The Mesh Ratio

The relationship between the mesh net grid spacing and the steps along the direction of propagation is given by the mesh ratio:

\[ r = \frac{\Delta x}{(\Delta z)^2} \]

The alternating direction implicit method is a finite difference technique that does not place a restriction on the mesh ratio.

Goals

To develop electromagnetic modeling techniques using the finite difference method that will allow accurate prediction of path loss, angular spread and time dispersion of fields in cylindrical tunnels. The technique should address the challenges of irregular boundary shapes, non-smooth boundaries, curved and bifurcating tunnels.

The Standard Parabolic Equation (SPE)

The SPE is an approximation that reduces the number derivatives in the Helmholtz equation.

\[ \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} + k^2 U = 0 \]

\[ -\frac{j}{2k_y} \left( \frac{\partial^2 U}{\partial y^2} + \frac{\partial^2 U}{\partial z^2} \right) = \frac{\partial U}{\partial x} \]  
(SPE)

(where \( U = u e^{-jkw} \) is the scalar potential)

The Finite Difference Method

The finite difference method approximates the derivatives of the SPE by using the lowest order terms of the Taylor series expansion. The field is discretized at the plane of propagation into a mesh net and is evaluated at spatial steps along the direction of propagation.

\[ \Delta x \quad \Delta y \quad \Delta z \]

Accuracy is increased by increasing the number of mesh points.

Typical Tunnel Geometries:

Cylindrical Tunnel

Irregular Surface

Branching Tunnel

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