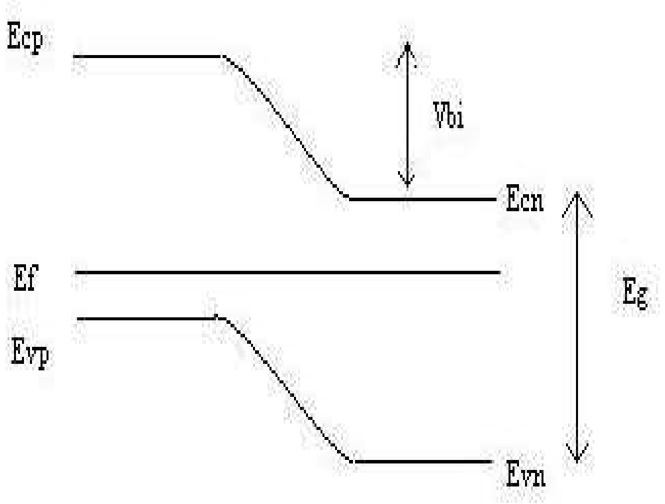
Homework 5

1. **Problem:** A silicon p-n junction is formed between n-type Si doped with $N_D = 10^{17}$ cm^{-3} and p-type Si doped with $N_A = 10^{16}$ cm^{-3}.
   (a) Sketch the energy band diagram. Label all axes and all important energy levels.
   (b) Find n_{n0}, n_{p0}, p_{p0}, and p_{n0}. Sketch the carrier concentration (of both electrons and holes) as a function of position.
   (c) Calculate the built-in potential V_{bi} in eV.
   Solution:

(a) The energy band diagram with labeled important energy levels and axes is



(b) Given $n_i = 1.5 \times 10^{10}$ cm^{-3},
in the quasi-neutral p-region,

$$p_{p0} = N_A = 10^{16} cm^{-3}. \tag{1}$$

In the quasi-neutral n-region,

$$n_{n0} = N_D = 10^{17} cm^{-3}. \tag{2}$$

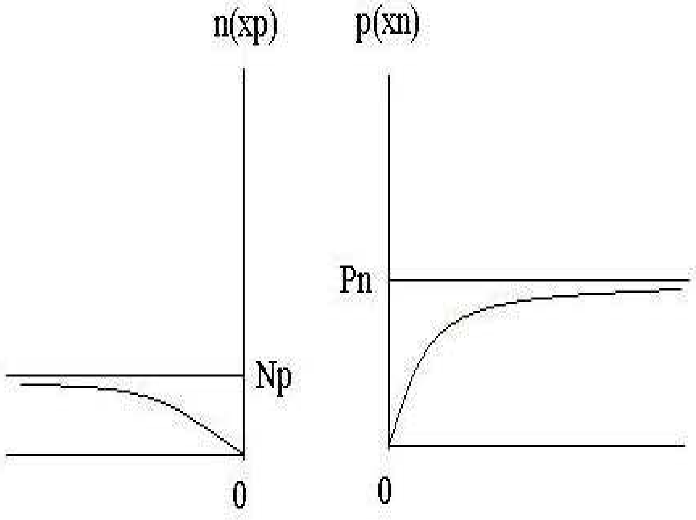
In the depletion region,

$$n_{p0} = \frac{n_i^2}{N_A} = 2.25 \times 10^4 cm^{-3} \tag{3}$$

for p-side and

$$p_{n0} = \frac{n_i^2}{N_D} = 2.25 \times 10^3 cm^{-3} \tag{4}$$

for n-side.
The diagram for carrier concentration is:



(c) Using Eq. (123) or (124) on lecture notes Part 2, the build-in potential is

$$V_{bi} = \frac{k_B T}{q} ln(\frac{N_A N_D}{n_i^2}) = \frac{k_B T}{q} ln(\frac{p_p}{p_n}) = 0.7543eV. \tag{5}$$

2. **Problem:** A Si p-n junction has dopant concentrations $N_D = 2 \times 10^{15}$ cm^{-3} and $N_A = 2 \times 10^{16}$ cm^{-3}. Calculate the built-in potential V_{bi} in eV and the total width of the depletion region $W = x_{n0} + x_{p0}$ at zero bias (that is, $V_a = 0$) and under a reverse bias $V_a = -8$V.
   Solution: (a) Given $n_i = 1.5 \times 10^{10}$ cm^{-3}, the build-in potential is:

$$V_{bi} = \frac{k_B T}{q} ln(\frac{N_A N_D}{n_i^2}) = \frac{k_B T}{q} ln(\frac{p_p}{p_n}) = 0.6709eV. \tag{6}$$

   (b) According to Eq. (140) & (141) in lecture notes Part 2, the depletion width with a bias V_a is give by:

$$W = \sqrt{\frac{2\epsilon_s(V_{bi} - V_a)}{q}(\frac{1}{N_A} + \frac{1}{N_D})}. \tag{7}$$

   For $V_a = 0V$, we have $W = 0.691\mu m$ and for $V_a = -8V$, we have $W = 2.475\mu m$.

3. **Problem:** A Si p-n junction is reverse-biased with $V_a = -10V$. Determine the percent change in junction (depletion) capacitance and built-in potential if the doping in the p region is increased by a factor of 2.
   Solution: Let us assume the N_A and N_D values from the Prob. 2, then the build-in potential is calculated by Eq. 6 and we have:

$$V_{bi} = \frac{k_B T}{q} ln(\frac{N_A N_D}{n_i^2}) = 0.6709eV, \tag{8}$$

   and

$$V'_{bi} = \frac{k_B T}{q} ln(\frac{2N_A N_D}{n_i^2}) = 0.6888eV. \tag{9}$$

Therefore, V_{bi} is increased by 2.67%.
From Eq. (184) on lecture note Part 2, the depletion capacitance can be calculated by

$$C_j = \frac{\epsilon A}{W}, \tag{10}$$

where A is the cross-sectional area of the junction and W is the depletion width calculated by:

$$W = \sqrt{\frac{2\epsilon_s(V_{bi} - V_a)}{q}(\frac{1}{N_A} + \frac{1}{N_D})} = 2.7549\mu m, \tag{11}$$

$$W' = \sqrt{\frac{2\epsilon_s(V'_{bi} - V_a)}{q}(\frac{1}{2N_A} + \frac{1}{N_D})} = 2.6938\mu m. \tag{12}$$

Therefore, the percent change in junction depletion capacitance is given by:

$$\frac{C'_j - C_j}{C_j} = \frac{W - W'}{W'} = 2.27\%. \tag{13}$$

4. **Problem:** Consider a p$^+$-n Si junction at $T = 300$ K with $N_A = 10^{18}$ cm^{-3} and $N_D = 10^{16}$ cm^{-3}. The minority carrier hole diffusion constant is $D_p = 12$ cm^2/s and the minority carrier hole lifetime is $\tau_p = 100$ ns. The cross-sectional area of the junction is $A = 10^{-4}$ cm^2. Calculate the reverse saturation current $I_s = AJ_s$. Calculate also the current at a forward bias $V_a = 0.5$ V.
Solution: Since $N_A \gg N_D$, it is an asymmetric junction and the total current is dominated by the most heavily-doped side of the junction. According to Eq. (151) in lecture notes Part 2, the saturation current density is given by

$$J_s = \frac{qD_p p_{n0}}{L_p} \tag{14}$$

where $L_p = (D_p \tau_p)^{1/2} = 1.095 \times 10^{-3}$cm and $p_{n0} = \frac{n_i^2}{N_D} = 2.25 \times 10^4$ cm^{-3}. So

$$J_s = 3.9492 \times 10^{-11} A/cm^2 \tag{15}$$

$$I_s = -AJ_s = -3.9492 \times 10^{-15} A. \tag{16}$$

For a forward bias $V_a = 0.5$ V, the current is:

$$I = I_s[e^{qV_a/k_BT} - 1] = 8.93 \mu A. \tag{17}$$

5. Which are the most important breakdown mechanisms in a reverse-biased p-n junction? Describe succintly (no more than one paragraph for each process) how they work and how they trigger the breakdown process. **Solution:** The important breakdown mechanisms are Zener and Avalanche. Refer to pages 108-117 in lecture notes Part 2.
6. **Problem:** (a) Calculate the maximum width of the depletion layer w_{max} (at the onset of inversion) and the maximum depletion charge $|Q_{d,max}|$ in p-type Si, GaAs, and Ge semiconductors of an MOS structure with $N_A = 10^{16}$ cm^{-3} and at $T = 300$ K.
   (b) Repeat the calculations for $T = 77$ K.
   Solution:

$$W_{max} = \sqrt{\frac{4\epsilon_r\epsilon_0 k_B T ln\frac{N_A}{n_i}}{q^2 N_A}} \tag{18}$$

and

$$Q_d = -qN_A W_{max}. \tag{19}$$

(a) At $T = 300$K,
For Si,

$$n_i = 1.5 \times 10^{10} cm^{-3} \tag{20}$$

$$\epsilon_r = 11.7. \tag{21}$$

From the above equations we find out W_{max} and $Q_{d,max}$

$$W_{max} = 0.2998\mu m \tag{22}$$

$$Q_{d,max} = -4.797 \times 10^{-8} \frac{C}{cm^2}. \tag{23}$$

For Ge,

$$n_i = 2.5 \times 10^{13} cm^{-3} \tag{24}$$

$$\epsilon_r = 16. \tag{25}$$

From the above equations we find out W_{max} and $Q_{d,max}$

$$W_{max} = 0.234\mu m \tag{26}$$

$$Q_{d,max} = -3.744 \times 10^{-8} \frac{C}{cm^2}. \tag{27}$$

For GaAs

$$n_i = 2 \times 10^{6} cm^{-3} \tag{28}$$

$$\epsilon_r = 13.2. \tag{29}$$

From the above equations we find out W_{max} and $Q_{d,max}$

$$W_{max} = 0.4109\mu m \tag{30}$$

$$Q_{d,max} = -6.574 \times 10^{-8} \frac{C}{cm^2}. \tag{31}$$

(b) At $T = 77$ K, note that

$$n_i(T) \propto T^{3/2} e^{-E_g/2k_B T}. \tag{32}$$

For Si,

$$n_i = 2.176 \times 10^{18} cm^{-3} \tag{33}$$

$$\epsilon_r = 11.7. \tag{34}$$

From the above equations we find out W_{max} and $Q_{d,max}$

$$W_{max} = 0.3652 \mu m \tag{35}$$

$$Q_{d,max} = -5.843 \times 10^{-8} \frac{C}{cm^2}. \tag{36}$$

For Ge,

$$n_i = 1.7529 \times 10^{4} cm^{-3} \tag{37}$$

$$\epsilon_r = 16. \tag{38}$$

From the above equations we find out W_{max} and $Q_{d,max}$

$$W_{max} = 0.3271 \mu m \tag{39}$$

$$Q_{d,max} = -5.234 \times 10^{-8} \frac{C}{cm^2}. \tag{40}$$

For GaAs,

$$n_i = 4.934 \times 10^{-30} cm^{-3} \tag{41}$$

$$\epsilon_r = 13.2. \tag{42}$$

From the above equations we find out W_{max} and $Q_{d,max}$

$$W_{max} = 0.45\mu m \tag{43}$$

$$Q_{d,max} = -7.2 \times 10^{-8} \frac{C}{cm^2}. \tag{44}$$

7. **Problem:**Determine the metal-semiconductor work function difference Φ_{MS} (in eV) in an MOS structure with p-type Si for the case where the gate is Al ($q\chi_M = 3.2$ eV), n$^+$ polysilicon (polycrystalline Si, assume it is identical to $'$normal$'$ Si), and p$^+$ polysilicon. Assume $N_A = 6 \times 10^{15}$ cm^{-3} and $T = 300$ K.
Solution: Given,

$$N_A = 6 \times 10^{15} cm^{-3} \tag{45}$$

$$T = 300K \tag{46}$$

$$\Phi_S = q\chi_S + \frac{E_g}{2} + \Phi_F \tag{47}$$

$$\Phi_S = q\chi_S + \frac{E_g}{2} + k_B T ln \frac{N_A}{n_i} \tag{48}$$

where $q\chi_S = 4.05$eV and $E_g = 1.11$eV.
For Al,

$$\Phi_M = q\chi_M = 3.2eV \tag{49}$$

$$\Phi_{MS} = \Phi_M - \Phi_S = -1.73eV. \tag{50}$$

For n$^+$ polysilicon gate,

$$\Phi_M = q\chi_S = 4.05eV \tag{51}$$

$$\Phi_{MS} = \Phi_M - \Phi_S = -0.889eV. \tag{52}$$

For p^{+} polysilicon gate,

$$\Phi_M = q\chi_M = 5.16eV \tag{53}$$

$$\Phi_{MS} = \Phi_M - \Phi_S = 0.221eV. \tag{54}$$