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A hybrid model predictive control strategy for nonlinear plant-wide control

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Abstract

A plant-wide control strategy based on integrating linear model predictive control (LMPC) and nonlinear model predictive control (NMPC) is proposed. The hybrid method is applicable to plants that can be decomposed into approximately linear subsystems and highly nonlinear subsystems that interact via mass and energy flows. LMPC is applied to the linear subsystems and NMPC is applied to the nonlinear subsystems. A simple controller coordination strategy that counteracts interaction effects is proposed for the case of one linear subsystem and one nonlinear subsystem. A reactor/separator process with recycle is used to compare the hybrid method to conventional LMPC and NMPC techniques. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

Linear model predictive control (LMPC) has been successfully applied to plant-wide control problems with hundreds of input and output variables [1]. LMPC can be expected to provide satisfactory performance if the controlled process is approximately linear over the typical range of operation. The industrial success of commercial techniques such as Dynamic Matrix Control demonstrates that LMPC can tolerate some degree of process nonlinearity. The standard approach for handling strong nonlinearities in the LMPC framework is to sacrifice performance by detuning the controller. However, some plants are sufficiently nonlinear to hinder the successful application of LMPC.

Nonlinear model predictive control (NMPC) has been proposed as an alternative to LMPC for plants with highly nonlinear behavior [2]. NMPC offers the same capabilities for interaction compensation and constraint handling as its linear counterpart. The key difference is that NMPC utilizes a nonlinear model to predict and optimize process performance. The use of NMPC for plant-wide control is problematic due to complications associated with dynamic modeling, state estimation and on-line optimization [3]. A nonlinear dynamic model of the entire plant is required for controller design. Such large-scale nonlinear models are extremely difficult to obtain using fundamental modeling and available techniques for empirical nonlinear modeling [4]. Another complication is that unmeasured state variables must be estimated from available on-line measurements. This requires the design of a nonlinear observer, which is a difficult task despite recent advances [5]. Even if a suitable nonlinear model is available, a nonlinear programming problem must be solved at each sampling period to generate the control moves. For large-scale systems the optimization problem may be computationally intractable due to the large number of decision variables and the complexity of the constraints resulting from the nonlinear model equations. While it can be argued that cheaper and faster computers soon will be available to solve plant-wide nonlinear optimization problems in real-time, a simple calculation in [6] has shown that a NMPC problem with 20 inputs and 20 outputs will not be able to be solved on-line until well into this century given expected advances in computer technology. As a result, the judicious use of modeling assumptions [7] and simplified controller formulations [6] are required even for problems of moderate size and complexity.

In this paper, we propose a plant-wide control strategy based on integrating LMPC and NMPC. The motivation for this approach is that most operating units in a typical chemical plant can be adequately described by linear dynamic models, while a small

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number of operating units account for the highly non-linear behavior. Unlike plant-wide control methods that employ time-scale decompositions [8,9], the proposed approach is based on decomposing the plant according to the degree of nonlinearity. A model predictive control system for the decomposed plant is constructed by applying LMPC to the linear subsystems and NMPC to the nonlinear subsystems. We present a simple controller coordination strategy for plants that can be decomposed into a single linear subsystem and a single nonlinear subsystem. A simple reaction/separation process is used to compare the hybrid method to conventional LMPC and NMPC techniques in terms of closed-loop performance and on-line computation.

2. Illustrative example

Consider a reaction/separation process which is designed to produce a product B by irreversible reaction of a reactant A. The effluent from a continuous stirred tank reactor is introduced to a distillation column where separation of the reactant and product occurs. The overhead stream enriched in A is recycled to the reactor assuming that inerts and light byproducts are not present in the system. Otherwise, an overhead vent stream would be required to avoid accumulation of these materials. The bottom stream enriched in B is recovered as the product. The basic reactor model obtained from [10] is well studied. The model is derived by assuming a first-order reaction, constant volume operation, and that the combined recycle and fresh feed stream temperature is maintained at a constant temperature by a fast regulatory controller. The resulting reactor model is comprised of state equations for the reactor concentration (C_A) and the reactor temperature T. The parameter values are same as those used in [10], except that the pre-exponential factor (k_0) is reduced from 7.2×10^{10} min^{-1} to 5.14×10^{10} min^{-1}. The control objective is to regulate the reactor temperature by manipulating the coolant temperature (T_c) assuming the coolant jacket dynamics are negligible. This scheme allows the reactor to be operated safely and effectively.

The distillation column consists of seven trays, a total condenser, and a reboiler. The effluent from the reactor is introduced as feed to the fourth tray. The assumptions used to derive the distillation column are discussed in [11]. In particular, the equimolal overflow assumption eliminates the need for energy balances. Tray-by-tray component balances [11] yield state equations for the liquid mole fraction of component A on each equilibrium stage (X_{An}), where $n = 1$ represents the condenser and $n = 9$ represents the reboiler. The control objective is to regulate the recycle mole fraction (X_{A1}) and the product mole fraction (X_{A9}) by manipulating the vapor rate (V) and the reflux rate (L). Overhead composition control is

desirable to maximize the amount of reactant recycled to the reactor, while control of the bottom composition is required to meet product purity requirements. In practice, V and L may be manipulated by adjusting the reboiler heat duty and the reflux valve position. The column model parameters are shown in Table 1.

Fig. 1 shows the open-loop responses of the outputs (T, X_{A1}, and X_{A9}) to step changes in the inputs (T_c, V). The initial operating point corresponds to an unstable steady state for the reactor. The solid line represents the response to a -3 K change in T_c at $t = 5$ min followed by $a + 10$ mol/min change in V at $t = 50$ min. The dashed line represents the response to $+3$ K change in T_c at $t = 5$ min. It is obvious that the reactor dynamics are highly nonlinear and much faster than the column dynamics.

The linear model for LMPC design is obtained by linearizing the nonlinear model equations about the nominal operating point and discretizing with a sampling interval $\Delta T = 10$ s. The resulting model has the standard state-space form:

$$x(k + 1) = Ax(k) + Bu(k)$$
$$y(k) = Cx(k) \tag{1}$$

where $x \in R^{11}$, $u \in R^3$ and $y \in R^3$. The LMPC controller is formulated to minimize the following infinite horizon objective function:

$$\min_{U(k)} \sum_{j=0}^{\infty} \{ [y(k + j|k) - y_s]^T Q[y(k + j|k) - y_s]$$
$$+ [u(k + j|k) - u_s]^T R[u(k + j|k) - u_s] \tag{2}$$
$$+ \Delta u^T(k + j|k) S \Delta u(k + j|k) \}$$

where: $y(k + j|k)$ and $u(k + j|k)$ are predicted values of the outputs and inputs, respectively; y_s and u_s are (possibly) time-varying target values for the outputs and inputs, respectively; and $\Delta u(k) = u(k) - u(k - 1)$. The decision variables are current and future values of the inputs:

$$U(k) = \begin{bmatrix} u(k|k) \\ u(k + 1|k) \\ \vdots \\ u(k + N - 1|k) \end{bmatrix} \tag{3}$$

Table 1
Distillation column model parameters

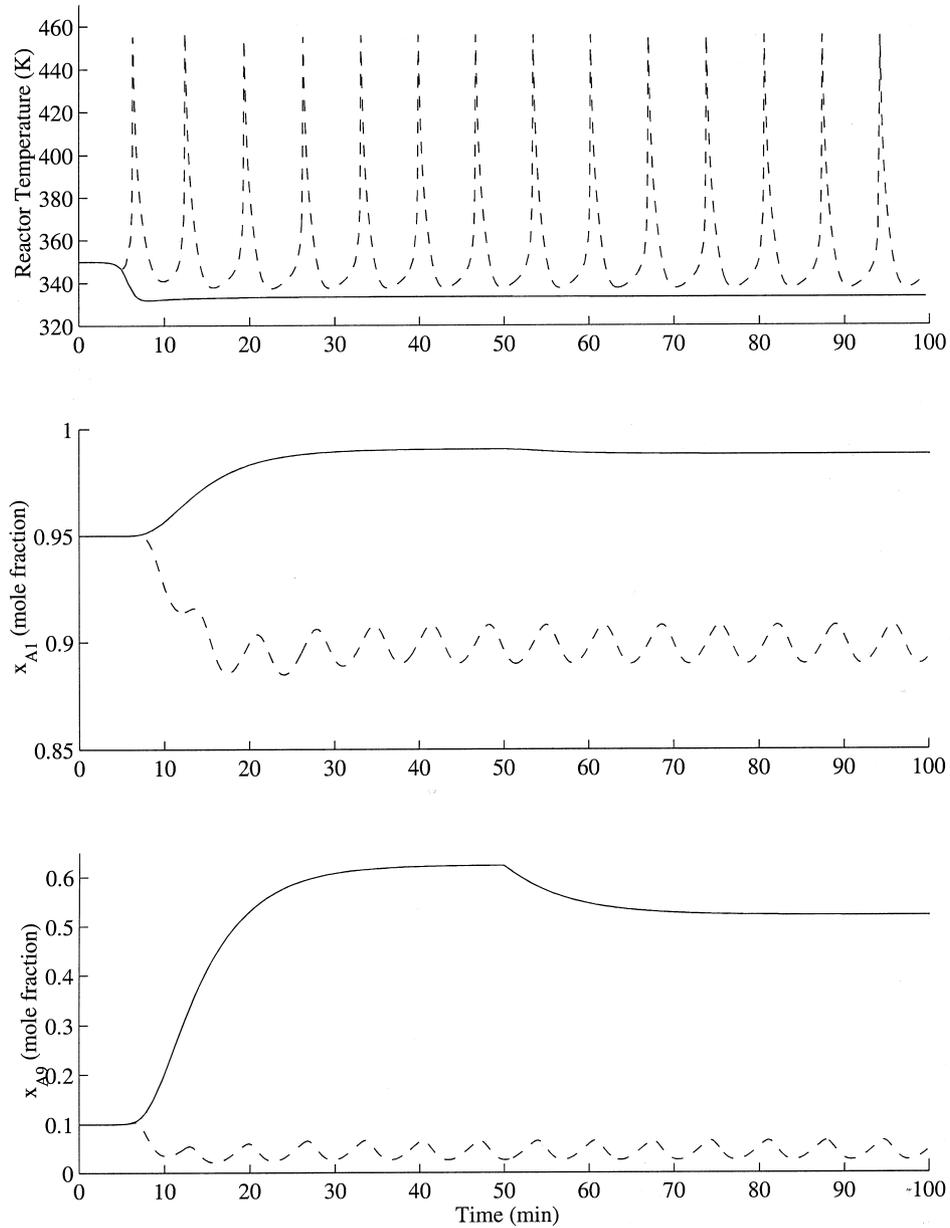|	
Relative volatility	$\alpha = 4$
Feed composition	$X_f = 0.5$
Feed rate	$F = 100$ mol min
Reflux rate	$L = 29.221$ mol min
Boil-up rate	$V = 84.199$ mol min
Condensor hold-up	$M_1 = 200$ mol
Tray hold-up	$M_i = 50$ mol $i = 2 \ldots 8$
Reboiler hold-up	$M_9 = 200$ mol
Top composition	$X_{A1} = 0.95$
Bottom composition	$X_{A9} = 0.1$

Fig. 1. Open-loop simulation for step changes in cooling jacket temperature and boil-up rate.

By assuming $u(k+j|k) = u_s$ for $j > N$, where the control horizon $N = 15$, a finite horizon formulation of (2) can be obtained [12]:

$$\min_{U(k)} [x(k+N|k) - x_s]^T \bar{Q}[x(k+N|k) - x_s]$$
$$+ \Delta u^T(k+N|k)S\Delta u(k+N|k)$$
$$+ \sum_{j=0}^{N-1} \{[x(k+j|k) - x_s]^T C^T Q C[x(k+j|k) - x_s] \qquad (4)$$
$$+ [u(k+j|k) - u_s]^T R[u(k+j|k) - u_s]$$
$$+ \Delta u^T(k+j|k)S\Delta u(k+j|k)\}$$

The terminal penalty matrix $\bar{Q}$ is obtained from solution of a Lyapunov equation [12]. The tuning matrices Q, R, and S are chosen as follows:

$$Q = \begin{bmatrix} 100 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 50 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix} \qquad (5)$$

$$S = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix}$$

The problem is solved subject to the input constraints:

$$\begin{bmatrix} 280\,\text{K} \\ 0\,\text{mol}/\min \\ 0\,\text{mol}/\min \end{bmatrix} \leqslant \begin{bmatrix} T_c \\ L \\ V \end{bmatrix} \leqslant \begin{bmatrix} 350\,\text{K} \\ 250\,\text{mol}/\min \\ 250\,\text{mol}/\min \end{bmatrix} \tag{6}$$

Output constraints are not considered in this example.

We assume the reactor temperature, recycle mole fraction, and product mole fraction can be measured on-line. The remaining state variables must be estimated from the available on-line measurements. Simultaneous state and disturbance estimation is performed by augmenting the process model with an output disturbance model [12]:

$$x(k + 1) = Ax(k) + Bu(k)$$
$$d(k + 1) = d(k)$$
$$y(k) = Cx(k) + d(k) \tag{7}$$

where $d \in R^3$ is a vector of disturbance variables. A Luenberger observer is used since the nominal operating point of the reactor is unstable. The estimated state variables are used to initialize the optimization problem at each time step, while the estimated disturbances are used to shift the target values x_s and u_s as discussed in [12].

Fig. 2 shows the closed-loop response obtained for setpoint changes of $+0.02$ and -0.05 in the column overhead and column bottom mole fractions, respectively. LMPC provides good performance because the reactor remains near the nominal point where the linear model was derived. The closed-loop response obtained for a $+5$ K change in the reactor temperature setpoint is shown in Fig. 3. Due to the strong reactor nonlinearities, the temperature tracking performance is very poor and the bottom mole fraction deviates significantly from its setpoint.

3. Integration of LMPC and NMPC

Setpoint changes in the recycle and product mole fractions are handled easily by LMPC because the column does not exhibit strong nonlinearities at moderate purities. However, LMPC yields unacceptably poor responses for reactor temperature setpoint changes due to the highly nonlinear reaction kinetics. As shown subsequently, this problem can be solved by applying NMPC to the entire plant. However, a more computationally efficient approach is to utilize nonlinear control only where necessary; *i.e.* apply NMPC to the reactor and apply LMPC to the column. This motivates the development of a new class of plant-wide control methods based on integrating LMPC and NMPC.

The proposed hybrid LMPC-NMPC control strategy consists of four steps:

1. Analysis of process nonlinearities.
2. Decomposition of the plant into linear and nonlinear subsystems.
3. Application of LMPC to the linear subsystems and NMPC to the nonlinear subsystems.
4. Coordination of the linear and nonlinear MPC controllers.

This paper focuses on the final two problems for plants that can be decomposed into a single linear subsystem and a single nonlinear subsystem. The reaction/separation system described above is an example of such a process. Our future work will focus on the first two problems and more complicated processes.

The linear subsystem model is assumed to have the form:

$$x_L(k + 1) = A_L x_L(k) + A_N x_N(k) + B_L u_L(k) \\ + B_N u_N(k) \tag{8}$$

$$y_L(k) = C_L x_L(k) + C_N x_N(k)$$

where the subscripts L and N denote variables associated with the linear subsystem and the nonlinear subsystem, respectively. The nonlinear subsystem model is assumed to have the form:

$$x_N(k + 1) = f[x_L(k), x_N(k), u_L(k), u_N(k)] \\ y_N(k) = h[x_L(k), x_N(k)] \tag{9}$$

In each subsystem model, variables from the other subsystem can be viewed as measured disturbances. For the reaction/separation process, the column is the linear subsystem and the reactor is the nonlinear subsystem. As compared to conventional NMPC, an immediate advantage of the proposed approach is that a nonlinear model is required only for the nonlinear subsystem. This eliminates the need for a full nonlinear plant model, which rarely is available or economically feasible to develop.

Solutions to the LMPC and NMPC problems must be computed sequentially to achieve a substantial reduction in computational effort as compared to standard NMPC. Sequential solution may be problematic because the linear and nonlinear subsystems are coupled via mass and energy flows. The most appropriate solution strategy depends on the type of couplings between the subsystems. Each plant configuration described below warrants a different approach:

1. The linear subsystem is coupled to the nonlinear subsystem but there is no material/energy transfer
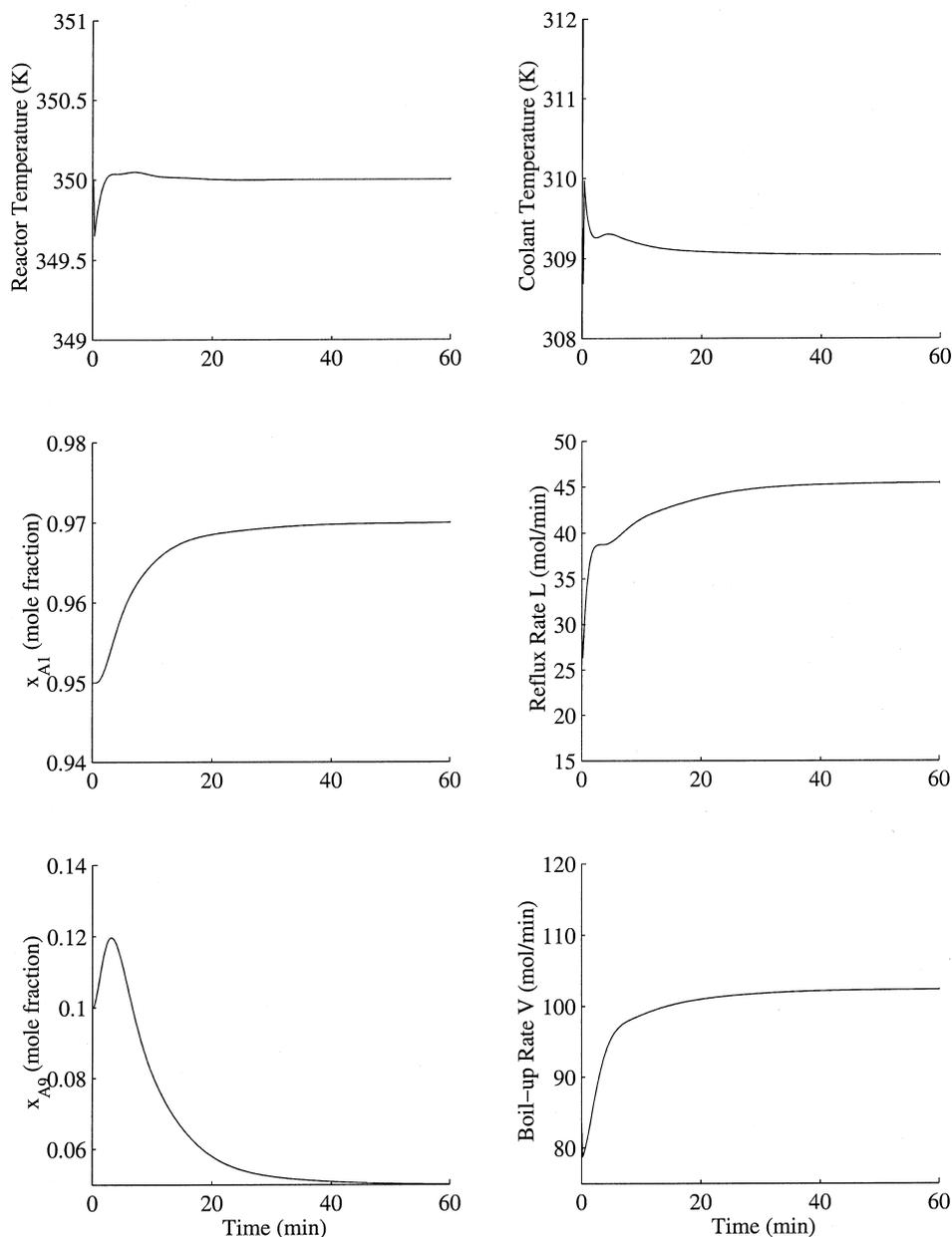
Fig. 2. LMPC for a setpoint change in the column mole fractions.

from the nonlinear subsystem to the linear subsystem. In this case, the LMPC problem is solved first and the LMPC solution is utilized to solve the NMPC problem.

2. The nonlinear subsystem is coupled to the linear subsystem but there is no material/energy transfer from the linear subsystem to the nonlinear subsystem. In this case, the NMPC problem is solved first and the NMPC solution is utilized to solve the LMPC problem.

3. The linear and nonlinear subsystems are fully coupled in the sense that there is material/energy transfer in both directions. This case is more complex due to the two directional coupling and

requires a more sophisticated controller coordination strategy.

The remainder of this section focuses on developing a controller coordination strategy for fully coupled systems such as the reaction/separation process. The first step is to compute a solution of the LMPC problem using a linear model of the entire plant rather than just the linear subsystem. The motivation for this approach is discussed below. The LMPC solution is used to compute the NMPC solution for the nonlinear subsystem.

To motivate the proposed controller coordination strategy, consider an alternative method in which the LMPC design is based on the linear subsystem model.
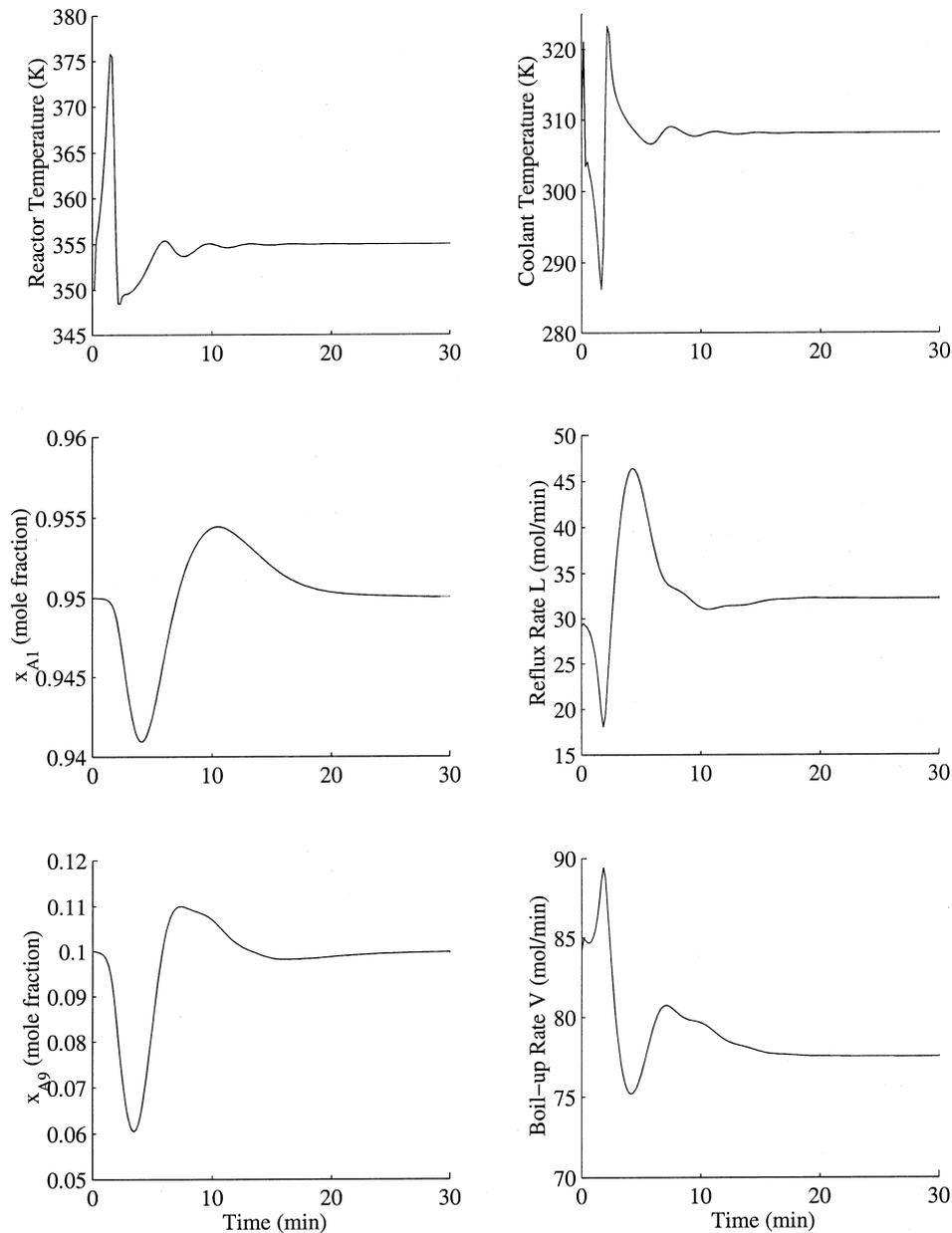
Fig. 3. LMPC for a setpoint change in the reactor temperature.

The following information is required from the non-linear subsystem to generate predictions over the LMPC control horizon N:

State variables : $x_N(k|k), \ldots, x_N(k + N - 1|k)$

Input variables : $u_N(k|k), \ldots, u_N(k + N - 1|k)$

The current value of the state vector $x_N(k|k)$ is available from on-line measurements and/or nonlinear state estimation. Future values of the state variables are not available until the NMPC problem is solved at the current time step, and the NMPC problem cannot be

solved until the LMPC problem is solved. A possible solution to this problem is to generate the unavailable future values from the NMPC solution obtained at the previous time step.

The information exchange problem between controllers can be partially mitigated if the LMPC design is based on a linear model of the entire plant. The advantage of this approach is that the dependence of the LMPC problem on the NMPC solution is completely eliminated. As a result, the LMPC problem can be solved independently. It is important to emphasize that only the LMPC solution for the linear subsystem actu-

ally is utilized; the manipulated input moves for the nonlinear subsystem are not implemented. The LMPC solution is used to compute the NMPC solution for the nonlinear subsystem. The disadvantages of this approach are that a linear approximation of the nonlinear subsystem is introduced and a larger LMPC problem must be solved.

The LMPC problem is formulated as described previously. The LMPC solution yields all the input and state information required to solve the NMPC problem:

State variables : $x_L(k|k), \ldots, x_L(k + P - 1|k)$

Input variables : $u_L(k|k), \ldots, u_L(k + P - 1|k)$

where P is the prediction horizon of the NMPC controller. The NMPC problem is formulated as:

$$\min_{U_N(k)} [y_N(k + P|k) - y_s]^T Q[y_N(k + P|k) - y_s] +$$

$$\sum_{j=0}^{P-1} \{[y_N(k + j|k) - y_s]^T Q[y_N(k + j|k) - y_s]$$

$$+ [u_N(k + j|k) - u_s]^T R[u_N(k + j|k) - u_s]$$

$$+ \Delta u_N^T(k + j|k) S \Delta u_N(k + j|k)\} \qquad (10)$$

where: $y_N(k + j|k)$ and $u_N(k + j|k)$ are predicted values of the nonlinear subsystem variables; y_s and u_s are target values for the nonlinear subsystem variables; and $\Delta u_N(k) = u_N(k) - u_N(k - 1)$. The decision variables are current and future values of the manipulated inputs in the nonlinear subsystem:

$$U_N(k) = \begin{bmatrix} u_N(k|k) \\ u_N(k + 1|k) \\ \vdots \\ u_N(k + M - 1|k) \end{bmatrix} \qquad (11)$$

where M is the NMPC control horizon. The problem is solved subject to input constraints and equality constraints derived from the nonlinear model.

For the present example, we assume the reactor concentration must be estimated from reactor temperature measurements. An important advantage of the hybrid approach is that a nonlinear observer can be designed for the reactor subsystem rather than the entire plant. Simultaneous state and disturbance estimation is performed using an augmented reactor subsystem model. In this case, an input disturbance model is employed to handle the reactor instabilities. Nonlinear observer design is facilitated by representing the reactor subsystem in the continuous-time form,

$$\dot{x}_N = \beta(y_N)x_N + \gamma(u_N + d_N) + \delta(x_L)$$

$$\dot{d}_N = 0$$

$$y_N = Cx_N \qquad (12)$$

where β and δ are nonlinear functions, γ is a constant, and $d_N \in R$ is a vector of input disturbance variables. Note that the model is linear in the unmeasured concentration C_A. The state affine model form allows the design of a simple nonlinear closed-loop observer [13]. The estimated state variables $\hat{x}_N(k)$ are used to initialize the NMPC problem at each time step, while the estimated disturbances $\hat{d}_N(k)$ are used to shift the input and output target values [2].

4. Simulation study

We compare the hybrid LMPC–NMPC method to standard LMPC and NMPC using the reaction/separation process described previously. The NMPC controller is formulated as in (10) except that the nonlinear model of the entire plant is used for controller design. The continuous-time nonlinear model is discretized using orthogonal collocation on finite elements [2] with a sampling period $\Delta t = 10$ s. Due to the number of state variables in the column model, the resulting NMPC problem is much larger than that encountered in the hybrid approach where NMPC is applied only to the reactor. The full-order NMPC problem is simplified by assuming full-state feedback and that the plant is not affected by unmeasured disturbances. This eliminates the need for a plant-wide nonlinear observer that generates estimates of the unmeasured state and disturbance variables. As a result, the simulation results for NMPC represent the best case senario in terms of setpoint tracking performance and on-line computation. The NMPC controller is tuned with $M = 1$, $P = 2$ and:

$$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 0.001 & 0 \\ 0 & 0 & 0.001 \end{bmatrix} \qquad (13)$$

$$S = \begin{bmatrix} 0.001 & 0 & 0 \\ 0 & 100 & 0 \\ 0 & 0 & 100 \end{bmatrix}$$

Longer control and prediction horizons increase the computation time dramatically, but they have very little effect on closed-loop performance new IP. The hybrid LMPC–NMPC controller is formulated as described in the previous section. The LMPC controller is tuned with $N = 15$ and:
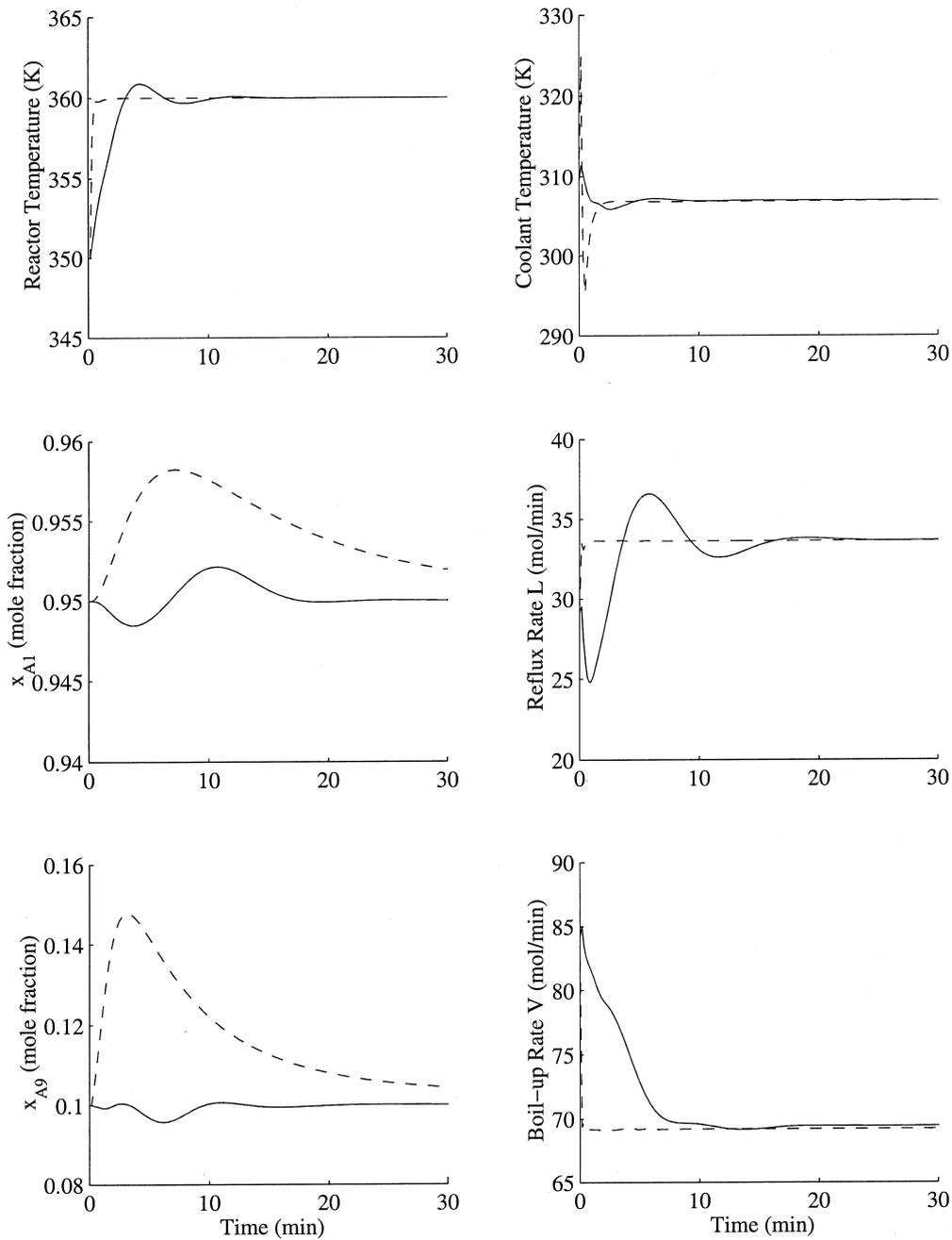
Fig. 4. Hybrid LMPC–NMPC (———) and NMPC (- - -) for a setpoint change in the reactor temperature.

$$Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 5 \times 10^4 & 0 \\ 0 & 0 & 5 \times 10^4 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.01 \end{bmatrix}, \quad S = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

The NMPC controller is tuned with $M = 1$, $P = 4$, $Q = 2$, $R = 0.001$ and $S = 0.001$.

Fig. 4 provides a comparison of the hybrid LMPC–NMPC and NMPC controllers for a +10 K change in the reactor temperature setpoint. Both controllers clearly outperform the standard LMPC controller (Fig. 2). The NMPC controller provides a faster temperature response due to the nonlinear formulation of the entire system and the shorter control and prediction horizons used ($M = 1$, $P = 2$). On the other hand, the hybrid controller yields much smaller deviations in the column mole fractions due to the longer control and prediction

horizons ($N = 15$, $P = \infty$) used in the LMPC controller. The sluggish NMPC performance for the column is due to the short control and prediction horizon lengths (10 and 20 s, respectively). As shown in Fig. 1, the dominant time constant of the column is about 15–20 min. Significant performance improvement is expected if the horizon lengths are dramatically increased. However, reasonable increases in the horizon lengths yield little improvement in performance but significantly increase the computational load.

A simultaneous setpoint change in the reactor temperature ($+20$ K), the recycle mole fraction ($+0.02$) and the product mole fraction (-0.05) is shown in Fig.

5. As before, the temperature response of the NMPC controller is significantly faster than that of the hybrid controller. The two controllers provide similar performance for the recycle mole fraction setpoint change, while the hybrid controller yields superior tracking of the product mole fraction setpoint change.

Even with the availability of increasingly powerful computers, the rigorous solutions of most plant-wide NMPC problems remains intractable. The major advantage of the hybrid method as compared to NMPC is computation time. For a typical 30 min MATLAB simulation on a DEC 433 workstation, the hybrid controller requires approximately 3 min of CPU time while
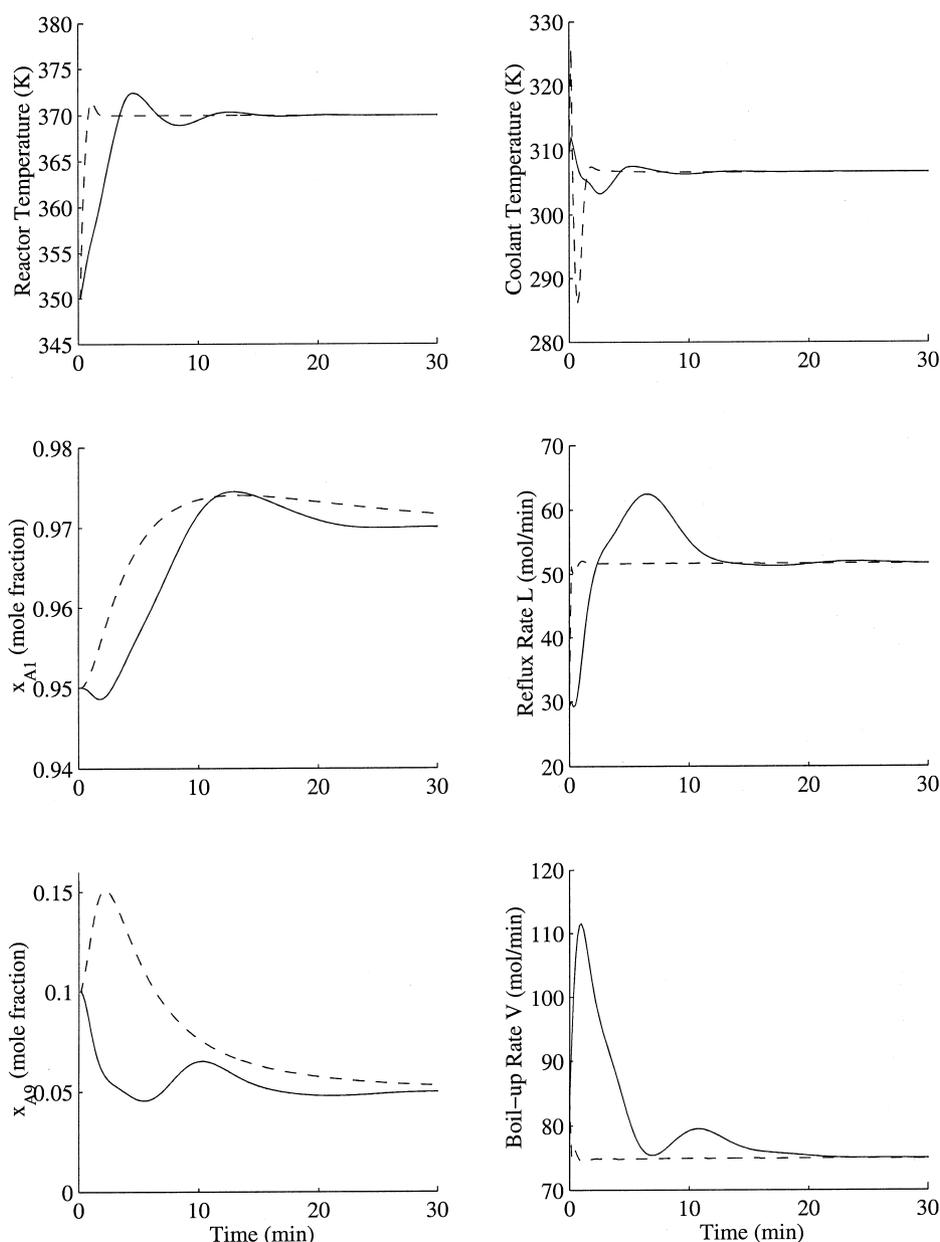

Fig. 5. Hybrid LMPC–NMPC (—) and NMPC (- - -) for setpoint changes in the reactor temperature and column mole fractions.

the NMPC controller requires about 40 minutes. It is important to note that NMPC execution times would be increased further by the introduction of a nonlinear state/disturbance estimator. These results demonstrate that the hybrid method can provide a suitable compromise between closed-loop performance and on-line computation for the class of nonlinear processes considered.

5. Conclusions and future work

This paper represents a first step in developing a comprehensive methodology for plant-wide control via integration of LMPC and NMPC. The proposed method involves decomposing the plant into linear and nonlinear subsystems and applying the appropriate MPC technology to each subsystem. A simple method for coordinating the LMPC and NMPC controllers for plants that can be decomposed into a single linear subsystem and a single nonlinear subsystem was presented. The proposed approach was compared to LMPC and NMPC using a prototypical reactor/separator process with recycle. Our future work will focus on stability analysis, the development of data driven techniques to perform the plant decomposition, and large-scale process applications.

Acknowledgements

References

[1] S.J. Qin, T.A. Badgwell, An overview of industrial model predictive control technology. In Chemical Process Control — V: Assessment and New Directions for Research, Tahoe City, CA, 1997, pp. 232–256.

[2] E.S. Meadows, J.B. Rawlings, Model predictive control, in: M.A. Seborg, D.E. Henson (Eds.), Nonlinear process control, chapter 5, Prentice Hall, Englewood Cliffs, NJ, 1997, pp. 233–310.

[3] M.A. Henson, Nonlinear model predictive control: current status and future directions, Comput. Chem. Engng. 23 (1998) 187–202.

[4] R.K. Pearson, B.A. Ogunnaike, Nonlinear process identification, in: M.A. Henson, D.E. Seborg (Eds.), Nonlinear process control, Prentice Hall, Englewood Cliffs, NJ(Chapter 2), 1997, pp. 11–110.

[5] K.R. Muske, T.F. Edgar, Nonlinear state estimation, in: M.A. Seborg, D.E. Seborg (Eds.), Nonlinear process control, Prentice Hall, Englewood Cliffs, NJ(Chapter 6), 1997, pp. 311–370.

[6] A. Zheng, A computationally efficient nonlinear MPC algorithm, in: Proc. American Control Conf., Albuquerque, NM, 1997, pp. 1623–1627.

[7] N.L. Bicker, J.H. Lee, Nonlinear modeling and state estimation for the Tennessee Eastman challenge process, Comput. Chem. Eng., 19:983- (1005) 1995.

[8] P.S. Buckley, Techniques of Process Control, Wiley, New York NY, 1964.

[9] M.L. Luyben, B.D. Tyreus, W.L. Luyben, Plantwide control design procedure, AIChE J (1997) 43.

[10] A. Uppal, W.H. Ray, A.B. Poore, On the dynamic behavior of continuous stirred tank reactors, Chem. Eng. Sci. 29 (1974) 967.

[11] M. Morari, E. Zafiriou, Robust Process Control, Prentice Hall, Englewood Cliffs NJ, 1989.

[12] K.R. Muske, J.B. Rawlings, Linear model predictive control of unstable processes, J. Process Control 3 (1993) 85–96.

[13] M.J. Kurtz, M.A. Henson, State and disturbance estimation for nonlinear systems affine in the unmeasured variables, Comput. Chem. Engng. 22 (1998) 1441–1459.