looking at the plots on page 127 of the Lecture Notes and following the discussion leading to the figure on page 138, could you draw a similar picture for the electron charge in the GaAs-side of the GaAs/AlGaAs heterojunction? Are there major differences between this junction and the MOS 'diode'?



Ge
Si
high doping
Ge
low doping
Si
10^20
10^0
10^−20
10^−40
−2
0
2
4
6
8
10
J_Zener (A/cm^2)
−V_applied (V)

Solution. The situation is quite analogous, strong reverse bias corresponding to inversion and forward bias to accumulation. The main differences stem from the fact under forward bias the large current flowing through the junction will prevent reaching complete accumulation. Also, under strong reverse bias there will be a large tunneling component of the current, much larger than in the case of the MOS diode since the barrier electrons see in tunneling from GaAs into AlGaAs is an order of magnitude smaller.