Homework 6

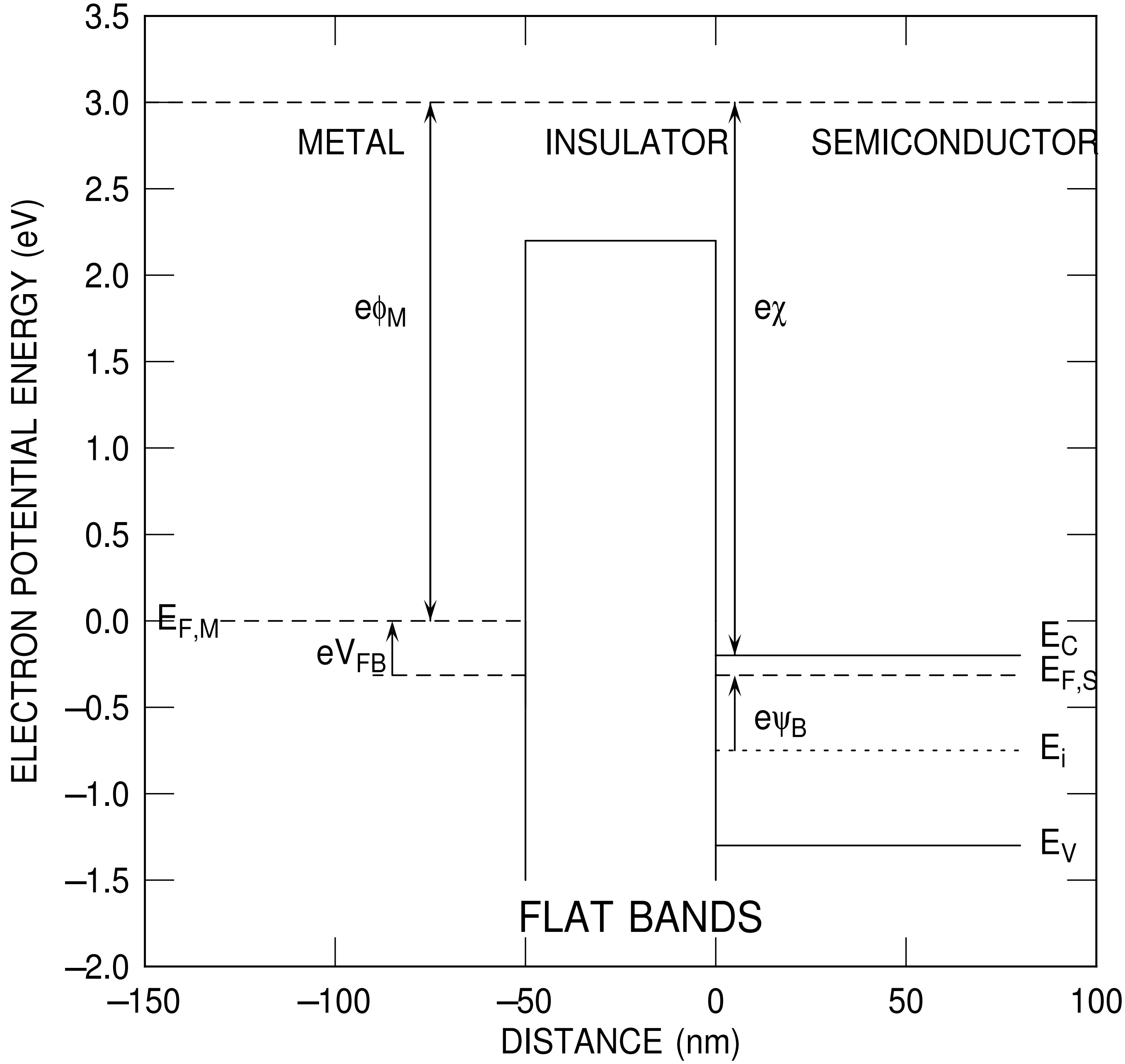
1. **Problem:**
(a) Using the figures on pages 133 and 134 of the Lecture Notes, Part II, as a guide, sketch the band diagram of an MOS capacitor with an n-type Si substrate in **i)** accumulation, **ii)** at flat band condition, and **iii)** at the onset of strong inversion (that is, $\psi_s = 2\psi_B$). Assume an Al gate with electron affinity $e\phi_M = 3.0$ eV, use for Si the work-function $e\chi$ = 3.2 eV, and assume the n-type substrate to be doped with $N_D = 3 \times 10^{17}$ donors/cm^3.
(b) What is the value of the flatband voltage V_{FB} and of the threshold voltage V_{T0} (see Eq. (247) of the Notes, Part III)?
(c) What is the value of the depletion capacitance C_D at the onset of strong inversion?

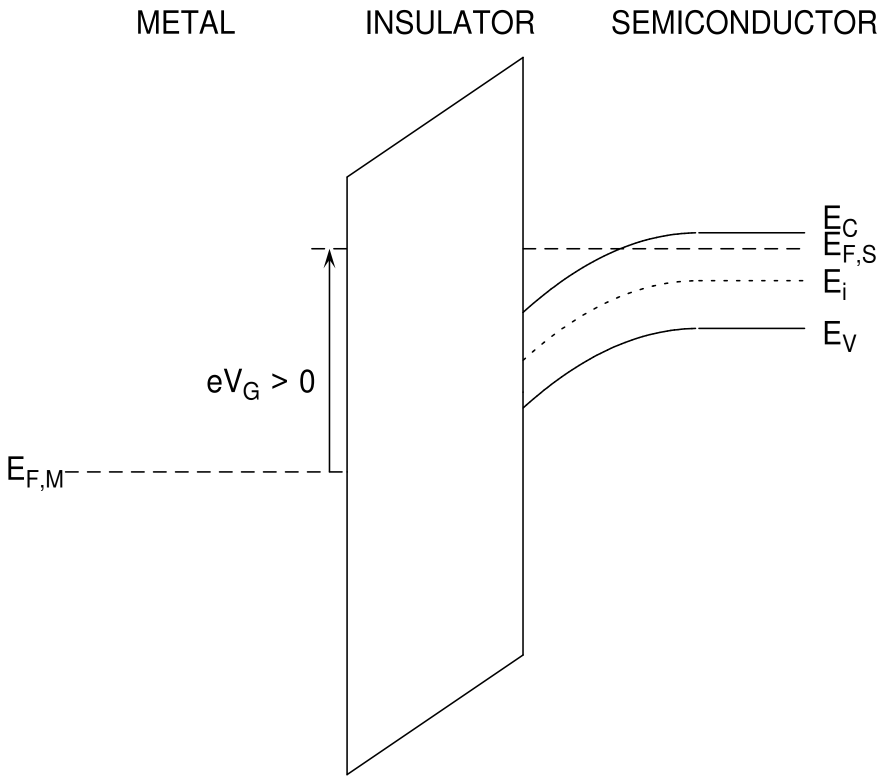
Solution:
(a) See the figures.
(b): Looking at the first figure, the distance between the metal Fermi level and the vacuum level (the topmost dashed horizontal line) is given by:

$$|V_{FB}| \ + \ e\phi_M \ = \ |V_{FB}| \ + \ 3.0 \text{ eV} \ . \tag{1}$$

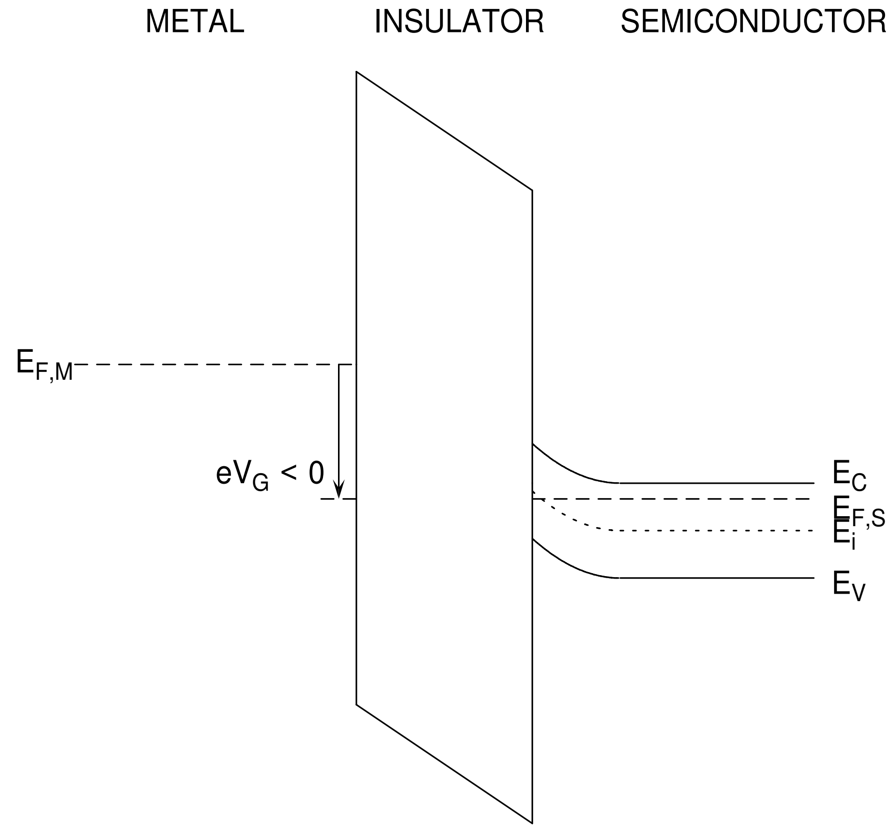
Similarly, the distance between the Si Fermi level and the vacuum level will be given by:

$$e\chi \ + \ E_i \ - \ \psi_B \ = 3.2 \ + \ 0.55 \ - \ 0.435 \text{ eV} \ = \ 3.315 \text{ eV} \ , \tag{2}$$


3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
–0.5
–1.0
–1.5
–2.0
ELECTRON POTENTIAL ENERGY (eV)
METAL
INSULATOR
SEMICONDUCTOR
eφM
eχ
EF,M
eVFB
EC
EF,S
eψB
Ei
EV
FLAT BANDS
–150
–100
–50
0
50
100
DISTANCE (nm)



ACCUMULATION



INVERSION

where $\psi_B = k_B T \ln(N_D/n_i)$ is the distance between the Fermi level and the intrinsic level E_i. From Eqns. (1) and (2), noticing that V_{FB} must be negative (since it raises the potential energy for electrons in the gate), we have V_{FB} = - 0.315 V.

Finally, the threshold voltage is given by Eq. (247) of the Notes with $\gamma = (2\epsilon_{Si} e N_D)^{1/2}$ (with N_D replacing N_A since we are dealing with an n-type substrate. Thus, using $C_{ox} = \epsilon_{ox}/t_{ox}$ and t_{ox} = 5 nm (from Problem 2):

$$C_{ox} = \frac{11.7 \times 8.85 \times 10^{-12} \text{F/m}}{5 \times 10^{-9} \text{m}} = 6.9 \times 10^{-3} \text{F/m}^2 \, . \quad (3)$$

To compute the body-factor γ let's convert the doping density to m^{-3}:

$$\gamma \;=\; \frac{(2 \times 1.6 \times 10^{-19} \times 11.7 \times 8.85 \times 10^{-12} \times 3 \times 10^{23})^{1/2}}{6.9 \times 10^{-4}} \;\approx\; \frac{9.22 \times 10^{-4}}{6.9 \times 10^{-3}} \;=\; 0.134 \text{ V}^{1/2} \tag{4}$$

Thus:

$$V_{T0} \;=\; V_{FB} \;+\; 2\psi_B \;+ \gamma\;(2\psi_B)^{1/2} \;\approx\; -0.315 \;+\; 0.87 \;+ 0.125 \;=\; 0.68 \text{ V} \tag{5}$$

(c) The depletion capacitance is given by:

$$C_D \;=\; \frac{\epsilon_{Si}}{W_D} \;, \tag{6}$$

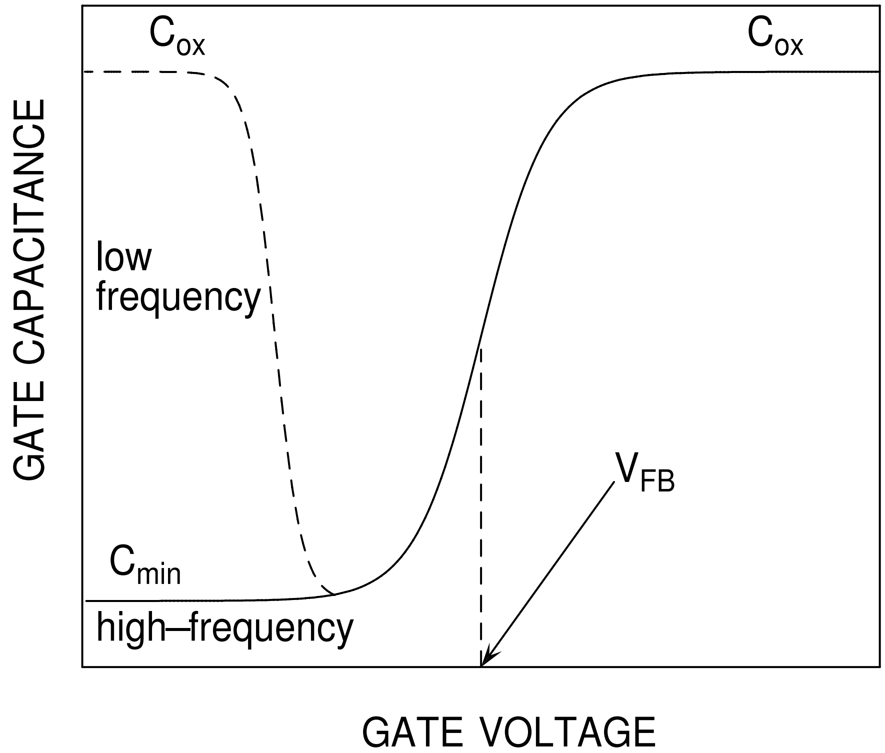
where $W_D = [2\epsilon_{Si}\psi_s/(eN_D)]^{1/2}$ is the depletion width. The minimum capacitance is obtained when the depletion width reaches its maximum value, for $\psi_s = 2\psi_B$. Using the value for ψ_B calculated above we have:

$$W_{D,max} \;=\; \left(\frac{4\epsilon_{Si}\psi_B}{eN_D}\right)^{1/2} \;\approx\; 61.3 \text{ nm} \;. \tag{7}$$

So:

$$C_{D,min} \;=\; \frac{11.7 \times 8.85 \times 10^{-12}\text{F/m}}{6.13 \times 10^{-8}\text{m}} \;=\; 1.7 \times 10^{-7} \text{ F/cm}^2 \tag{8}$$

2. **Problem:**
Assuming an oxide thickness t_{ox} of 5 nm and a gate area of 100 μm^2, sketch the capacitance-voltage $(C - V)$ characteristics of the MOS capacitor of problem 1, as in the figure at page 140 of the Lecture Notes. Recall that this capacitor is on an n-type substrate!
Solution:
See the following figure:



Note that we have already calculated $C_{ox} \approx 6.9 \times 10^{-3}$ F/m^2 and $C_D \approx 1.7 \times 10^{-3}$ F/m^2. Since the area of the capacitor is 100 μm$^2 = 10^{-6}$cm^{-2}, we have $C_{ox} \approx$ 0.69 pF, $C_D \approx$ 0.17 pF, so that the minimum capacitance at high frequency in inversion will be $C_{min} = 1/(1/C_{ox} + 1/C_D) \approx 0.14$ pF.

3. **Problem:**
   Consider a Si nMOSFET with the following parameters:
   substrate doping (p-type): $N_A = 5 \times 10^{16}$ cm^{-3}
   channel length L = 0.5 μm
   gate width W = 5 μm
   electron mobility μ_n = 600 cm^2/Vs
   oxide thickness t_{ox} = 15 nm
   flatband voltage V_{FB} = 0.
   Using the simplified model of the Lecture Notes, pages 159-162, especially Eq. (255), plot as accurately as you

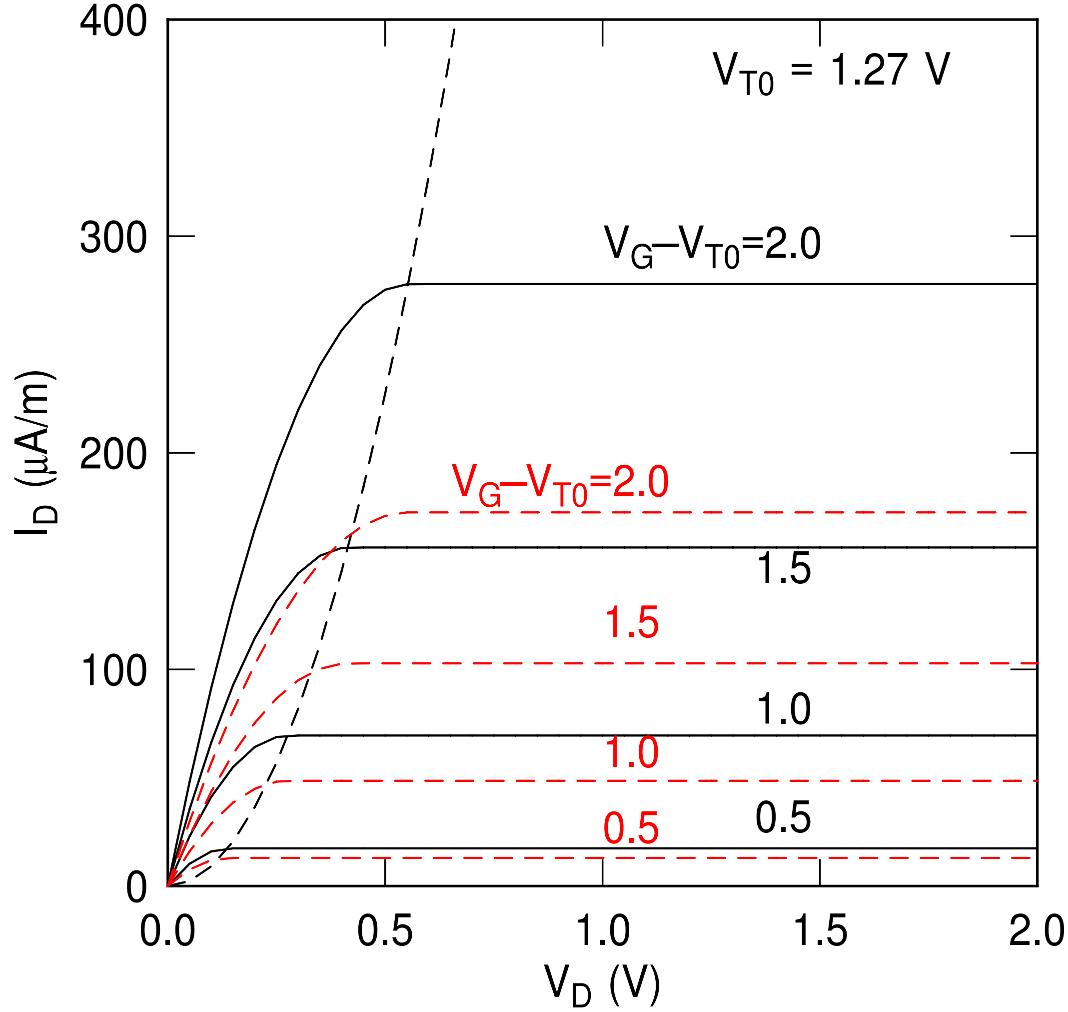
can the $I_D - V_D$ characteristics of the device assuming that the source contact is grounded (that is, V_S=0). More specifically, plot I_D-vs-V_D for $V_G - V_{T0} =$ 0.0, 0.5, 1.0. 1.5, and 2.0 V for V_D ranging from 0 to 5 V. Indicate as best as you can the separation between the linear and saturated region by computing $V_{D,sat}$ for the various values of $V_G - V_{T0}$. Also indicate the value of the threshold voltage V_{T0}.

Solution:

The figure below shows the drain current as a function of drain bias in four curves (black, solid lines) parametrized by the gate overdrive $V_G - V_{T0}$. Note that $V_{T0} =$ 1.27 V and $n =$ 1.32. The parabola indicated by a dashed black line is given by the expression:

$$I_D = \mu \, C_{ox} \, \frac{W}{L} \, \frac{n}{2} \, V_D^2 \, , \tag{9}$$

and it separates the linear (left) from the saturated region (right). When V_D exceeds $V_{D,sat}$ – so, at the right of the dashed parabola – Eq. (255) is not valid, as explained at page 160, immediately after Eq. (255), because the channel is pinched off. Many of you, especially those who have employed a computer to evaluate numerically the current, have forgotten this fact and have used Eq. (255) also when $V_D > V_{D,sat}$, getting negative values for the current. Not good...



4. **Problem:**
As in problem 3 above, plot the same $I_D - V_D$ characteristics, but now account for the degradation of the electron mobility with increasing V_G via Eq. (298) of the Lecture Notes. To do this, just replace μ_n in Eq. (255) with μ_{eff} given by Eq. (298). Use a value of K such that $KC_{ox}/(2\epsilon_s) = 0.5$ V^{-1}.
Solution:
The drain current is shown in the figure above by red dashed lines.

5. **Problem:**

Consider a Si nMOSFET with the following parameters:
substrate doping (p-type): $N_A = 5 \times 10^{16}$ cm^{-3}
channel length $L = 1.0$ μm
gate width $W = 10$ μm
electron mobility $\mu_n = 600$ cm^2/Vs
oxide thickness $t_{ox} = 4$ nm
threshold voltage $V_{T0} = 1$ V.

Using, as in the previous problem, the simplified model of the Lecture Notes, pages 159-162, especially Eq. (255), calculate the width of a similar pMOSFET giving the same saturated current for the same gate overdrive (that is, at the same value of $|V_G - V_{T0}|$). Assume for the hole mobility a value of $\mu_p = 200$ cm^2/Vs. If you wish, you may assume also for the threshold voltage of the pMOSFET the value of -1 V. Recall that in dealing with the pFET all polarities (V_D and V_G) are switched.

Do you really need to know the entire set of parameters characterizing the devices in order to reach your answer?

Solution:

No, you don't. Since the threshold voltage has the same magnitude (just of opposite polarity), the only reason why the two currents (for the n and p FETs) differ at a given gate overdrive is due to the different mobilities for the two devices. Since the hole mobility is three times smaller than the electron mobility, the p-channel FET must be three times wider, that is $W = 30$ μm, in order to carry the same current.