Electromagnetic Spatial Degrees of Freedom in 2-D Random Scattering Environments

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1. Introduction
The Multiple-Input Multiple-Output (MIMO) system for wireless communications has potential for large capacity gains over single antenna systems. This capacity gain depends on the electromagnetic (EM) degrees of freedom (DOF) of the volumes involved. Our objective is to predict the EM DOF of a system in scattering environments. A numerical technique to evaluate the EM spatial DOF and some simulation results in two dimensions are presented here.

2. Independent Channels — Defining EM Spatial DOF

Notation:
- $\Gamma$ - Tensor Green’s Functions; $\eta$ - Intrinsic Impedance

Field Function in the Receive Volume Due to the Source Function in the Transmit Volume:

$$F(r) = \left[ E(r), \eta H(r) \right] = \int_{T} \left[ \Gamma_{Ei}(r,r'), \Gamma_{Hi}(r,r') \right] \left[ J(r'), M(r') \right] dr'$$

Basis Representations and Connection Strengths:

$$S(r') = \sum b_{n} g(n, r'), \quad F(r) = \sum d_{i} f_{i}(r), \quad d_{i} = \sum g_{i} b_{i}$$

$$g_{ij} = \int_{T} f_{i}^{*}(r') g(n, r') dr'$$

Operator Eigenvalue Problems:

$$\hat{\mathbf{x}} \mathbf{g} = \mathbf{g} \mathbf{z}, \quad \{ \mathbf{g} \} : \text{Orthonormal Basis of Source Space}$$

$$\hat{\mathbf{x}} \mathbf{f} = \mathbf{f} \mathbf{z}, \quad \{ \mathbf{f} \} : \text{Orthonormal Basis of Field Space}$$

$$\mathbf{f} = \int_{T} \Gamma^{*}(r',r) \mathbf{f}(r') dr'$$

Optimal Bases: One-To-One Excitation

Maximum Connection Strengths

Definition: The number of electromagnetic spatial degrees of freedom is the maximum number of optimal receive modes that can have power stronger than noise level when the transmit power is fixed.

3. DOF vs. Transmit/Receive Volume

1. Channel quality deteriorates logarithmically along the channel index.
2. DOF number generally increases with both transmit volume and receive volume.
3. DOF is related to the Wavevector-Aperture-Product (WAP).

4. DOF vs. Scattering Region

1. DOF number increases with scattering region size.
2. Quality difference between channels decreases.

5. Large Volume Performance

DOF number tends to saturate as the receive volume size increases.

6. DOF vs. Transmit Power

1. DOF number increases logarithmically with the transmit power in scattering environments.
2. DOF number rarely changes with the transmit power in free space due to lack of scattering.

7. Conclusion

1. A numerical technique to evaluate EM spatial DOF
2. Validation of some common sense notions on the DOF in MIMO systems