Direct Adaptive Control of Partially Known Nonlinear Systems

Richard B. McLain, Michael A. Henson, and Martin Pottmann

***Abstract*— A direct adaptive control strategy for a class of single-input/single-output nonlinear systems is presented. The major advantage of the proposed method is that a detailed dynamic nonlinear model is not required for controller design. The only information required about the plant is measurements of the state variables, the relative degree, and the sign of a Lie derivative which appears in the associated input–output linearizing control law. Unknown controller functions are approximated using locally supported radial basis functions that are introduced only in regions of the state space where the closed-loop system actually evolves. Lyapunov stability analysis is used to derive parameter update laws which ensure (under certain assumptions) the state vector remains bounded and the plant output asymptotically tracks the output of a linear reference model. The technique is successfully applied to a nonlinear biochemical reactor model.**

***Index Terms*—Bioreactor control, direct adaptive control, Lyapunov stability, radial basis functions.**

I. Introduction

MOST advanced control strategies require a suitable dynamic model of the system to be controlled. It is interesting to note that some biological control systems are believed to operate without the use of explicit models [2], [11], [15]. Meanwhile, biological systems provide high performance control of considerably more complex nonlinear systems than those encountered in technological applications [4], [10], [19]. This suggests the modeling step may be eliminated entirely if a satisfactory method for direct construction of the nonlinear controller is available [6]. For linear systems, model reference adaptive control provides such a framework [17].

Several difficulties are encountered when attempting to develop direct adaptive controllers for nonlinear systems. Determination of a suitable controller structure is the first consideration [14]. Lightbody and Irwin [9] include a linear, fixed-gain controller in parallel with a nonlinear adaptive controller. Sanner and Slotine [16] use a derivative of the desired trajectory combined with a proportional-derivative control component which consists of a linear combination of tracking error state variables and an adaptive term to recover the unknown controller functions. In this paper, the nonlinear control law is generated by approximating on-line the unknown functions of the associated input–output linearizing control law [7]. Derivation of stable parameter update laws is another challenging problem. We utilize Lyapunov theory to derive stable update laws [8].

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II. Class of Nonlinear Systems

The unknown nonlinear system is assumed to have the general form,

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned} \tag{1}$$

where x is an n-dimensional vector of measured state variables, and u and y are scalar manipulated input and controlled output variables, respectively. The associated input–output linearizing control law is [5], [7],

$$\begin{aligned} u &= \frac{-L_f^r h(x) - \gamma_r L_f^{r-1} h(x) - \cdots - \gamma_1 h(x) + \gamma_1 y_{\rm sp}}{L_g L_f^{r-1} h(x)} \\ &\equiv \frac{-\alpha^*(x) + \gamma_1 y_{\rm sp}}{\beta^*(x)} \end{aligned} \tag{2}$$

where $y_{\rm sp}$ is the setpoint, r is the relative degree, γ_i are adjustable tuning parameters, $L_f^i h(x)$ and $L_g L_f^{r-1} h(x)$ are Lie derivatives, and $\alpha^*(x)$ and $\beta^*(x)$ represent the "true" controller functions. This control law yields the closed-loop dynamics

$$y^{(r)} + \gamma_r y^{(r-1)} + \cdots + \gamma_1 y = \gamma_1 y_{\rm sp} \tag{3}$$

which can be made stable by suitable choice of the γ_i. Additional assumptions are required to ensure internal stability of the closed-loop system due to the presence of an $(n-r)$-dimensional nonlinear system called the "zero dynamics." A sufficient condition for bounded tracking is that these dynamics are exponentially stable and Lipschitz continuous [18].

We consider the problem where the nonlinear system (1) only is partially known, and the input–output linearizing control law cannot be synthesized directly. The objective is to construct on-line estimates of the unknown controller functions $\alpha^*(x)$ and $\beta^*(x)$. For systems of relative degree one, the function $\beta^*(x) = L_g h(x)$ often is known from basic conservation relations. As an example, consider a continuous biochemical reactor described by the equations [1]

$$\begin{aligned} \dot{X} &= r_1(X, S) - DX \\ \dot{S} &= r_2(X, S) + D(S_f - S) \end{aligned} \tag{4}$$

where X and S are the biomass and substrate concentrations, respectively, D is the dilution rate, S_f is the feed substrate concentration, and $r_1(X, S)$ and $r_2(X, S)$ are unknown functions associated with the reaction kinetics. If D and X are chosen as the manipulated input and the controlled output, respectively, then the system has relative degree one and the function $L_g h(x) = -X$ is known regardless of the form of the reaction kinetics. As shown in Section III, we are able to construct the nonlinear controller by knowing just the sign of $\beta^*(x) = L_g L_f^{r-1} h(x)$.

III. Direct Adaptive Control

A. Radial Basis Function Network

The unknown controller functions $\alpha^*(x)$ and $\beta^*(x)$ are approximated as

$$\begin{aligned} \alpha^*(x) &\cong \sum_{i=1}^{N} \alpha_i \phi_i(x) = \alpha^T \phi(x) \\ \beta^*(x) &\cong \sum_{i=1}^{N} \beta_i \phi_i(x) = \beta^T \phi(x) \end{aligned} \tag{5}$$

where α and β are vectors of time-varying controller parameters, $\phi(x)$ is a vector of basis functions, and N is the number of basis functions employed. The resulting control law has the form

$$u = \frac{-\alpha^T \phi(x) + \gamma_1 y_{\rm sp}}{\beta^T \phi(x)}. \tag{6}$$

A wide variety of basis functions have been proposed for multivariate function approximation [3]. We utilize a locally supported radial basis function of the form [13]

$$\begin{aligned} \phi(r) &= \begin{cases} (1-r)^4(1+4r+3r^2+0.75r^3), & r \in [0,1] \\ 0, & \text{elsewhere} \end{cases} \\ r^2 &= \sum_{j=1}^{n} \frac{(x_j - x_{c_j j})^2}{a_j^2} \end{aligned} \tag{7}$$

where x_c is the basis function center and a_j are scaling parameters. As compared to radial basis functions such as the Gaussian [16], the locally supported basis function (7) offers several potential advantages including 1) on-line adaptation is simplified because only a subset of the controller parameters have to be updated at any particular time and 2) knowledge about previous operating regimes can be retained because adaptation only affects the controller locally.

B. Relative Degree One Systems

The objective is to recursively update the controller parameters such that the plant output asymptotically tracks the output of a linear reference model. The following reference model is suitable for nonlinear systems of relative degree one:

$$\dot{y}_m = -\gamma_1 y_m + \gamma_1 y_{\rm sp} \tag{8}$$

where y_m is the model output and $\gamma_1 > 0$ is a controller tuning parameter. Two assumptions are invoked to facilitate Lyapunov design of the parameter update laws. The first assumption that $\beta^T\phi(x) \neq 0$ ensures the nonlinear control law (6) remains well defined. Because the sign of $\beta^*(x)$ is assumed to be known, this condition usually can be satisfied by careful initialization of the controller parameters β. The second assumption ensures the existence of "true" controller parameters α^* and β^* such that model matching is achieved

$$\alpha^{*T}\phi(x) = L_f h(x) + \gamma_1 h(x), \quad \beta^{*T}\phi(x) = L_g h(x). \tag{9}$$

This implies that perfect estimation of the controller functions throughout the entire state space is possible. This assumption does not strictly hold in practice, although results for globally supported radial basis functions suggest the controller functions can be approximated arbitrarily well on a compact set if a sufficient number of basis functions are employed [12].

Using (9), the derivative of the output along the system trajectories can be written as

$$\begin{aligned} \dot{y} = L_f h(x) + L_g h(x) u &= -\gamma_1 h(x) + \alpha^{*T}\phi(x) \\ &+ \beta^{*T}\phi(x)u + \beta^T\phi(x)u - \beta^T\phi(x)u. \end{aligned} \tag{10}$$

Assuming $\beta^T\phi(x) \neq 0$, substitution of the control law (6) yields

$$\dot{y} = -\gamma_1 h(x) + \gamma_1 y_{\rm sp} - \Psi_1^T\phi(x) - \Psi_2^T\phi(x)u \tag{11}$$

where $\Psi_1 \equiv \alpha - \alpha^*$ and $\Psi_2 \equiv \beta - \beta^*$ are parameter error vectors. The dynamics of the tracking error $e \equiv y_m - y$ can be written as

$$\dot{e} = -\gamma_1 e + \Psi_1^T\phi(x) + \Psi_2^T\phi(x)u. \tag{12}$$

The form of the error dynamics suggests the following parameter update laws [17]:

$$\dot{\psi}_1 = \dot{\alpha} = -\eta_1 e\phi(x), \quad \dot{\psi}_2 = \dot{\beta} = -\eta_2 e\phi(x)u \tag{13}$$

where $\eta_i > 0$ are adjustable adaptation gains.

Closed-loop stability is analyzed using the Lyapunov function

$$V = \frac{e^2}{2} + \frac{1}{2\eta_1}\psi_1^T\psi_1 + \frac{1}{2\eta_2}\psi_2^T\psi_2. \tag{14}$$

The derivative of V along trajectories of the error system is: $\dot{V} = -\gamma_1 e^2 \leq 0$. This establishes that e, ψ_1, and ψ_2 are bounded, and e is square integrable [8]. The exponential stability and Lipschitz continuity assumptions imposed on the zero dynamics ensure x is bounded and e is uniformly continuous [18]. It follows from Barbalat's Lemma [8] that $\lim_{t\to\infty} e(t) = 0$.

C. Higher Relative Degree Systems

We now extend the direct adaptive control technique to nonlinear systems of relative degree two and higher. The appropriate reference model is

$$y_m^{(r)} = -\gamma_r y_m^{(r-1)} - \cdots - \gamma_1 y_m + \gamma_1 y_{\rm sp} \tag{15}$$

where the γ_i are controller tuning parameters chosen such that $s^r + \gamma_r s^{r-1} + \cdots + \gamma_1$ is a Hurwitz polynomial. We assume $\beta^T\phi(x) \neq 0$ and the existence of "true" controller parameters α^* and β^* that achieve model matching

$$\begin{aligned} \alpha^{*T}\phi(x) &= L_f^r h(x) + \gamma_r L_f^{r-1} h(x) + \cdots + \gamma_1 h(x) \\ \beta^{*T}\phi(x) &= L_g L_f^{r-1} h(x) \end{aligned}. \tag{16}$$

Following the previous development, the dynamics of the tracking error can be written as

$$e^{(r)} = -\gamma_r e^{(r-1)} - \cdots - \gamma_1 e + \Psi_1^T \phi(x) + \Psi_2^T \phi(x) u. \quad (17)$$

The gradient update laws (13) do not ensure Lyapunov stability because the transfer function

$$M(s) \equiv \frac{1}{s^r + \gamma_r s^{r-1} + \cdots + \gamma_1} \quad (18)$$

associated with the error dynamics is not strictly positive real [17]. This difficulty is overcome using the augmented error approach [18]. Define the parameter error Ψ and the regressor Φ as

$$\Psi \equiv \begin{bmatrix} \Psi_1 \\ \Psi_2 \end{bmatrix}, \quad \Phi \equiv \begin{bmatrix} \phi \\ \phi u \end{bmatrix}. \quad (19)$$

Now the error dynamics can be written as $e = M(s)[\Psi^T \Phi]$, which represents the filtering of the time domain signal $\Psi^T \Phi$ by the stable transfer function $M(s)$. The "true" and estimated values of the controller parameters are defined as

$$\theta^* \equiv \begin{bmatrix} \alpha^* \\ \beta^* \end{bmatrix}, \quad \theta \equiv \begin{bmatrix} \alpha \\ \beta \end{bmatrix}. \quad (20)$$

The augmented error e_1 is defined as [18]

$$e_1 \equiv e + \theta^T M(s)[\Phi] - M(s)[\theta^T \Phi]. \quad (21)$$

This relation allows e_1 to be computed from measurable signals. In general, $e_1 \neq e$ because the estimated parameters vary with time. By contrast, the "true" parameters are constant so $\theta^{*T} M(s)[\Phi] - M(s)[\theta^{*T}\Phi] = 0$. By subtracting this equation from (21), an alternative representation of e_1 which is more convenient for analysis is obtained

$$e_1 = e + \Psi^T M(s)[\Phi] - M(s)[\Psi^T \Phi] = \Psi^T M(s)[\Phi]. \quad (22)$$

The form of this error equation suggests the following normalized gradient update law [17]:

$$\dot{\Psi} = \dot{\theta} = \frac{-\eta e_1 \xi}{1 + \xi^T \xi} \quad (23)$$

where $\xi \equiv M(s)[\Phi]$ is the filtered regressor. Closed-loop stability can be analyzed using the procedure presented by Sastry and Isidori [18].

D. Comparison with Linear Model Reference Adaptive Control

Linear model reference adaptive control (LMRAC) techniques are based on a set of standard assumptions which ensure closed-loop stability [17]. We show the proposed nonlinear model reference adaptive control (NMRAC) strategy requires analogous, but generally stronger, assumptions. LMRAC techniques require a minimum phase transfer function model in which the sign of the high-frequency gain is known. The NMRAC strategy is restricted to nonlinear systems which are minimum phase in the sense discussed in Section II. In the Appendix, we show the assumption that the sign of $\beta^*(x)$ is known is a nonlinear generalization of the high-frequency gain condition. The definition of the reference model in LMRAC techniques requires knowledge of the system order (n) and the relative degree (r). The relative degree also must be known in the NMRAC strategy to define the reference model. Knowledge of the system order is replaced by the stronger assumption that all n state variables are measurable. Both techniques require the existence of nominal controller parameters which achieve exact model matching. As discussed previously, this assumption is considerably stronger in the nonlinear case since unknown nonlinear functions are approximated by linearly parameterized basis functions.

TABLE I
NOMINAL OPERATING CONDITIONS

Variable	Value	Variable	Value
μ_m	0.48 h^{-1}	K_m	1.2 g/L
K_i	15 g/L	$Y_{X/S}$	0.4 g/g
D	0.2 h^{-1}	S_f	20 g/L
X	7.64 g/L	S	0.895 g/L

E. Radial Basis Function Generation

The previous development is based on the assumption that the number of radial basis functions (N) is fixed. To reduce computational demands, only basis functions which are centered "near" the closed-loop trajectories should be utilized. We address this problem by using a modification of center generation procedures proposed in [13] and [16]. Potential locations for basis function centers are placed on a regular grid in the state space. A particular basis function is activated only if the closed-loop system evolves "near" its center. In the two-dimensional example considered in the next section, the four basis functions surrounding the current operating point are activated if they are not presently contained in the network. We have found this scheme provides a reasonable compromise between the number of basis functions and the smoothness of the control moves.

IV. SIMULATION EXAMPLE

We apply the direct adaptive control strategy to the bioreactor model (4) where the reaction rate functions have the form [1]

$$r_1(X,S) = \frac{\mu_m S}{K_m + S + \frac{S^2}{K_i}} X, \qquad r_2(X,S) = -\frac{1}{Y_{X/S}} r_1(X,S). \quad (24)$$

Nominal operating conditions are shown in Table I. The manipulated input and controlled output are chosen as the dilution rate (D) and the biomass concentration (X), respectively. The system has relative degree one and the unknown input–output linearizing control law is

$$D = \frac{-r_1(X,S) - \gamma_1 X + \gamma_1 y_{\rm sp}}{-X} \equiv \frac{-\alpha^*(x) + \gamma_1 y_{\rm sp}}{\beta^*(x)} \quad (25)$$

where the state vector is defined as $x \equiv [X \ \ S]^T$. The associated zero dynamics can be shown to be locally stable via Jacobian linearization.

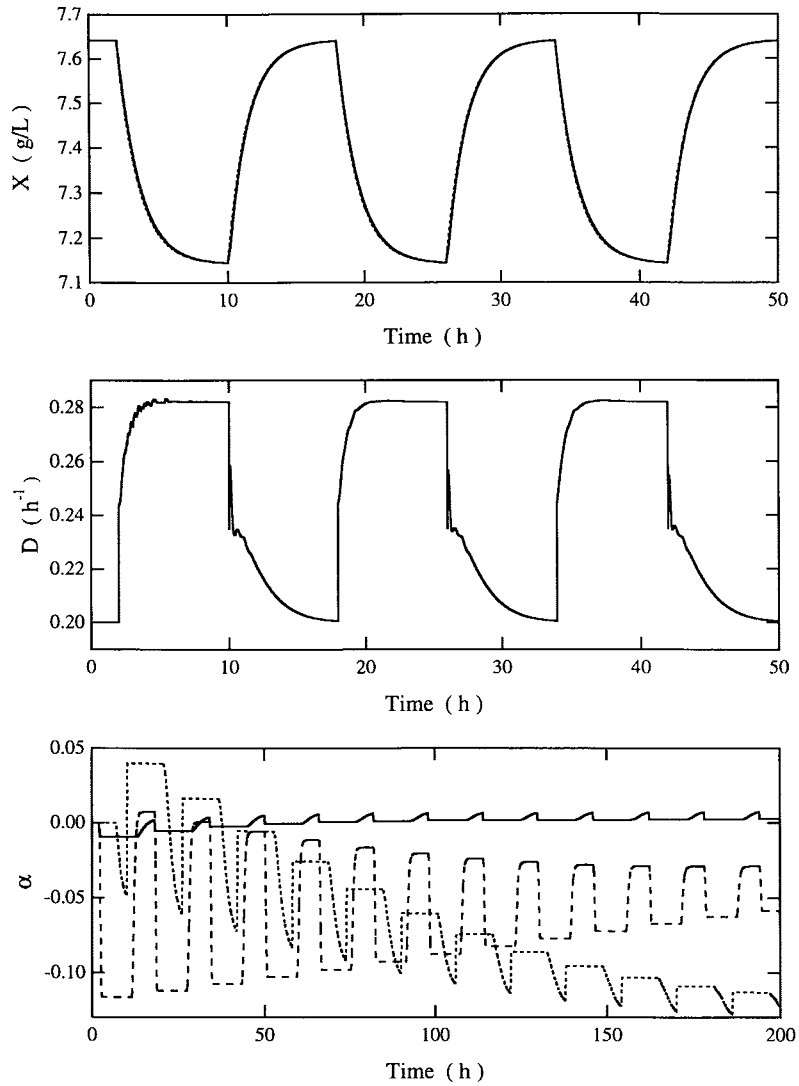

Fig. 1. Repeated setpoint change.

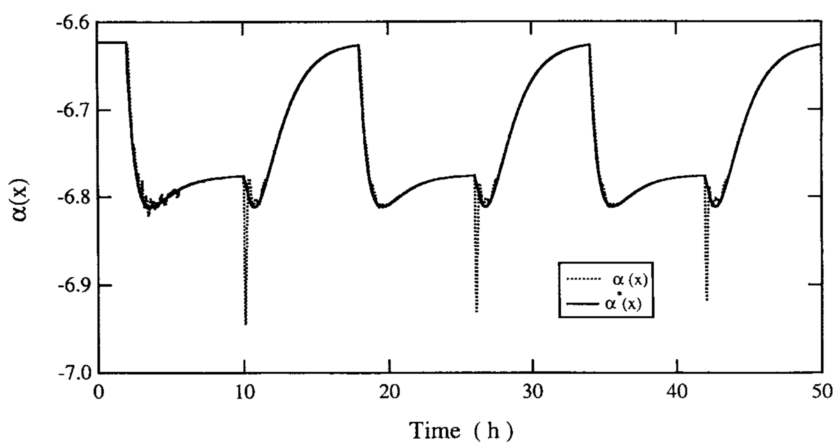

Fig. 2. On-line function approximation for repeated setpoint change.

As discussed in Section II, it is reasonable to assume the function $\beta^*(x)$ is known and the function $\alpha^*(x)$ is unknown. Therefore, the nonlinear control law has the form (6) where $\beta^T\phi(x) = -X$. Radial basis functions are used to construct an on-line estimate of $\alpha^*(x)$ using the procedure described in Section III-E. The mesh size for centers is 0.05 g/L and 0.1 g/L for X and S, respectively, and the basis functions are scaled with $a_1 = 0.2$ g/L and $a_2 = 0.4$

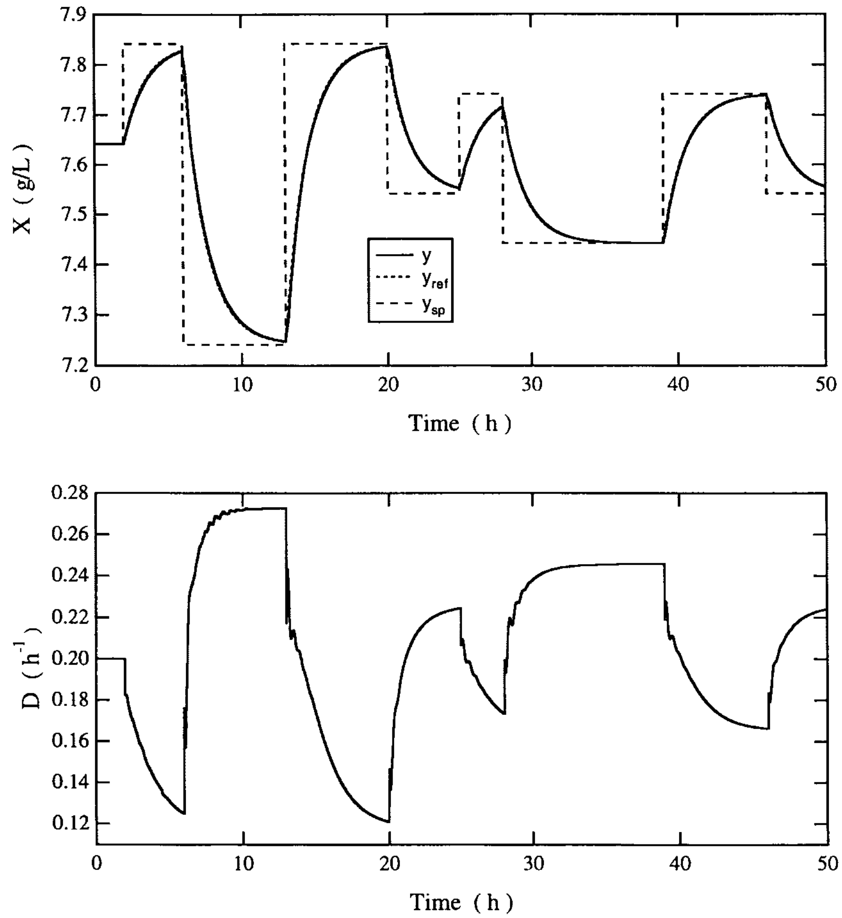

Fig. 3. Random setpoint changes.

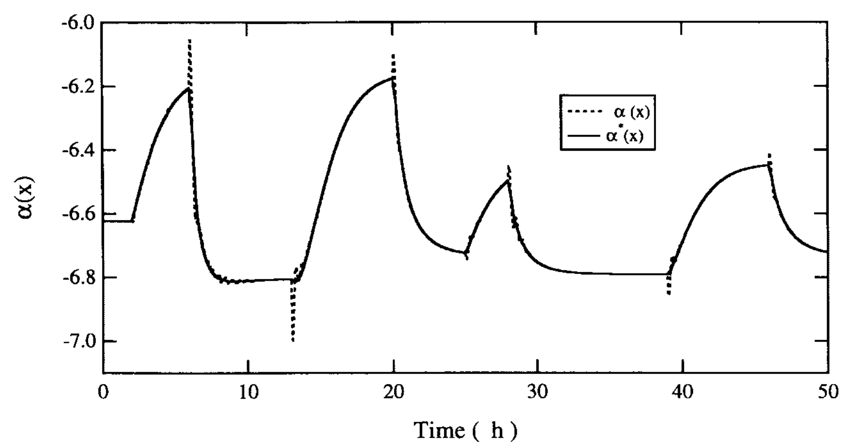

Fig. 4. On-line function approximation for random setpoint changes.

g/L. The controller parameters α_i of the four basis functions surrounding the nominal operating point are initialized such that $D(0)$ is equal to the nominal value in Table I. The remaining controller parameters α_i are initialized to zero when the corresponding basis functions are introduced to the network. The desired setpoint response is described by the reference model (8) with $\gamma_1 = 0.67$ h^{-1}. The parameter update law is (13) where $\eta_1 = 50$ (the β update law is not needed).

Fig. 1 shows the servo performance for repeated setpoint changes between the nominal biomass concentration (7.64 g/L) and a lower value (7.14 g/L). A total of 47 basis functions are activated for this test. The adaptive nonlinear controller provides such outstanding tracking that it is difficult to distinguish between the outputs of the plant and the reference model. The controller produces reasonable dilution rate changes, and the control moves become slightly smoother as training progresses. Three randomly chosen controller parameters shown

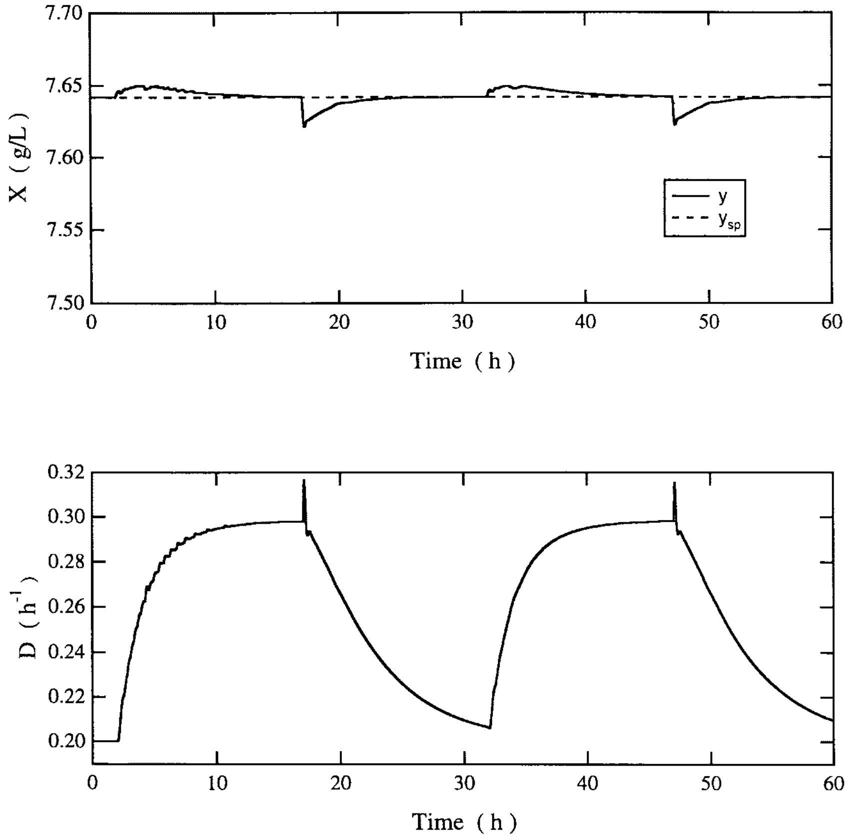

Fig. 5. Repeated disturbance.

for a longer test run appear to be converging. These correspond to $\alpha(3)$, $\alpha(10)$, and $\alpha(44)$ located in the two dimensional state space at $[7.65,\ 0.80]^T$, $[7.55,\ 1.2]^T$, and $[7.15,\ 2.2]^T$, respectively. Fig. 2 shows the on-line approximation of the true controller function $\alpha^*(x)$ by the estimated function $\alpha(x)$. After the initial training phase the function is approximated very accurately, with the exception of "spikes" which appear in the approximated function for positive setpoint changes.

Fig. 3 shows the servo performance for setpoint changes of random magnitude and duration. A total of 53 basis functions are activated. As before, it is difficult to distinguish between the outputs of the plant and the reference model. Reasonably smooth dilution rate changes are produced even though the input changes $\pm 40\%$ from its nominal value. Fig. 4 shows the controller function approximation for this case. With the exception of a few "spikes" in the estimated function, outstanding approximation is obtained.

Fig. 5 shows the regulatory performance for repeated feed substrate concentration disturbances between the nominal value (20 g/L) and a larger value (22 g/L). A total of 46 basis functions are activated. The controller provides excellent disturbance rejection as the biomass concentration is maintained within 0.02 g/L of the setpoint. The input is well behaved and becomes slightly smoother with continued training. Fig. 6 shows the regulatory performance for random feed substrate concentration disturbances. A total of 22 basis functions are activated. The controller provides excellent disturbance rejection as before.

V. Summary and Conclusions

We have proposed a nonlinear adaptive control strategy which does not require a detailed dynamic model of the process to be controlled. The technique is applicable to single-input, single-output nonlinear systems with full-state feedback and stable zero dynamics. The only structural information required is the relative degree and the sign of the Lie derivative $L_g L_f^{r-1} h(x)$ which appears in the associated input–output linearizing control law. Unknown controller functions are approximated with locally supported radial basis functions that are linearly parameterized. Basis functions are introduced only in regions of the state space where the closed-loop system actually evolves. Parameter update laws which ensure (under certain assumptions) the plant output asymptotically tracks the output of a linear reference model and the state vector remains bounded are derived via Lyapunov stability analysis. The strategy provides good servo and regulatory performance when applied to a two-dimensional biochemical reactor model.

Appendix

We show the assumption that the sign of $\beta^*(x)$ is known is a nonlinear generalization of the high-frequency gain condition in linear model reference adaptive control schemes. The transfer function for a linear system of order n and relative degree r can be written as

$$\frac{y(s)}{u(s)} = k\frac{s^{n-r} + b_{n-r-1}s^{n-r-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0} \tag{26}$$

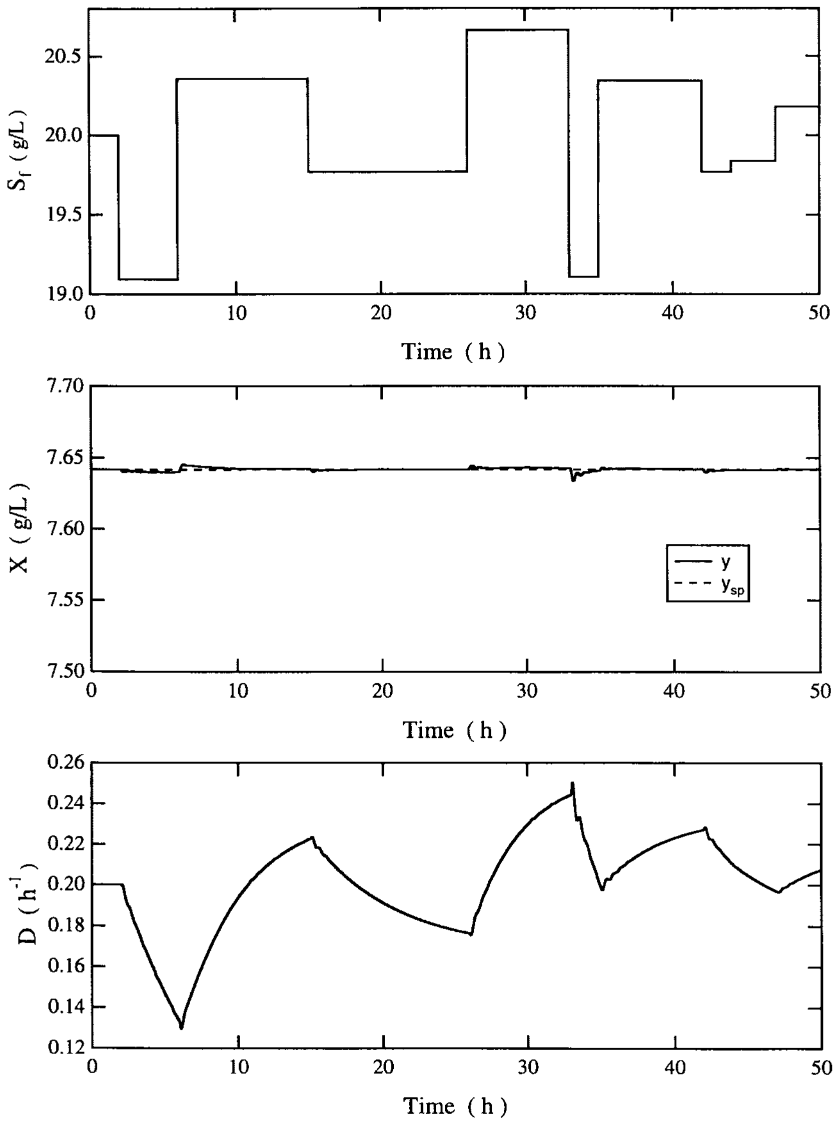

Fig. 6. Random disturbances.

where k is the high-frequency gain. A minimal state-space realization is

$$\begin{aligned} \dot{x} &= Ax + bu \\ y &= cx \end{aligned} \tag{27}$$

where the matrix A, b, and c are defined in [7]. Using these matrices, it is easy to show that

$$y^{(r)} = cA^r x + cA^{r-1} bu \tag{28}$$

where $cA^{r-1}b = k$. It follows from [7] that $L_g L_f^{r-1} h(x) = k$ for a linear state-space system in the form (27). In this sense, $L_g L_f^{r-1} h(x)$ is the nonlinear generalization of the high-frequency gain k.

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