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State and disturbance estimation for nonlinear systems affine in the unmeasured variables

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Abstract

The problem of estimating unmeasured state and disturbance variables for nonlinear process control applications is considered. We assume that the process model is affine with respect to the unmeasured disturbances and any unmeasured state variables. The disturbances are considered as additional state variables, and a nonlinear observer is designed for the augmented state-space system. The observability of the augmented system is analyzed, and a constructive procedure for calculating the nonlinear observer gains is proposed. Full-order and reduced-order observers are constructed for both the full-state and partial-state feedback cases. Stability results for the nonlinear observer are presented. An output feedback controller is obtained by combining the nonlinear observer with an input–output linearizing controller. The proposed estimation strategy is compared to linear estimation techniques using a fluidized bed reactor model. © 1998 Elsevier Science Ltd. All rights reserved

Keywords: nonlinear systems; state estimation; chemical reactors; feedback; linearization

1. Introduction

High-performance estimation and control techniques are needed for chemical processes due to increased demands on productivity, product quality, and environmental responsibility. Most control systems in the process industries are based on linear models despite the fact that a large number of chemical processes are inherently nonlinear. Estimation and control strategies based directly on nonlinear models can be expected to provide significantly improved performance for highly nonlinear processes. Nonlinear model predictive control (Bequette, 1991; Rawlings *et al*., 1994) and feedback linearization (Henson and Seborg, 1991) are the most widely studied nonlinear control techniques for chemical process applications. Both approaches require full-state feedback when a state-space model is used for controller design. In many process control applications, the entire state vector cannot be measured on-line and unmeasured state variables must be estimated from available measurements to implement the nonlinear controller.

Several nonlinear process control strategies which use an open-loop observer to reconstruct unmeasured state variables have been proposed (Daoutidis and Kravaris, 1992; Henson and Seborg, 1991). Open-loop observers have several obvious disadvantages including: (i) the error dynamics cannot be altered; (ii) biased estimates may be obtained if the process has multiple steady states; and (iii) unbounded estimates may result for unstable processes. The design of closed-loop nonlinear observers is an active area of research, and a variety of techniques are now available (Bastin and Gevers, 1988; Gauthier *et al*., 1992; Krener and Isidori, 1983; Misawa and Hedrick, 1989; Nicosia *et al*., 1989; Zeitz, 1987). Several of these methods have been applied to nonlinear models of chemical processes (Gibon-Fargeot *et al*., 1994; Kantor, 1989; Limqueco *et al*., 1991). However, available techniques suffer from at least one of the following shortcomings: (i) the underlying assumptions are restrictive and difficult to verify; (ii) the observer design procedure is complex; or (iii) the computational requirements are high. Consequently, there is considerable motivation to develop nonlinear observers which are easy to design and implement.

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Input–output linearization provides a very computationally efficient means to enact nonlinear control. Unfortunately, unmeasured disturbances present a problem in that a linear input–output map is achieved only if disturbance variables satisfy a restrictive matching condition (Isidori, 1989). For single-input, single-output systems, the matching condition requires that each disturbance has a less direct effect on the controlled output than does the manipulated input. In many process applications, the matching condition is not satisfied and exact linearization of the closed-loop system is not possible. As a result, input–output linearizing controllers can yield unacceptable performance and robustness (Henson and Seborg, 1994). Hence, it is important to develop feedback linearizing controllers which provide more effective rejection of unmeasured disturbances. Improved disturbance rejection also could enhance the performance of other control techniques such as nonlinear model predictive control.

Several control strategies which attempt to address the problem of unmatched disturbances have been proposed. Sliding mode control (Colantonio *et al.*, 1995; Sira-Ramirez, 1989) and almost disturbance decoupling (Marino *et al.*, 1989) techniques provide approximate disturbance decoupling, but require high gain feedback that is unacceptable in most process control applications. In robust nonlinear control strategies (Arkun and Calvet, 1992; Behtash, 1990), unmeasured disturbances are viewed as unmodeled, state-dependent perturbations. The controller is designed to handle a large class of such perturbations, and therefore it tends to yield poor performance for any particular disturbance. If viewed as unknown, constant parameters, disturbances can be estimated on-line using nonlinear adaptive control methods (Henson and Seborg, 1991; Sastri and Isidori, 1989). Although they can provide good performance, nonlinear adaptive controllers often are difficult to design and implement.

In this paper, an alternative method to estimate unmeasured state and disturbance variables for a specific class of nonlinear systems is proposed. The model form and some specific process examples are given in Section 2. The state and disturbance estimation problems are considered in Sections 3 and 4, respectively. The combined state/disturbance estimation problem is explored in Section 5. In each case, the nonlinear estimation technique is compared to a linear estimation method using a nonlinear model of a fluidized-bed reactor. Stability of the observer is discussed in Section 6. Finally, Section 7 provides a summary and conclusions.

2. Class of nonlinear systems

The state/disturbance estimator design is based on the nonlinear state-space model,

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + p(x)d, \\ y_m &= Cx, \end{aligned} \tag{1}$$

where x is an n-dimensional vector of state variables, u and y_m are the manipulated input and measured outputs, respectively, and d is a q-dimensional vector of unmeasured disturbances that are estimated. The estimation techniques are developed for the single-input case, but they are easily extended to multiple-input systems. Note that the model is affine with respect to the estimated disturbances (d), and the measured outputs (y_m) are assumed to be a linear combination of the state variables (x). It is assumed that y_m is available continuously, or these measurements have sampling times much less than the dominant time constant of the process such that they can be considered continuous. If this condition does not hold, then it may be necessary to design a discrete-time nonlinear observer (Moraal and Grizzle, 1995; Song and Grizzle, 1995). The design of nonlinear observers for sampled data systems is not considered here.

If only state estimation is considered, then the disturbance vector d is assumed to remain constant, and the functions $f(x)$, $g(x)$, and $p(x)$ are assumed to be affine with respect to the unmeasured state variables:

$$\begin{aligned} f(x) &= f_1(y_m) + f_2(y_m)x, \\ g(x) &= g_1(y_m) + g_2(y_m)x, \\ p(x) &= p_1(y_m) + p_2(y_m)x. \end{aligned} \tag{2}$$

We denote the controlled output as $y = h(x)$ and assume y depends only on measured signals: $h(x) = h(y_m)$. The resulting system is affine with respect to the unmeasured state variables. Note that all bilinear models have this form. It should be mentioned that the factorization of the functions $f(x)$, $g(x)$, and $p(x)$ may not be unique. It is desirable to make $f_2(y_m)$, $g_2(y_m)$, and $p_2(y_m)$ as simple as possible to simplify the observer gain calculations.

In the case of disturbance estimation alone, the entire state vector is assumed to be measured ($y_m = x$). The disturbances are considered as unmeasured state variables and an observer is constructed for the following augmented model:

$$\begin{aligned} \dot{x} &= f(x) + g(x)u + p(x)d, \\ \dot{d} &= 0, \\ y_m &= Cx. \end{aligned} \tag{3}$$

Although the estimator design is based on the assumption of constant disturbances, the technique also is applicable to systems with slowly varying disturbances. It also is possible to include *a priori* knowledge about the class of disturbances which will be encountered by using a different type of disturbance model. The basic observer design procedure will remain the same; however, the conditions for observability will change. The performance of the observer will depend on how closely the chosen model matches the actual disturbances. Which unmeasured disturbances are chosen to be estimated depends on the system to be controlled and the type of controller used. In the input–output linearization approach, disturbances that do *not* satisfy the matching condition are estimated. The matching condition implies that the manipulated input has a more "direct" effect on the controlled output than does the disturbance. The concept of "directness" is formalized by introducing the relative degree r of the manipulated input (u) and the relative degree ρ_i of the ith disturbance (d_i) (Henson and Seborg, 1991). The disturbance is said to satisfy the matching condition if $\rho_i > r$. Hence, each disturbance contained in the vector d has the property that $\rho_i \leq r$. If another technique such as nonlinear model predictive control is used, then knowledge of which disturbances cause the most deleterious effects determines the choice of the vector d.

In the case where both unmeasured state and disturbance variables are estimated, the functions $f(x)$ and $g(x)$ are assumed to be affine with respect to the unmeasured state variables as in equation (2). Furthermore, the function $p(x)$ is assumed to depend only on measured outputs: $p(x) = p(y_m)$. The resulting system is affine with respect to the unmeasured variables. We refer to this problem as disturbance estimation under partial-state feedback.

It is important to note that the observer design procedures can be applied to nonlinear systems in which some of the unmeasured variables do not appear linearly. This is accomplished by linearizing the model only with respect to the unmeasured state and disturbance variables, thereby producing a state-affine model of the form required. This approach generally will provide a more accurate description of the nonlinear system than traditional Jacobian linearization. We have used this technique to estimate substrate concentration (which appears nonlinearly) in a nonlinear bioreactor model (Kurtz and Henson, 1996). It is, of course, preferable to measure as many "nonlinear" variables as possible.

2.1. Process examples

Although the class of systems considered is restrictive, a number of chemical processes have the affine form discussed above. For example, continuous stirred tank reactors (CSTR) which involve first-order kinetics yield model equations that are affine in the reactor concentrations. Consider a CSTR with the irreversible reaction $A \rightarrow B$ given by the following equations (Henson and Seborg, 1991):

$$\dot{C}_A = \frac{q}{V}(C_{A_f} - C_A) - k_0 \exp\left(-\frac{E}{RT}\right) C_A,$$

$$\dot{T} = \frac{q}{V}(T_f - T) + \frac{(-\Delta H)}{\rho C_p} k_0 \exp\left(-\frac{E}{RT}\right) C_A + \frac{UA}{\rho C_p V}(T_c - T),$$

$$\dot{T}_c = \frac{q_c}{V_c}(T_{c_f} - T_c) + \frac{UA}{\rho_c C_{pc} V_c}(T - T_c), \qquad (4)$$

where C_A is the concentration of reactant A, T is the reactor temperature, T_c is the coolant temperature, C_{A_f} is the feed concentration, T_f is the feed temperature, q_c is the coolant flow rate, and T_{c_f} is the coolant feed temperature. Usually, the variable to be controlled is reactor temperature ($y = T$) while the manipulated input is chosen as the coolant flow rate ($u = q_c$). If the reactor temperature is measured ($y_m = T$), then the reactor concentration C_A, the coolant temperature T_c and the disturbances $d = [C_{A_f}\ T_f\ T_{c_f}]^T$ appear linearly. State and disturbance estimation for this reaction system is discussed by Kurtz and Henson (1995).

As a second example, consider a fluidized bed reactor (FBR) in which the oxidation of benzene to maleic anhydride and carbon oxides takes place (Perrier, 1982):

$$\begin{array}{ccc} \text{Benzene} & \leftrightarrow & \text{Maleic Anhydride} \\ & \searrow \quad \swarrow & \\ & \text{Carbon Oxides} & \end{array}$$

This system has been described by a simple two phase model under the assumption of a perfectly mixed dense phase. The resulting dimensionless equations are as follows (Aoufoussi *et al.*, 1992):

$$\begin{aligned} \dot{x}_1 &= a_1(d_1 - x_1) + a_2(x_2 - x_1) + a_3[k_{1f}\Delta H_{r1}\xi_1(x_1) \\ &\quad + k_{3f}\Delta H_{r3}\xi_3(x_1)]x_3 + a_3 k_{2f}\Delta H_{r2}\xi_2(x_1)\,x_4, \\ \dot{x}_2 &= c_1 + a_2 b_1(x_1 - x_2) + b_2(d_2 - x_2)\,u, \\ \dot{x}_3 &= a_4(d_3 - x_3) - a_5[k_{1f}\xi_1(x_1) + k_{3f}\xi_3(x_1)]\,x_3, \\ \dot{x}_4 &= a_4(d_4 - x_4) + a_5[k_{1f}\xi_1(x_1)\,x_3 - k_{2f}\xi_2(x_1)\,x_4], \end{aligned} \qquad (5)$$

where x_1 is the dimensionless reactor temperature, x_2 is the dimensionless wall temperature, x_3 is the dimensionless benzene mole fraction, x_4 is the dimensionless maleic anhydride mole fraction, u is the dimensionless coolant flow rate, d_1 represents the inlet reactor temperature, d_2 represents the inlet coolant temperature, d_3 represents the inlet benzene mole fraction, and d_4 represents the inlet maleic anhydride mole fraction. The reaction rate expressions ξ_i are described as:

$$\xi_i(x_1) = \exp\left(\frac{\beta_i x_1}{1 + (x_1/\beta_m)}\right). \qquad (6)$$

A full description of the dimensionless variables and parameter values can be found in Kendi and Doyle (1996). The controlled output usually is chosen as the reactor temperature ($y = x_1$). If reactor temperature is measured, all the other state variables (x_2, x_3, x_4) appear linearly. In addition, all the disturbance variables (d_1–d_4) appear linearly, thus making the FBR system an ideal candidate for the proposed method. The simulation studies performed in subsequent sections are based on this FBR model.

2.2. Related work

It should be noted that the design of observers and output feedback controllers for the class of systems described above has been explored elsewhere. Praly (1992) designs an output feedback controller using a Lyapunov technique which necessitates the inclusion of a strong detectability condition as well as growth conditions on the Lyapunov function candidate. Pomet *et al.* (1993) design a Lyapunov-based output feedback controller, but do not explicitly consider the problem of observer design. Gibon-Fargeot *et al.* (1994) propose a nonlinear observer which resembles the linear Kalman filter. Although the design yields a stable observer, a potentially computationally expensive calculation is required to determine the observer gains. As compared to these techniques, the proposed method offers several important advantages including: (i) technical assumptions such as persistent excitation and Lipschitz continuity are not required; and (ii) the design procedure is similar to that employed for linear systems, thus making the observer easy to construct, implement, and tune.

3. State estimation

First we consider the estimation of unmeasured state variables only (i.e. disturbances are not considered). Estimates of the state variables are generated by a nonlinear closed-loop observer. In this section, we present the basic observer design procedure, discuss the observability of the affine model, and propose two methods to calculate the nonlinear observer gain. To simplify the subsequent analysis, the model consisting of (1) and (2) is rewritten as,

$$\dot{x} = \alpha(u, y_m)\, x + \beta(u, y_m),$$
$$y_m = Cx, \tag{7}$$

where $\alpha(u, y_m) = f_2(y_m) + g_2(y_m)u$ and $\beta(u, y_m) = f_1(y_m) + g_1(y_m)\,u$.

3.1. Basic design procedure

The objective of the observer design is to exactly linearize the estimation error dynamics in the sense described below. The observer is chosen to have the form

$$\dot{\hat{x}} = \alpha(u, y_m)\hat{x} + \beta(u, y_m) + k(u, y_m)\,[y_m - C\hat{x}] \tag{8}$$

where $k(u, y_m)$ is an $n \times m$ nonlinear observer gain matrix which depends only on the manipulated input and the measured outputs. By defining the estimation error as $e = x - \hat{x}$, it is easy to show that the observer error dynamics are:

$$\dot{e} = [\alpha(u, y_m) - k(u, y_m)\, C]\, e = A_0(u, y_m)\, e. \tag{9}$$

Note that the error dynamics resulting from a linear observer are obtained if the model (7) is linear ($\alpha(u, y_m) = A$, $\beta(u, y_m) = bu$) and the observer gain is constant ($k(u, y) = k$). The objective is to design the gain $k(u, y_m)$ such that the matrix $A_0(u, y_m)$ has specified eigenvalues that are invariant with respect to u and y_m. The existence and computation of such an observer gain are discussed below.

The proposed approach is considerably less demanding than alternative observer design techniques based on exact linearization of the error dynamics (Isidori, 1989; Krener and Isidori, 1983). In these methods, additional assumptions are required to guarantee the existence of the observer and the construction of the necessary coordinate transformations often is intractable. Consequently, the proposed method is applicable to a larger class of process control systems. On the other hand, our technique is restricted to nonlinear models which are affine in the unmeasured state variables and it is more difficult to prove stability (see Section 6).

3.2. Observability

The existence of observer gains which achieve the desired objective is related to the observability of the nonlinear system. The nonlinear system (1) *without* input is locally observable (Hermann and Krener, 1977) at the point $\bar{x}$ if the following matrix has rank n when $x = \bar{x}$,

$$W_0(x) = \begin{bmatrix} \dfrac{\partial}{\partial x} h_m(x) \\ \dfrac{\partial}{\partial x} L_f\, h_m(x) \\ \vdots \\ \dfrac{\partial}{\partial x} L_f^{n-1}\, h_m(x) \end{bmatrix} \tag{10}$$

where $h_m(x)$ is the (possibly nonlinear) output measurement map. By analogy to the linear case, $W_0(x)$ is called the *nonlinear observability matrix*. If the system is observable for all $\bar{x}$ in a set S, then we say that the system is observable on the set S. The system *with* input is considerably more difficult to analyze because an input $u(t)$ can cause two different initial conditions to produce identical output trajectories. In this case, we say that $u(t)$ is a singular input (Gibon-Fargeot *et al.*, 1994).

By treating u and y_m as *known* signals, the nonlinear observability matrix for (7) can be written as:

$$W_0(u, y_m) = \begin{bmatrix} C \\ C\alpha(u, y_m) \\ \vdots \\ C\alpha^{n-1}(u, y_m) \end{bmatrix}. \tag{11}$$

If the system remains at rest ($u(t) = \bar{u}$, $y_m(t) = \bar{y}_m$), the observability matrix W_0 is independent of time, and the nonlinear system is *globally* observable if and only if $W_0(\bar{u}, \bar{y}_m)$ is full rank. For the general time-varying case, the nonlinear system is *locally* observable at the point $(\bar{u}, \bar{y}_m)$ if and only if $W_0(\bar{u}, \bar{y}_m)$ has rank n. In most cases, observability cannot be checked a priori since values of $u(t)$ and $y_m(t)$ are not known.

Table 1. Parameter values for the fluidized bed reactor model

Parameter	Value	Parameter	Value
β_m	19.1172	b_1	12.0169
β_1	1.1626	b_2	0.1388
β_2	0.6747	c_1	0.2646
β_3	1.1626	ΔH_{r1}	1.5165e + 06
a_1	0.0014	ΔH_{r2}	1.1521e + 06
a_2	0.01	ΔH_{r3}	2.6885e + 06
a_3	0.0005457	k_{1f}	0.1186e-04
a_4	0.6265	k_{2f}	0.0525e-04
a_5	12847	k_{3f}	0.0509e-04
T^*	33.1115 K	y^*	2
F^*	1 m^3/s	T_f	633 K

3.3. *Calculation of the observer gain*

Recall that the nonlinear observer (8) yields the error dynamics (9). We seek a nonlinear observer gain $k(u, y_m)$ which allows the spectrum of $A_0(u, y_m)$ to be assigned arbitrarily. The spectrum should be invariant with respect to u and y_m to guarantee certain stability properties of the observer (see Section 6). If the system is time invariant, these properties can be achieved if and only if $W_0(\bar{u}, \bar{y}_m)$ is full rank. In the general time-varying case, the spectrum is assignable if and only if $W_0(u, y_m)$ is full rank for all (u, y_m) in the compact set in which the system evolves.

Assuming the nonlinear system is observable, the remaining problem is to calculate the observer gain. If the model is low dimensional, the gain often can be determined analytically by setting the characteristic polynomial of the error dynamics (9) equal to a desired polynomial,

$$\det[\lambda I + k(u, y_m)\, C - \alpha(u, y_m)] = \lambda^n + \gamma_{n-1}\lambda^{n-1} + \cdots + \gamma_0 \qquad (12)$$

where the γ_i are observer tuning parameters chosen to make the polynomial Hurwitz. This method is used in most of the subsequent simulation examples. However, an analytical solution is difficult to obtain for higher-dimensional systems. An alternative approach is to calculate the gain at each sampling instant by solving equation (12) using the current values of u and y_m. As compared to the analytical technique, this method is computationally expensive since a linear observer design problem must be solved at each sampling instant. This approach is demonstrated when simultaneous state and disturbance estimation is considered.

In order to simplify the gain calculation, it may be possible to reduce the order of the observer by eliminating a subset of the state equations when forming the affine model. The reduced-order observer still utilizes all available measurements when possible, but only uses the chosen subset of measured outputs to form the estimation error. This order reduction is demonstrated in the following simulation example.

3.4. *Simulation example*

The proposed state estimation technique is evaluated using the fluidized-bed reactor model (5). Nominal parameters and initial conditions for the model are given in Tables 1 and 2, respectively. Since disturbances are not considered in this example, the term $p(x)\,d$ can be incorporated into $f(x)$ in the model equation (1).

3.4.1. Nonlinear controller design. The objective is to control reactor temperature (x_1) with the coolant flow rate (u). It is easy to verify that the relative degree $r = 2$; therefore, the input–output linearizing controller has the following form (Isidori, 1989)

$$u = \frac{v - L_f^2 h(x)}{L_g L_f h(x)}. \qquad (13)$$

The "new" input v is chosen as (Henson and Seborg, 1991)

$$v = -\frac{3}{\varepsilon} L_f h(x) - \frac{3}{\varepsilon^2}(y - y_0) + \frac{1}{\varepsilon^3}\int_0^t (y_{sp} - y)\, d\tau \qquad (14)$$

where y_{sp} is the setpoint, y_0 is the nominal value of the output, and $\varepsilon > 0$ is the controller tuning parameter. Explicit expressions for the Lie derivatives $L_f^2 h(x)$ and $L_g L_f h(x)$ can be found in (Aoufoussi *et al.*, 1992).

Table 2. Nominal values for the fluidized bed reactor model

Physical variable	Nominal value	Dimensionless variable
Bed temperature (T_r)	703.175 K	$x_1 = (T_r - T_f)/T^*$
Wall temperature (T_w)	572.437 K	$x_2 = (T_w - T_f)/T^*$
Benzene mole fraction (y_{bz})	0.47686	$x_3 = y_{bz}/y^*$
Maleic anhydride mole fraction (y_{ma})	0.76672	$x_4 = y_{ma}/y^*$
Inlet reactor temperature (T_{r0})	633 K	$d_1 = (T_{r0} - T_f)/T^*$
Inlet coolant temperature (T_{c0})	303 K	$d_2 = (T_{c0} - T_f)/T^*$
Inlet benzene mole fraction (y_{bz0})	2	$d_3 = y_{bz0}/y^*$
Inlet maleic anhydride mole fraction (y_{ma0})	0	$d_4 = y_{ma0}/y^*$
Coolant flow rate (F_c)	0.65437 m^3/s	$u = F_c/F^*$

Full-state feedback is required to implement the control law. If any state variables are unmeasured, then an observer is used to provide estimates of those variables for the controller. The resulting control law can be represented as,

$$u = \frac{v - L_f^2 h(y_m, \hat{x})}{L_g L_f h(y_m, \hat{x})} \tag{15}$$

where the design of v is modified accordingly. The controller parameter is chosen as $\varepsilon = 10$ s. Saturation constraints are imposed on the input to keep it within the physically meaningful range (0–1). However, no constraint handling technique, such as anti-reset windup (Kendi and Doyle, 1996), is used in the simulations that follow.

3.4.2. Nonlinear observer design. We consider the problem of estimating the benzene mole fraction (x_3) and the maleic anhydride mole fraction (x_4) from measurements of reactor temperature (x_1) and wall temperature (x_2). A reduced-order observer is obtained by basing the estimation error on the reactor temperature only. It is important to note that the measurement of the wall temperature is used elsewhere (i.e. in the controller and observer equations), but it is not used to drive the observer. The order reduction is realized by effectively eliminating the x_2 state equation. That is, the state vector of the observer is defined as:

$$\hat{x} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_3 \\ \hat{x}_4 \end{bmatrix}. \tag{16}$$

The x_2 state equation can be eliminated because it offers no additional information beyond what the x_1 state equation provides. Working from the fluidized bed reactor model (5), the matrices in the observer (8) are as follows:

$$\alpha(u, y_m) = \begin{bmatrix} 0 & a_3[k_{1f}\Delta H_{r1}\xi_1(x_1) + k_{3f}\Delta H_{r3}\xi_3(x_1)] & a_3 k_{2f}\Delta H_{r2}\xi_2(x_1) \\ 0 & -a_4 - a_5[k_{1f}\xi_1(x_1) + k_{3f}\xi_3(x_1)] & 0 \\ 0 & a_5 k_{1f}\xi_1(x_1) & -a_4 - a_5 k_{2f}\xi_2(x_1) \end{bmatrix},$$

$$\beta(u, y_m) = \begin{bmatrix} a_1(d_1 - x_1) + a_2(x_2 - x_1) \\ a_4 d_3 \\ a_4 d_4 \end{bmatrix},$$

$$C = [1 \quad 0 \quad 0],$$

$$y_m = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}. \tag{17}$$

Note that the matrices are not a function of the input due to the elimination of the x_2 state equation. It is easy to show by the tests discussed above that the reduced-order system is locally observable around the nominal values given in Table 2 and, therefore, yield a locally exponentially stable observer (see Section 6). The nonlinear observer gains are calculated analytically by equating the characteristic polynomial of the error dynamics to the characteristic polynomial $\lambda^3 + \gamma_2\lambda^2 + \gamma_1\lambda + \gamma_0$, where the γ_i are chosen to give roots at $r = [-1 \quad -1.1 \quad -1.05]$. These poles are chosen such that the dominant time constant of the observer is much faster than the controller time constant. Distinct roots are used to allow a more direct comparison with the linear observer described below. The gain matrix $k(u, y_m)$ calculated for the nonlinear observer is shown in the Appendix.

The nonlinear observer design procedure is analogous to that used for linear systems (Brogan, 1991) and maintains the computational simplicity of its linear counterpart. Therefore, it is meaningful to compare the proposed observer and a conventional linear observer. The linear observer is designed by linearizing the nonlinear model (via Jacobian approximation) at the nominal operating point shown in Tables 1 and 2. An order reduction is performed as in the nonlinear case. Specifically, the wall temperature measurement is treated as an additional measured input rather than a state variable. The MATLAB routine PLACE is used to calculate the gains of the linear observer. This routine does not allow pole multiplicities greater than the number of outputs; thus, the poles are chosen to have distinct values: $r = [-1 \quad -1.1 \quad -1.05]$.

3.4.3. Simulation results. The open-loop responses of the linear and nonlinear observers to an initial condition error are shown in Fig. 1. The initial error is $+0.1$ for the benzene mole fraction and -0.2 for the maleic anhydride mole fraction. The responses are comparable for the benzene estimates, but slightly better estimates of maleic anhydride mole fraction are obtained with the nonlinear observer. Figure 2 shows results for the same initial condition error with a -0.1 change in coolant flow rate at $t = 0$. The estimates of the linear observer do not converge to the true values since the system moves from the region where the linear model is valid. Note the large error produced by the linear observer for the maleic anhydride mole fraction. The nonlinear observer, however, rapidly converges to the true values.

Closed-loop responses of the observer/controller combinations are shown in Figs 3 and 4. Stabilization of the nominal point in the presence of a $+0.1$ initial condition error in the benzene mole fraction and a -0.2 initial condition error in the maleic anhydride mole fraction is shown in Fig. 3. Reactor temperature

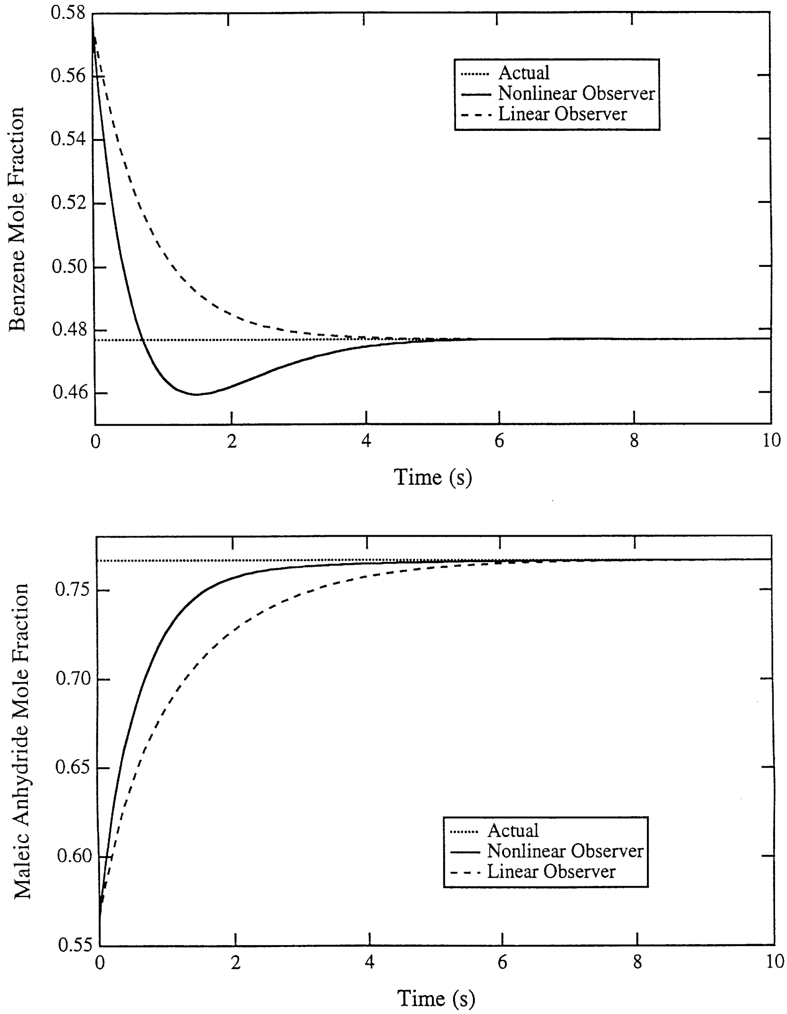

Fig. 1. Open-loop responses to an initial condition error.

is effectively controlled with the controller utilizing the nonlinear observer. The response of the controller utilizing the linear observer is oscillatory, but convergence to the desired setpoint eventually is achieved. However, the state estimates produced by the linear observer are much less accurate despite the small changes in reactor temperature. The error in the maleic anhydride mole fraction is especially large, indicating the possibility of problems for more severe tests such as setpoint or disturbance changes. On the other hand, the nonlinear observer quickly converges to the true values, thus yielding an output that only slightly deviates from the setpoint.

The responses of the observer/controller combinations to a $+33$ K setpoint change and the same initial condition errors as in the previous test are shown in Fig. 4. The response obtained when the controller has full-state feedback also is shown. The nonlinear observer yields an output response almost identical to that in the full-state feedback case. When the linear observer is employed, the closed-loop system is unstable due to the input encountering the saturation constraints. This behavior is attributable to the poor estimates produced by the linear observer which, in turn, is a result of the linear model used for observer design.

4. Disturbance estimation

Now we consider the estimation of unmeasured disturbances under full-state feedback. The disturbances are treated as unmeasured state variables and estimated using a nonlinear observer designed for the augmented system (3). The design is simplified considerably by exploiting the affine structure of the augmented system.

4.1. Basic design procedure

As in the state estimation problem, it is convenient to rewrite the augmented system to simplify the

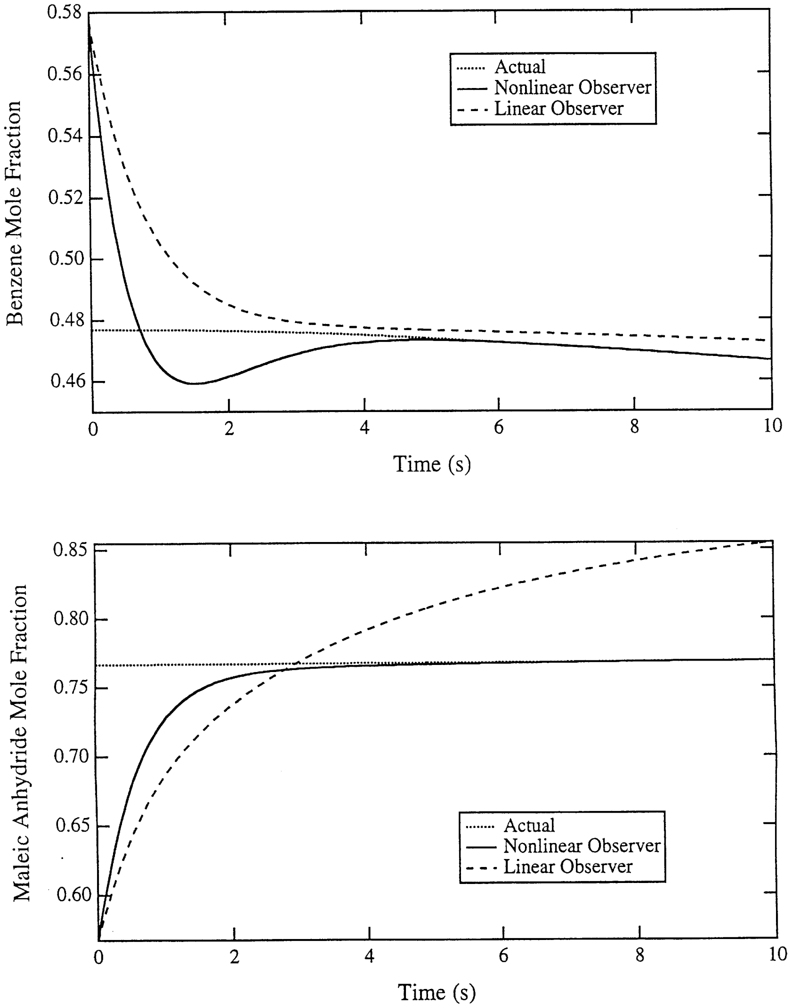

Fig. 2. Open-loop responses to an initial condition error and an input change.

development of the observer. The model (3) can be written as,

$$\dot{z} = \tilde{\alpha}(u, y_m)\, z + \tilde{\beta}(u, y_m),$$
$$y_m = \tilde{C} z, \tag{18}$$

where z is the $(n+q)$-dimensional vector of augmented state variables defined by

$$z \equiv \begin{bmatrix} x \\ d \end{bmatrix}. \tag{19}$$

Because we are considering full-state feedback ($y_m = x$), the matrices in equation (18) are

$$\tilde{\alpha}(y_m) = \begin{bmatrix} 0 & p(y_m) \\ 0 & 0 \end{bmatrix},$$
$$\tilde{\beta}(u, y_m) = \begin{bmatrix} f(y_m) + g(y_m)\, u \\ 0 \end{bmatrix}, \tag{20}$$
$$\tilde{C} = [I_n \quad 0].$$

The nonlinear observer for the augmented system (18) is constructed as,

$$\dot{\hat{z}} = \tilde{\alpha}(u, y_m)\, \hat{z} + \tilde{\beta}(u, y_m) + k(u, y_m)\, (y_m - \tilde{C}\hat{z}) \tag{21}$$

where $\hat{z}$ denotes the estimated state vector and k is a nonlinear observer gain matrix that depends only on measurable signals (u, y_m). If the estimation error is defined as $e \equiv z - \hat{z}$, then the observer error dynamics can be written as:

$$\dot{e} = [\tilde{\alpha}(u, y_m) - k(u, y_m)\, \tilde{C}]\, e \equiv A_0(u, y_m)\, e \tag{22}$$

As before, the objective is to design the observer gain $k(u, y_m)$ such that the matrix $A_0(u, y_m)$ has arbitrary, constant eigenvalues. It should be noted that the dimension of the augmented system does not necessarily determine the complexity of the observer design problem. Under full-state feedback, a number of chemical process systems have

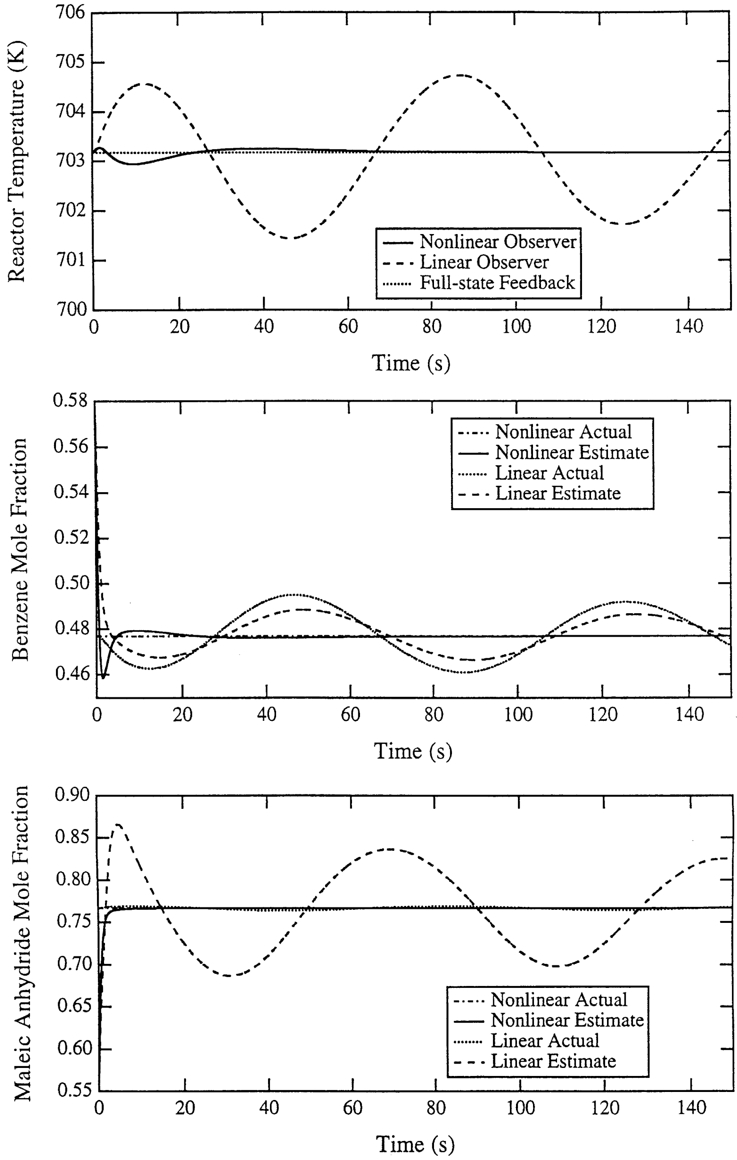

Fig. 3. Closed-loop stabilization in the presence of an initial condition error.

a special form, or can be put into this form by order reduction, such that the observer gains can be more readily determined analytically. This is discussed in Section 4.3.

4.2. Observability

We seek to determine conditions under which the eigenvalues of the matrix $A_0(u, y_m)$ can be placed arbitrarily. As expected, the conditions are related to the observability of the augmented nonlinear system (18)–(20). The augmented system is locally observable at the point $\bar{x}$ if the nonlinear observability matrix $W_0(x)$ has full rank at $x = \bar{x}$ (i.e. rank $(W_0) = n + q$) (Isidori, 1989). Treating u and y_m as known signals, the observability matrix assumes the following form in this case:

$$W_0(y_m) = \begin{bmatrix} I_n & 0 \\ 0 & p(y_m) \\ 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \end{bmatrix}. \tag{23}$$

Since $p(y_m) \in \Re^{n \times q}$, it follows that the observability matrix is full rank if and only if $p(y_m)$ has rank q. A necessary condition for this to hold is that $n \geq q$. Hence, the number of estimated disturbances cannot exceed the number of state variables. In general, W_0 is

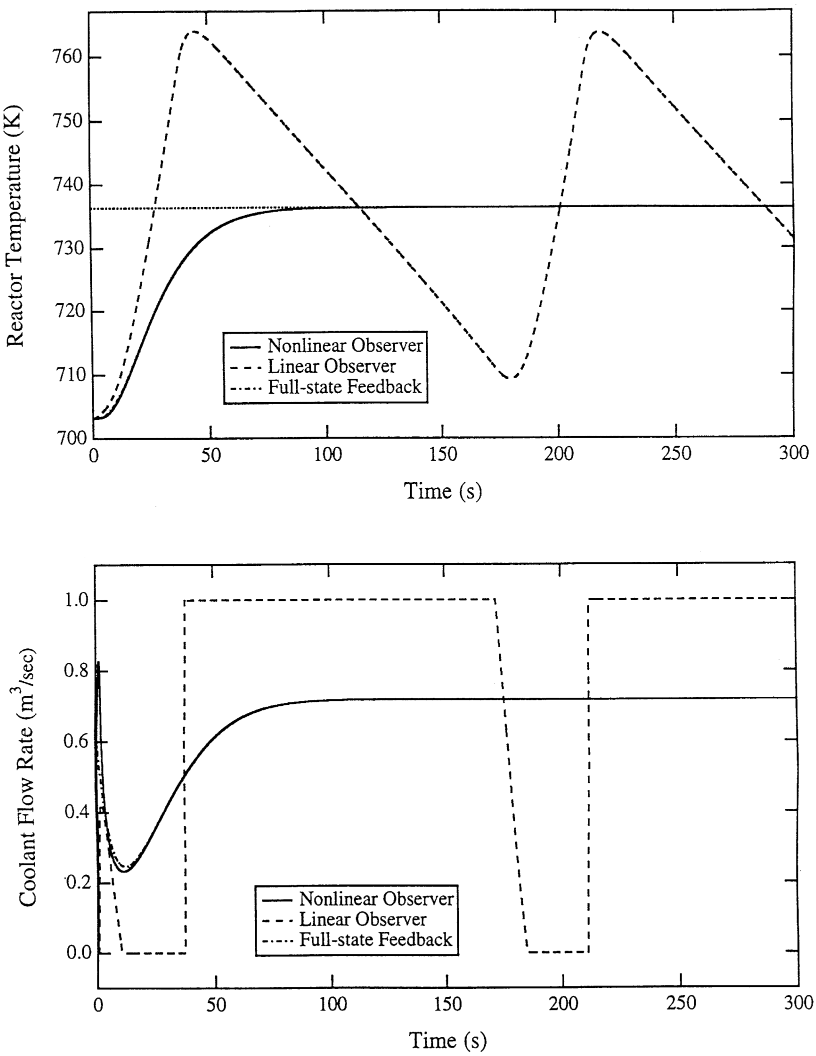

Fig. 4. Closed-loop responses to + 33 K setpoint change and an initial condition error.

an implicit function of time, and the system is locally observable at the point $\bar{y}_m$ if and only if $p(\bar{y}_m)$ has rank q. The system is observable on a set S if $p(y_m)$ has rank q for all $y_m \in S$.

4.3. Reduced-order observer

The full-order observer presented above uses all the state equations of the augmented system. As a result, the observer has dimension $n + q$. It is possible to construct a reduced-order observer if the number of state variables (n) is strictly greater than the number of estimated disturbances (q). Using observability arguments, it is easy to show that the minimal order possible is $2q$. Under full-state feedback, the order reduction allows the error matrix to be partitioned as,

$$A_0(u, y_m) = \begin{bmatrix} A_{11}(u, y_m) & A_{12}(u, y_m) \\ A_{21}(u, y_m) & A_{22}(u, y_m) \end{bmatrix} \tag{24}$$

where each matrix A_{ij} is square. For some systems this block structure greatly simplifies the calculation of the observer gain matrix, as discussed below.

The order reduction is performed by using a subset of the process state vector to form the augmented state vector,

$$w = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_l \\ d \end{bmatrix},$$

where we have assumed, without loss of generality, that the first l state variables are chosen. The reduced-order augmented system is represented as,

$$\begin{aligned} \dot{w} &= \breve{\alpha}(u, y_m)\, w + \breve{\beta}(u, y_m), \\ \breve{y}_m &= \breve{C} w, \end{aligned} \tag{25}$$

where $w \in \Re^{l+q}$ and the "$\breve{\ }$" is used to denote submatrices of the full-order matrices in equation (18). The matrices in equation (25) have the following form for the full-state feedback case ($y_m = x$):

$$\breve{\alpha}(u, y_m) = \begin{bmatrix} 0 & \breve{p}(y_m) \\ 0 & 0 \end{bmatrix},$$

$$\breve{\beta}(u, y_m) = \begin{bmatrix} \breve{f}(y_m) + \breve{g}(y_m)u \\ 0 \end{bmatrix}, \tag{26}$$

$$\breve{C} = [I_l \quad 0].$$

Observability of the reduced-order augmented system can be checked using the methods presented earlier. More precisely, the observability test determines whether a particular order reduction is valid. It follows that the system is observable if and only if the reduced-order matrix $\breve{p}(y_m)$ has rank q. This result indicates that it is *always* possible to perform an order reduction under full-state feedback as long as $n > q$ and the original system is observable. If $p(y_m)$ has rank q then an $l \times q$ ($q \le l < n$) submatrix of $p(y_m)$ with rank q can be extracted. The reduced-order system is formed based on this submatrix of p.

There is a particular class of systems that admits an order reduction such that the observer gain calculation is simplified considerably. This is possible when $\breve{p}(y_m)$ is either an upper or lower triangular matrix. By choosing particular elements of the gain matrix $k(u, y_m)$, the matrix $A_0(u, y_m)$ has the following form when $p(y_m)$ is upper triangular,

$$\alpha(u, y_m) - \begin{bmatrix} k_1(u, y_m) \\ k_2(u, y_m) \end{bmatrix} [I_l \quad 0]$$

$$= \begin{bmatrix} -k_{11}^1 & \cdots & 0 & \breve{p}_{11} & \cdots & \breve{p}_{1q} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -k_{qq}^1 & 0 & \cdots & \breve{p}_{qq} \\ -k_{11}^2 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & -k_{qq}^2 & 0 & \cdots & 0 \end{bmatrix}$$

$$\equiv M(u, y_m) \tag{27}$$

where the dependence of the matrix elements on u and y_m has been omitted for convenience. The following property of matrix determinants is very useful for this system:

If

$$T = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

with A, B, C, D all upper or lower triangular matrices of equal dimension j, then:

$$\det(T) = \det\left(\begin{bmatrix} a_{11} & b_{11} \\ c_{11} & d_{11} \end{bmatrix}\right) \det\left(\begin{bmatrix} a_{22} & b_{22} \\ c_{22} & d_{22} \end{bmatrix}\right) \cdots \det\left(\begin{bmatrix} a_{jj} & b_{jj} \\ c_{jj} & d_{jj} \end{bmatrix}\right).$$

Because the matrix $\lambda I - M$ has this special form, it follows that:

$$\det(\lambda I - M) = (\lambda^2 + k_{11}^1 \lambda + p_{11} k_{11}^2) \times (\lambda^2 + k_{22}^1 \lambda + p_{22} k_{22}^2) \cdots$$

By factoring the desired polynomial into quadratics, the observer gains are easily calculated by equating the corresponding coefficients. Note that there is considerable freedom as to which roots are assigned to which gains.

Another potential disadvantage of the full-order observer is that unmeasured disturbances that would be decoupled by a standard input–output linearizing controller may no longer be decoupled due to interactions between the controller and the estimator. In the full-state feedback case, it may be possible to perform the order reduction such that n–q of these disturbances remain decoupled despite the introduction of the estimator.

4.4. Simulation example

The proposed disturbance estimation technique is evaluated using the fluidized bed reactor model. The input–output linearizing control law based on the augmented model (18) is the same as that derived in the case of state estimation since constant disturbances are assumed. The implementation of the controller, however, is different because full-state feedback is used, and disturbance estimates generated by the nonlinear observer are used directly in the control law.

4.4.1. Nonlinear observer design. We estimate the inlet reactor temperature (d_1) and inlet benzene mole fraction (d_3) from the available state measurements. If the entire process state vector is used to drive the observer, then the matrix $p(y_m)$ in equation (20) is

$$p(y_m) = \begin{bmatrix} a_1 & 0 \\ 0 & 0 \\ 0 & a_4 \\ 0 & 0 \end{bmatrix}, \tag{28}$$

It is clear that estimates of d_1 and d_3 can be generated by using only the x_1 and x_3 state equations. That is, by choosing the reduced state vector w as,

$$w = \begin{bmatrix} x_1 \\ x_3 \\ d_1 \\ d_3 \end{bmatrix} \tag{29}$$

a reduced-order observer can be constructed with the reduced-order disturbance matrix:

$$\breve{p}(y_m) = \begin{bmatrix} a_1 & 0 \\ 0 & a_4 \end{bmatrix}. \tag{30}$$

By utilizing only the diagonal elements of the gain matrices $k_1(u, y_m)$ and $k_2(y, y_m)$, the error dynamics of the reduced-order system can be put into the special form (27), thus making the calculation of the observer gains almost trivial. In this case, the gain matrix is,

$$k(u, y_m) = \begin{bmatrix} k_{11}^1 & 0 \\ 0 & k_{22}^1 \\ k_{11}^2 & 0 \\ 0 & k_{22}^2 \end{bmatrix} = \begin{bmatrix} -(r_1 + r_2) & 0 \\ 0 & -(r_3 + r_4) \\ \frac{r_1 r_2}{a_1} & 0 \\ 0 & \frac{r_3 r_4}{a_4} \end{bmatrix} \quad (31)$$

where the r_i are the desired poles of the error dynamics. For the simulation studies in this section, the poles are chosen as $r = [-1 \quad -1.1 \quad -1.05 \quad -1.15]$. As before, the reason for the choice of distinct values is to conform to the design of the linear observer so that a more direct comparison is possible. It is interesting to note that, in this case, the error dynamics (22) depend on constant matrices. Therefore, placement of the eigenvalues in the open left-half plane guarantees global exponential stability. In general, this condition does not hold and stability of the observer requires stronger conditions (see Section 6).

For the sake of comparison, a linear estimator is designed using a linear model obtained via Jacobian approximation of the reduced-order augmented

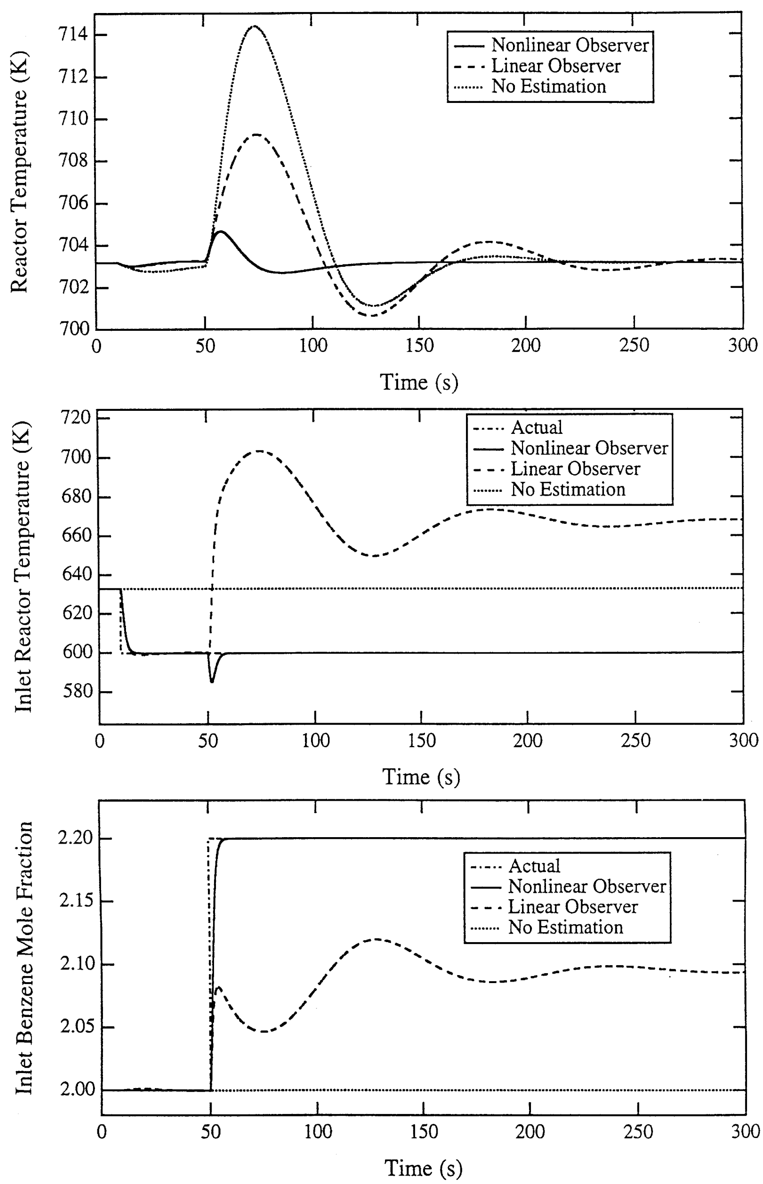

Fig. 5. Closed-loop responses for changes in estimated disturbances.

model (25)–(26) at the nominal operating point. The poles of the estimator are placed at $r = [-1 \quad -1.1 \quad -1.05 \quad -1.15]$.

4.4.2. Simulation results. Figures 5 and 6 compare the disturbance rejection performance of three alternative control schemes, all of which utilize the input–output linearizing controller. The control systems differ only with respect to the type of disturbance estimator used. The three schemes considered use: (i) a nonlinear estimator; (ii) a linear estimator; and (iii) no estimator. Figure 5 shows closed-loop responses for disturbances which are explicitly estimated. A -33.1 K change in the inlet reactor temperature occurs at $t = 10$ s, followed by a $+0.2$ change in the inlet benzene mole fraction at $t = 50$ s. Each of the controller/estimator combinations handle the inlet reactor temperature disturbance quite easily. However, the control system with the nonlinear disturbance estimator clearly outperforms the other schemes for the inlet concentration change as a result of improved disturbance estimates. The responses of the controller/estimator combinations to a $+66.2$ K change in the inlet coolant temperature are shown in Fig. 6. It is important to note that this disturbance is not measured or estimated. While all three estimators produce good responses, this test indicates that the nonlinear estimator has some desirable robustness characteristics.

5. State/disturbance estimation

Finally, we consider simultaneous estimation of state variables and unmeasured disturbances which appear linearly in the process model. This case is referred to as "partial-state feedback" since it is assumed that some subset of the state variables is directly measurable (or inferred from measurements). The augmented system can be written as equation (18) with the matrices having the form:

$$\tilde{\alpha}(u, y_m) = \begin{bmatrix} f_2(y_m) + g_2(y_m)u & p(y_m) \\ 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} \alpha(u, y_m) & p(y_m) \\ 0 & 0 \end{bmatrix},$$

$$\tilde{\beta}(u, y_m) = \begin{bmatrix} f_1(y_m) + g_1(y_m)u \\ 0 \end{bmatrix} \equiv \begin{bmatrix} \beta(u, y_m) \\ 0 \end{bmatrix}, \tag{32}$$

$$\tilde{C} = [C \quad 0].$$

Assuming the system is observable, the observer is constructed as in the previous section on disturbance estimation, thereby yielding the error dynamics (22). The gain calculation also is performed as before.

5.1. Observability

Observability of the augmented system (18) and (32) is guaranteed locally at the point $(\bar{u}, \bar{y}_m)$ if and only if the following conditions hold: (*i*) the original system *without disturbances* is observable at $(\bar{u}, \bar{y}_m)$; and (*ii*) the following matrix is full rank (i.e. has rank $n + q$):

$$\begin{bmatrix} \alpha(\bar{u}, \bar{y}_m) & p(\bar{y}_m) \\ C & 0 \end{bmatrix}. \tag{33}$$

This result can be proven using standard rank condition tests [6], which require that the following matrix is full rank for all values of the variable λ:

$$\begin{bmatrix} \alpha(\bar{u}, \bar{y}_m) - \lambda I_n & p(\bar{y}_m) \\ 0 & -\lambda I_q \\ C & 0 \end{bmatrix}. \tag{34}$$

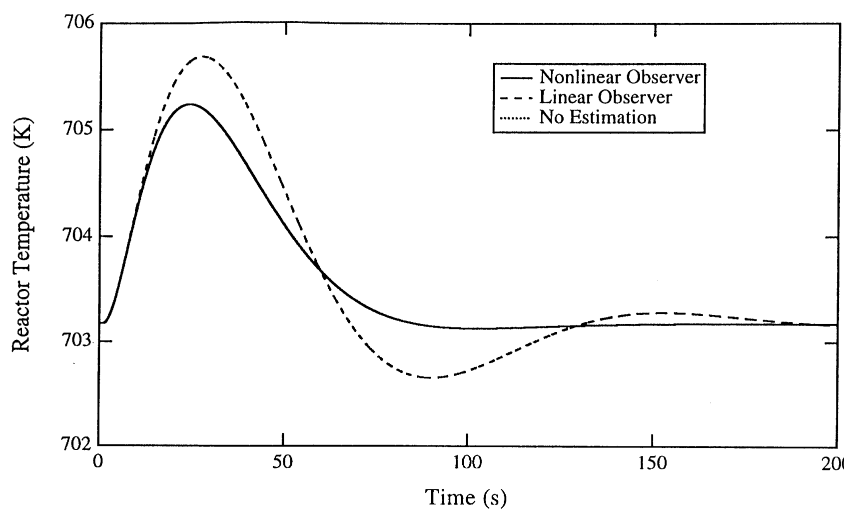

Fig. 6. Closed-loop responses for a change in an unestimated disturbance.

The first condition guarantees that the first column of matrix (34) has rank n. This means an additional q independent columns must be obtained from the second column, which is ensured if λ is nonzero. The second condition is needed to guarantee a total of $(n+q)$ independent columns if $\lambda = 0$. It is obvious that a necessary condition for the second condition to be true is that the matrix $p(\bar{y}_m)$ has rank q. Thus, the number of measured state variables must be greater than or equal to the number of disturbances that are estimated.

5.2. Reduced-order observer

Note that the full-order observer has dimension $n+q$. It may be possible to construct a reduced-order observer if the number of measured outputs (m) is strictly greater than the number of estimated disturbances (q). In this case, it can be shown that the minimal order possible is $2q+n-m$. Assuming that l state variables are chosen to form the closed-loop observer, the reduced-order augmented system is represented by equation (25) where the matrices have the following form:

$$\breve{\alpha}(u, y_m) = \begin{bmatrix} \breve{f}_2(y_m) + \breve{g}_2(y_m)u & \breve{p}(y_m) \\ 0 & 0 \end{bmatrix} \equiv \begin{bmatrix} \alpha'(u, y_m) & \breve{p}(y_m) \\ 0 & 0 \end{bmatrix},$$
$$\breve{\beta}(u, y_m) = \begin{bmatrix} \breve{f}_1(y_m) + \breve{g}_1(y_m)u \\ 0 \end{bmatrix}, \tag{35}$$
$$\breve{C} = [C' \quad 0].$$

The "$\breve{}$" is used to denote submatrices of the full-order matrices in equation (32).

In the partial-state feedback case, it cannot be guaranteed that an order reduction can be performed such that the reduced-order system is observable. The requirements for observability are: (i) the reduced-order system without disturbances must be observable; and (ii) the following matrix must have full rank $l+q$:

$$\begin{bmatrix} \alpha'(\bar{u}, \bar{y}_m) & \breve{p}(\bar{y}_m) \\ C' & 0 \end{bmatrix}. \tag{36}$$

Thus, the first step is to perform the reduction such that the reduced-order system without disturbances is observable. The next step is to check the observability of the reduced-order system with disturbances. This procedure is demonstrated below.

5.3. Simulation example

The proposed method for simultaneous estimation of unmeasured state and disturbances variables is evaluated using the fluidized-bed reactor model (5). The input–output linearizing controller design is the same as that described in Section 3.4. The only difference is that disturbance estimates are utilized directly in the control law.

5.3.1. Nonlinear observer design. Consider the problem of constructing estimates of the unmeasured state variables (x_3, x_4) and disturbances (d_1, d_3) from the measured outputs (x_1, x_2). The resulting augmented system is observable if the following matrix is full rank:

$$\begin{bmatrix} 0 & 0 & a_3[k_{1f}\Delta H_{r1}\xi_1(x_1) + k_{3f}\Delta H_{r3}\xi_3(x_1)] & a_3 k_{2f}\Delta H_{r2}\xi_2(x_1) & a_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -a_4 - a_5[k_{1f}\xi_1(x_1) + k_{3f}\xi_3(x_1)] & 0 & 0 & a_4 \\ 0 & 0 & a_5 k_{1f}\xi_1(x_1) & -a_4x_4 - a_5k_{2f}\xi_2(x_1) & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}. \tag{37}$$

It is obvious that the matrix is rank deficient and, therefore, the augmented system is unobservable. However, the system that results when x_3, x_4, and d_3 are estimated is observable based on the techniques described above. These three variables can be estimated with a reduced-order observer which utilizes only x_1 to drive the error dynamics. This is possible because the x_2 state equation does not provide new information on the estimated variables. The x_2 measurement, however, is used elsewhere in the observer.

The order reduction is performed by defining the following state vector:

$$w = \begin{bmatrix} x_1 \\ x_3 \\ x_4 \\ d_3 \end{bmatrix}. \tag{38}$$

This gives the following matrices for design of the reduced-order observer (35):

$$\breve{\alpha} = \begin{bmatrix} 0 & a_3[k_{1f}\Delta H_{r1}\xi_1(x_1) + k_{3f}\Delta H_{r3}\xi_3(x_1)] & a_3 k_{2f}\Delta H_{r2}\xi_2(x_1) & 0 \\ 0 & -a_4 - a_5[k_{1f}\xi_1(x_1) + k_{3f}\xi_3(x_1)] & 0 & a_4 \\ 0 & a_5 k_{1f}\xi_1(x_1) & -a_4 x_4 - a_5 k_{2f}\xi_2(x_1) & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\breve{\beta} = \begin{bmatrix} a_1(d_1 - x_1) + a_2(x_2 - x_1) \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \breve{C} = [1 \quad 0 \quad 0 \quad 0]. \tag{39}$$

The nonlinear observer gains are calculated at each time step using the MATLAB linear pole-placement algorithm PLACE. The poles are placed at $r = [-1 \quad -1.1 \quad -1.05 \quad -1.15]$. This approach is simple to implement, but it is computationally expensive as compared to the analytical method described earlier.

It is important to note that if nonlinear observer design is not possible because of *structural* unobservability, then linear observer design also will not be possible. This is a consequence of the nonlinear observability tests reducing to the linear case when

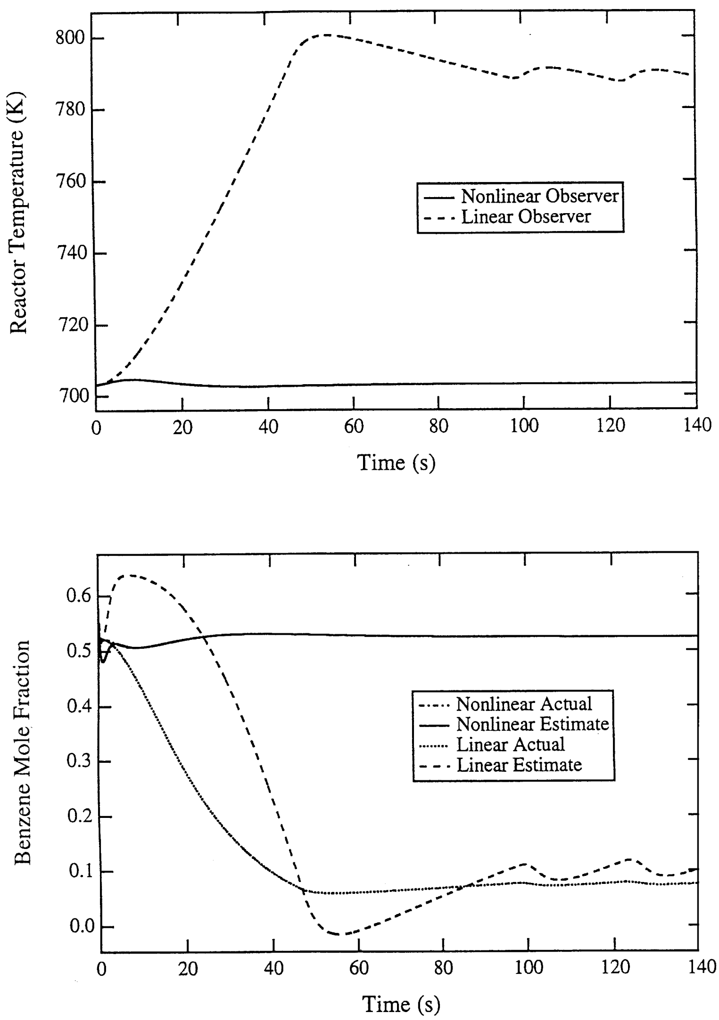

Fig. 7. Closed-loop responses for an initial condition error and a change in an estimated disturbance.

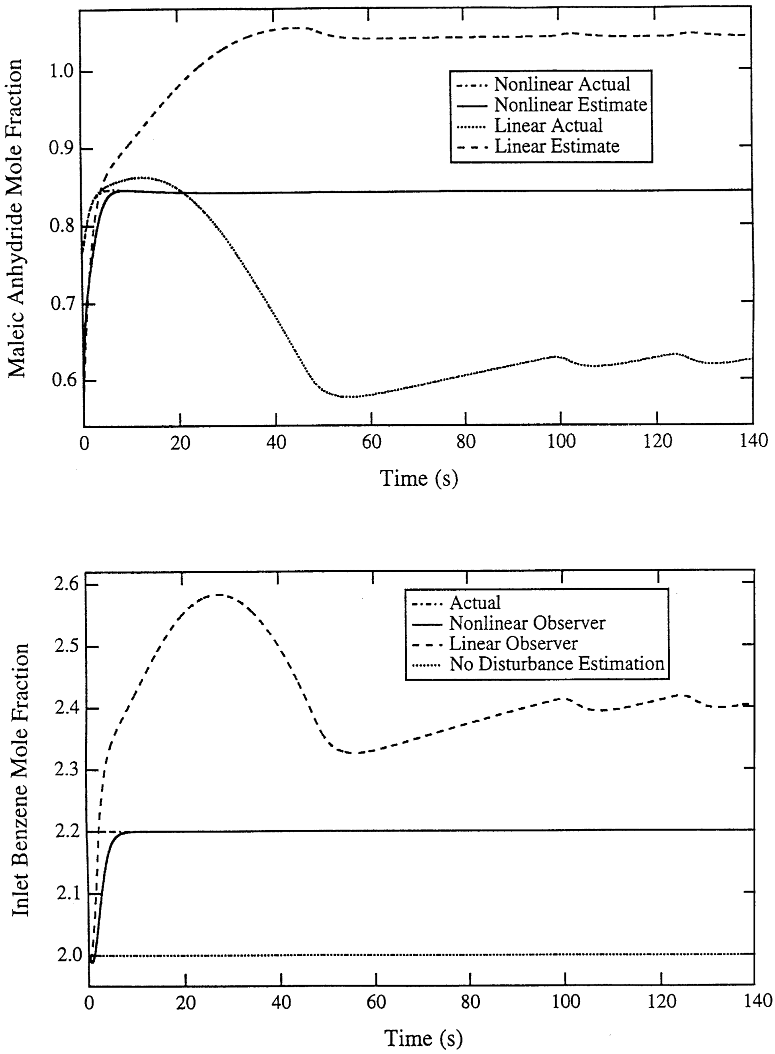

Fig. 8. Estimates for Fig. 7.

dealing with a linear model (Isidori, 1989). Consequently, the linear observer is designed based on the Jacobian linearization of the *reduced-order* system described above. The observer poles are placed in the same locations as the nonlinear observer poles.

5.3.2. Simulation results. We compare three control systems, each of which uses the input–output linearizing controller. The first system uses the nonlinear observer to provide state and disturbance estimates, while the second system uses the linear observer for simultaneous state and disturbance estimation. The third system uses the nonlinear observer to provide state estimates, but disturbance estimation is not attempted.

The responses of the three closed-loop systems to a simultaneous initial condition error and disturbance are shown in Figs 7 and 8. The initial condition error is $+0.1$ in the benzene mole fraction and -0.2 in the maleic anhydride mole fraction, while the disturbance is a $+0.2$ change in the inlet benzene mole fraction at $t = 0$. As discussed below, the response obtained with the nonlinear observer without disturbance estimation is not shown in this case. The nonlinear observer with disturbance estimation produces a vastly superior response as compared to the linear observer (Fig. 7). This behavior is attributable to the nonlinear observer producing more accurate state and disturbance estimates (Fig. 8), which allows the closed-loop system to avoid input constraints. The saturation constraints cause the undesirable response shown for the linear observer. The response of the closed-loop system utilizing a nonlinear observer only for state estimation is not shown because it is even worse than the response obtained with the linear observer. For instance, the linear observer with disturbance estimation yields oscillations around 790 K while the nonlinear observer without disturbance estimation

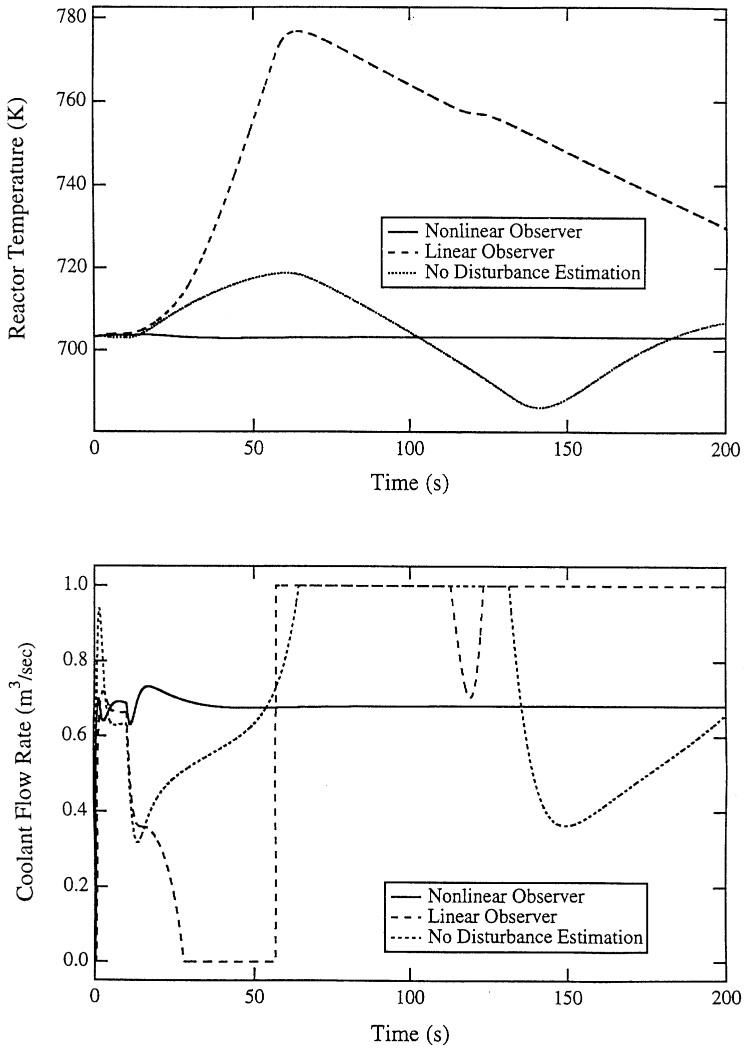

Fig. 9. Closed-loop responses for an initial condition error and a change in an unestimated disturbance.

produces oscillations at a higher value of approximately 900 K.

In Fig. 9, the three control systems are subjected to a $+33.1$ change in the inlet reactor temperature at $t = 10$ s in addition to the same initial condition error used in the previous test. It is important to note that this disturbance is not measured or estimated. The inputs generated by the control systems which utilize the linear observer with disturbance estimation and the nonlinear observer without disturbance estimation encounter the saturation constraints, resulting in poor performance. By contrast, the nonlinear observer with disturbance estimation easily handles the disturbance. The estimates are not shown since all the observers, whether linear or nonlinear, necessarily yield biased predictions. This test indicates the proposed method has some reasonable robustness to plant/model mismatch.

6. Stability of the nonlinear observer

The simulation results presented in the previous sections demonstrate that the proposed estimation technique can lead to excellent closed-loop performance. We now investigate the stability of the nonlinear observer. More precisely, conditions under which the error dynamics given by equation (9) are stable around the zero steady state are explored. The results can be extended to disturbance estimation and combined state/disturbance estimation in a straightforward manner. We are mainly concerned about stability under a constant input, although the results can be extended to a time-varying input if observability is retained. We assume that the nonlinear system is observable, which allows the observer gain matrix $k(u, y_m)$ to be chosen such that the matrix $A_0(u, y_m)$ has constant eigenvalues with negative real part.

The first result demonstrates that the observer (8) is stable in a small region around a nominal point.

Theorem 1. *The zero steady state of the observer error dynamics* (9) *is locally stable at any point* $(\bar{u}, \bar{y}_m)$ *where the nonlinear system* (7) *is observable.*

The proof follows by examination of the error dynamics $\dot{e} = A_0(\bar{u}, \bar{y}_m)\ e$ obtained by linearizing around the point $(u, y, e) = (\bar{u}, \bar{y}_m, 0)$. The observer design guarantees that $\lambda[A_0(\bar{u}, \bar{y}_m)] < 0$, which yields local exponential stability. There are counter-examples (Khalil, 1996; Slotine and Li, 1991) that show the eigenvalue condition on $A_0(u, y_m)$ is *not* sufficient to ensure global stability of the error dynamics. Therefore, additional assumptions are required to guarantee stability in a larger operating region.

The next result uses the theory of slowly varying systems to extend the region of stability.

Theorem 2. *If the following conditions hold*:

1. y_m *is slowly varying,*
2. $A_0(y')$ *is continuously differentiable in* y',
3. $A_0(y')$ *is bounded for all* y',
4. $(\partial/\partial y')A_0(y')$ *is bounded for all* y',
5. *The nonlinear system* (7) *is observable,*

then the zero steady state of (9) *is exponentially stable.*

The proof of Theorem 2 follows directly from Lemma 5.12 and Example 5.13 in (Khalil, 1996) and the fact that the eigenvalues of $A_0(y')$ are chosen to lie in the open left-half plane for every y'. Global exponential stability is achieved if the conditions hold globally. If the conditions hold in some region, then exponential stability is guaranteed only if the system evolves within that region. It should be noted that for many chemical processes, the system outputs evolve only within a finite region (e.g. temperature does not escape to infinity). Thus, condition (2) will imply conditions (3) and (4) since a continuous function of bounded variables is itself bounded.

To use Theorem 2, it is necessary to characterize when y_m is "slowly varying". A bound on $\dot{y}_m$ is found by constructing the following Lyapunov function based on the linear, "frozen" system (Khalil, 1996)

$$V = x^T P(y')x$$

where y' is a "frozen point" of the system. One of the properties of the "frozen" matrix $P(y')$ is that its derivative with respect to y' is bounded:

$$\left\| \frac{\partial}{\partial y'} P(y') \right\| \leq c$$

By taking the derivative of V with respect to time and determining the conditions required for negative definiteness, it can be shown that $\dot{y}_m$ must be strictly less than $1/c$. This condition, unfortunately, is difficult to verify in practice.

Other stability results are potentially applicable. For example, the error dynamics are globally exponentially-stable if the matrix $A + A^T$ has eigenvalues that remain in the open left-half plane (Slotine and Li, 1991). Unfortunately, this result is not particularly useful because it is difficult (and sometimes impossible) to design the observer gain matrix k such that the condition is satisfied. Additional stability theorems for linear time-varying systems can be found in (Hahn, 1963, 1967). However, the conditions imposed usually are very difficult to verify, especially for high-dimensional systems.

7. Summary and conclusions

A state and disturbance estimation technique for an important class of nonlinear systems has been proposed. By considering unmeasured disturbances as additional state variables, a nonlinear closed-loop observer is designed to provide estimates of unmeasured state and disturbance variables. The approach is restricted to process models in which the unmeasured variables appear (or can be made to appear) in an affine manner. This class of models includes all bilinear systems, as well as a number of chemical reaction processes. The design of full-order and reduced-order observers was discussed, and the stability of the nonlinear observer was analyzed. The proposed technique was shown to provide significantly improved open-loop and closed-loop performance as compared to linear methods when applied to a fluidized bed reactor model.

Acknowledgements

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Appendix

The observer gains are more easily represented by defining the following functions:

$$a(x_1) = a_3[k_{1f}\Delta H_{r1}\xi_1(x_1) + k_{3f}\Delta H_{r3}\xi_3(x_1)],$$
$$b(x_1) = a_3 k_{2f}\Delta H_{r2}\xi_2(x_1),$$
$$c(x_1) = -a_4 - a_5[k_{1f}\xi_1(x_1) + k_{3f}\xi_3(x_1)],$$
$$d(x_1) = a_5 k_{1f}\xi_1(x_1),$$
$$e(x_1) = -a_4 - a_5 k_{2f}\xi_2(x_1)$$

Then the observer matrix has the following form:

$$A_0(x_1) = \begin{bmatrix} -k_1(x_1) & a(x_1) & b(x_1) \\ -k_2(x_1) & c(x_1) & 0 \\ -k_3(x_1) & d(x_1) & e(x_1) \end{bmatrix}.$$

The associated characteristic polynomial is,

$$\lambda^3 + [k_1 - c - e]\lambda^2 + [bk_3 + ak_2 - c(k_1 - e) - k_1 e]\lambda + [cek_1 + bdk_2 - bck_3 - aek_2]$$

where the dependence of the functions on x_1 has been omitted for simplicity. This is equated to the desired characteristic polynomial, $\lambda^3 + \gamma_2\lambda^2 + \gamma_1\lambda + \gamma_0$, to give the following expressions for the observer gains,

$$k_1(x_1) = \gamma_2 + c(x_1) + e(x_1),$$
$$k_2(x_1) = \frac{c(x_1)\gamma_1 + \gamma_0 + c^2(x_1)\gamma_2 + c^3(x_1)}{b(x_1)d(x_1) - a(x_1)e(x_1) + a(x_1)c(x_1)},$$
$$k_3(x_1) = \frac{c(x_1)e(x_1)(\gamma_2 + c(x_1) + e(x_1)) + (b(x_1)d(x_1) - a(x_1)e(x_1))k_2(x_1) - \gamma_0}{b(x_1)c(x_1)}.$$

As long as the system is observable, the denominators in the k_2 and k_3 equations are bounded away from zero and the gains are well defined.