Midterm Exam #1
ChE 4198
Fall 2009

Problem 1 (100 pts). Consider an isothermal drum in which a binary mixture is separated. The feed stream is a saturated liquid with mass flow rate F, mass fraction x_F, and temperature T. A heating rate $Q(t)$ is applied to the mixture to produce a saturated liquid stream with mass flow rate L, mass fraction x_L, and temperature T and a saturated vapor stream with mass flow rate V, mass fraction x_V, and temperature T. The liquid and vapor mass holdups in the drum are denoted H_L and H_V, respectively. The heat of vaporization is denoted ΔH_V.

1. (20 pts) Perform dynamic mass balances on the drum to derive the following model equations:

$$\begin{aligned}
\frac{dH_L}{dt} &= -\frac{dH_V}{dt} + F - L - V \\
\frac{dx_L}{dt} &= \frac{1}{H_L}\left[x_L\frac{dH_V}{dt} - \frac{d(H_V x_V)}{dt} + F(x_F - x_L) - V(x_V - x_L)\right]
\end{aligned}$$

2. (20 pts) Perform an energy balance on the drum and show that the equations in part 1 can be simplified as follows when the reference temperature for enthalpy $T_R = T$, the vapor liquid equilibrium is modeled as $x_V = kx_L$ and vapor holdup is negligible such that $\frac{dH_V}{dt} \approx 0$ and $\frac{d(H_V x_V)}{dt} \approx 0$.

$$\begin{aligned}
\frac{dH_L}{dt} &= F - L - \frac{Q}{\Delta H_V} = f_1(Q) \\
\frac{dx_L}{dt} &= \frac{1}{H_L}\left[F(x_F - x_L) - \frac{Q}{\Delta H_V}(k-1)x_L\right] = f_2(H_L, x_L, Q)
\end{aligned}$$

3. (10 pts) Given the parameter values $F = 10$, $x_F = 0.5$, $L = 5$, $k = 4$ and $\Delta H_V = 10$, show that the model equations in part 2 yield the following steady state: $\bar{x}_L = 0.2$ and $\bar{Q} = 50$.

4. (20 pts) Linearize the model equations in part 2 about the steady state in part 3 to obtain the following linearized equations:

$$\begin{aligned} \frac{dH_L'}{dt} &= 0.1Q' \\ \frac{dx_L'}{dt} &= -25x_L' - 0.06Q' \end{aligned}$$

5. (15 pts) Compute the dynamic response of the liquid holdup $H_L(t)$ to the rectangular pulse input $Q'(s) = \frac{10}{s}(1 - e^{-s})$. Determine $\lim_{t\to\infty} H_L(t)$.

6. (15 pts) Compute the dynamic response of the liquid composition $x_L(t)$ to the step input $Q'(s) = \frac{10}{s}$. Determine $\lim_{t\to\infty} x_L(t)$.

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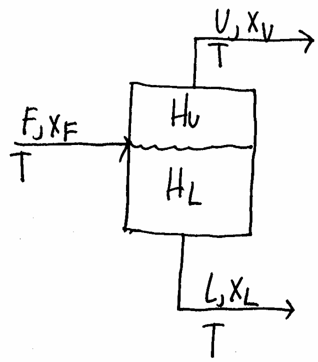
(1)



$$\frac{d}{dt}(H_L + H_V) = F - L - V \Rightarrow \frac{dH_L}{dt} = -\frac{dH_V}{dt} + F - L - V$$

$$\frac{d}{dt}(H_L x_L + H_V x_V) = F x_F - L x_L - V x_V$$

$$H_L \frac{dx_L}{dt} + x_L \frac{dH_L}{dt} = -\frac{d}{dt}(H_V x_V) + F x_F - L x_L - V x_V$$

$$H_L \frac{dx_L}{dt} = -x_L\left(-\frac{dH_V}{dt} + F - L - V\right) + F x_F - L x_L - V x_V - \frac{d}{dt}(H_V x_V)$$

$$= x_L \frac{dH_V}{dt} - \frac{d(H_V x_V)}{dt} + F(x_F - x_L) - V(x_V - x_L)$$

$$\frac{dx_L}{dt} = \frac{1}{H_L}\left[x_L \frac{dH_V}{dt} - \frac{d(H_V x_V)}{dt} + F(x_F - x_L) - V(x_V - x_L)\right]$$

(2) $$\frac{d}{dt}\left[H_L C_{pL}(T - T_R) + H_V C_{pV}(T - T_R)\right] = F C_{pL}(T - T_R) - L C_{pL}(T - T_R) - V C_{pV}(T - T_R) + Q - V \Delta H_V$$

Take: $T_R = T \implies Q = V \Delta H_V \implies V = Q/\Delta H_V$

Given: $\frac{dH_V}{dt} \cong 0 \quad \frac{d(H_V x_V)}{dt} \cong 0 \qquad x_V = k x_L$

$$\frac{dH_L}{dt} = F - L - Q/\Delta H_V = f_1(Q)$$

$$\frac{dx_L}{dt} = \frac{1}{H_L}\left[F(x_F - x_L) - \frac{Q}{\Delta H_V}(k-1)x_L\right] = f_2(H_L, x_L, Q)$$

(3) $F = 10,\ x_F = 0.5,\ L = 5,\ k = 4,\ \Delta H_V = 10$

$$0 = F - L - \bar{Q}/\Delta H_V \implies \bar{Q} = (F - L)\Delta H_V = (10-5)(10) = 50$$

$$0 = \frac{1}{\bar{H}_L}\left[F(x_F - \bar{x}_L) - \frac{\bar{Q}}{\Delta H_V}(k-1)\bar{x}_L\right]$$

$$\implies \bar{x}_L = \frac{F x_F}{F + \frac{\bar{Q}}{\Delta H_V}(k-1)} = \frac{(10)(0.5)}{10 + \frac{50}{10}(4-1)} = \frac{5}{10+15} = 0.2$$

(4) $$\frac{dH_L'}{dt} = \cancelto{0}{f_1(\bar{Q})} + \frac{\partial f_1}{\partial Q}\Big|_{ss} Q' = -\frac{1}{\Delta H_V} Q' = -\frac{1}{10} Q'$$

$$\frac{dx_L'}{dt} = \cancelto{0}{f_2(\bar{H}_L, \bar{x}_L, \bar{Q})} + \cancelto{0}{\frac{\partial f_2}{\partial H_L}\Big|_{ss}} H_L' + \frac{\partial f_2}{\partial x_L}\Big|_{ss} x_L' + \frac{\partial f_2}{\partial Q}\Big|_{ss} Q'$$

$$= -\frac{F + \bar{Q}/\Delta H_V (k-1)}{\bar{H}_L} x_L' - \frac{\frac{k-1}{\Delta H_V}\bar{x}_L}{\bar{H}_L} Q'$$

(5)

Given: $\bar{H}_L = 1$

$$\frac{dX_L'}{dt} = -\frac{10 + \frac{50}{10}(4-1)}{1} X_L' - \frac{\frac{(4-1)}{10}(0.2)}{1} Q' = -25 X_L' - 0.06 Q'$$

(5) $Q'(s) = \frac{10}{s}(1 - e^{-s})$

$$s H_L'(s) - \cancel{H_L'(0)}^{\,0} = -\frac{1}{10} Q'(s) = -\frac{1}{10}\frac{10}{s}(1-e^{-s}) = -\frac{1}{s}(1-e^{-s})$$

$$H_L'(s) = -\frac{1}{s^2}(1-e^{-s}) = -\frac{1}{s^2} + \frac{e^{-s}}{s^2}$$

$$H_L'(t) = -t + (t-1)S(t-1)$$

$$H_L(t) = H_L'(t) + \bar{H}_L = -t + (t-1)S(t-1) + 1$$

$$\lim_{t\to\infty} H_L(t) = -t + (t-1) + 1 = 0$$

(6) $Q'(s) = \frac{10}{s}$

$$s X_L'(s) - \cancel{X_L'(0)}^{\,0} = -25 X_L'(s) - 0.06\, Q'(s)$$

$$X_L'(s) = -\frac{0.06}{(s+25)} Q'(s) = -\frac{0.06}{s+25}\frac{10}{s} = -\frac{0.6}{s(s+25)} = -\frac{0.6/25}{s(\frac{1}{25}s+1)}$$

$$X_L'(t) = -\frac{0.6}{25}(1 - e^{-25t})$$

$$X_L(t) = X_L'(t) + \bar{X}_L = -\frac{0.6}{25}(1-e^{-25t}) + 0.2$$

$$\lim_{t\to\infty} X_L(t) = -\frac{0.6}{25} + 0.2 = 0.176$$