Cascade Control

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Introduction

- **Feedback control**
  - Corrective action taken regardless of disturbance source
  - Corrective action not taken until after the output has deviated from the setpoint
  - Problematic for processes with large time constants and/or long time delays

- **Feedforward control**
  - Corrective action can be taken before the output has deviated from the setpoint
  - Disturbance must be explicitly measured
  - Typically requires a process model
Cascade Control

- Cascade control can provide improved performance for unmeasured disturbances

- Basic characteristics
  - A second measurement and a second controller are used in an inner feedback loop
  - The setpoint for the inner control loop is provided by an outer control loop designed to regulate the controlled output
  - The disturbance can be more rapidly identified with the secondary measurement than with the primary measurement

- Particularly useful when the disturbance is associated with the manipulated input
Flow Control Example

- **Control problem**
  - Regulate flow rate through control value despite changes in upstream and/or downstream pressures

- **Control alternatives**
  - Specify the value position to indirectly achieve the desired flow rate
  - Utilize a measurement of the flow rate to design a feedback controller that explicitly regulates the flow rate

- **Applications**
  - Any control problem in which a flow rate is used as a manipulated variable
  - Outer controller provides the setpoint for the inner flow controller in a cascade arrangement
Furnace Example

- Conventional control
  - Directly regulate hot oil temperature with fuel gas value position
  - Poor performance for changes in fuel gas supply pressure
  - Disturbance is related to the manipulated input

- Cascade control
  - Add an inner controller that regulates fuel gas pressure by manipulating the value position
  - Hot oil temperature controller provides setpoint for inner controller
  - Improved performance for changes in fuel gas supply pressure
Chemical Reactor Example

- **Secondary (inner) controller**
  - Manipulated variable – cooling water makeup value position
  - Controlled output – cooling jacket temperature
  - Disturbance – cooling water inlet temperature

- **Primary (outer) controller**
  - Manipulated variable – cooling jacket temperature setpoint
  - Controlled output – reactor temperature
  - Disturbance – reactant feed flow rate
Closed-Loop Transfer Function

- General formula for closed-loop transfer functions

\[
\frac{Z}{Z_i} = \frac{\Pi_f}{1 + \Pi_e}
\]

- \(Z\) = any variable in feedback system
- \(Z_i\) = any input variable in feedback system
- \(\Pi_f\) = product of all transfer functions between \(Z\) and \(Z_i\)
- \(\Pi_e\) = product of all transfer functions in feedback loop

- Closed-loop transfer function for inner disturbance \((D_2)\)

\[
\frac{Y_2}{\tilde{Y}_{sp2}} = \frac{G_{c2}G_vG_{p2}}{1 + G_{m2}G_{c2}G_vG_{p2}} \equiv G_1 \quad \frac{Y_2}{D_2} = \frac{G_{d2}}{1 + G_{m2}G_{c2}G_vG_{p2}} \equiv G_2
\]

\[
\frac{Y_1}{D_2} = \frac{G_{2}G_{p1}}{1 + G_{m1}G_{c1}G_{1}G_{p1}} = \frac{G_{d2}G_{p1}}{1 + G_{m2}G_{c2}G_vG_{p2} + G_{m1}G_{c1}G_{c2}G_vG_{p2}G_{p1}}
\]
Closed-Loop Stability

- Characteristics equation for cascade control

\[ 1 + G_{m2} G_{c2} G_v G_{p2} + G_{m1} G_{c1} G_{c2} G_v G_{p2} G_{p1} = 0 \]

- Characteristic equation for conventional control (\( G_{m2} = 0, G_{c2} = 1 \))

\[ 1 + G_{m1} G_{c1} G_v G_{p2} G_{p1} = 0 \]

- If inner loop has faster dynamics then the outer loop, then cascade control usually has improved stability characteristics
Closed-Loop Stability Example

- **Transfer functions**

  \[ G_v = \frac{5}{s + 1} \quad G_{p1} = \frac{4}{(4s + 1)(2s + 1)} \quad G_{p2} = 1 \]

  \[ G_{d1} = \frac{1}{3s + 1} \quad G_{d2} = 1 \quad G_{m1} = 0.05 \quad G_{m2} = 0.2 \]

- **Inner loop for cascade control** \((K_{c2} = 4)\)

  \[ \frac{Y_2}{\tilde{Y}_{sp2}} = \frac{G_{c2} G_v G_{p2}}{1 + G_{m2} G_{c2} G_v G_{p2}} = \frac{(4)\left(\frac{5}{s + 1}\right)(1)}{1 + (0.2)(4)\left(\frac{5}{s + 1}\right)(1)} = \frac{4}{0.2s + 1} \]

- **Inner loop for conventional control**

  \[ \frac{Y_2}{\tilde{Y}_{sp2}} = \frac{G_{c2} G_v G_{p2}}{1 + G_{m2} G_{c2} G_v G_{p2}} = \frac{(1)G_v(1)}{1 + (0)(1)G_v G_{p2}} = G_v = \frac{5}{s + 1} \]
Closed-Loop Stability Example cont.

- Characteristic equation for cascade control

\[ 1 + G_{m2}G_{c2}G_vG_{p2} + G_{m1}G_{c1}G_{c2}G_vG_{p2}G_{p1} = 0 \]

\[ 8s^3 + 46s^2 + 31s + 5 + 4K_{c1} = 0 \]

\[ K_{c1,u} = 43.3 \]

- Characteristic equation for conventional control

\[ 1 + G_{m1}G_{c1}G_vG_{p2}G_{p1} = 0 \]

\[ 8s^3 + 14s^2 + 7s + 1 + K_{c1} = 0 \]

\[ K_{c1,u} = 11.3 \]

- Can use larger gains in outer control loop with cascade control than with conventional control
Cascade Controller Design

- **Inner loop**
  - Typically a P or PI controller
  - Tuned first with outer controller in manual
  - Tuned to provide fast response to setpoint changes

- **Outer loop**
  - Typically a PI or PID controller
  - Tuned second with inner controller in automatic
  - Tuned to provide acceptable responses to setpoint changes and/or disturbances
  - Must be retuned if inner loop is retuned
Direct Synthesis Design

- **Inner loop**

\[ \frac{Y_2}{\tilde{Y}_{sp2}} = \frac{G_{c2} G_v G_{p2}}{1 + G_{m2} G_{c2} G_v G_{p2}} \equiv G_{d2} \quad \Rightarrow \quad G_{c2} = \frac{1}{G_{p2} G_v} \frac{G_{d2}}{1 - G_{m2} G_{d2}} \]

If \( G_{m2} = 1 \) \( \Rightarrow \) \( G_{c2} = \frac{1}{G_{p2} G_v} \frac{G_{d2}}{1 - G_{d2}} \)

- **Outer loop**

\[ \frac{Y_1}{Y_{sp1}} = \frac{K_{m1} G_{c1} G_{1} G_{p1}}{1 + G_{m1} G_{c1} G_{1} G_{p1}} = \frac{K_{m1} G_{c1} G_{d2} G_{p1}}{1 + G_{m1} G_{c1} G_{d2} G_{p1}} = G_{d1} \]

\[ G_{c1} = \frac{1}{G_{p1} G_{d2}} \frac{G_{d1}}{K_{m1} - G_{m1} G_{d1}} \]

If \( G_{m1} = K_{m1} \) \( \Rightarrow \) \( G_{c1} = \frac{1}{K_{m1} G_{p1} G_{d2}} \frac{G_{d1}}{1 - G_{d1}} \)
Cascade Controller Design Example

- Transfer functions

\[
G_v = \frac{5}{s+1} \quad G_{p1} = \frac{4}{10s+1} \quad G_{p2} = 1 \\
G_{d1} = 1 \quad G_{d2} = 1 \quad G_{m1} = 1 \quad G_{m2} = 1
\]

- Inner controller design

\[
G_{d2} = \frac{1}{0.2s+1} \quad \Rightarrow \quad G_{c2} = \frac{1}{G_{p2} G_v} \frac{G_{d2}}{1-G_{d2}} = \frac{s+1}{5} \frac{1}{0.2s} = \frac{s+1}{s} = K_c \frac{\tau_I s + 1}{\tau_I s}
\]

- Outer controller design

\[
G_{d1} = \frac{1}{5s+1} \quad \Rightarrow \quad G_{c1} = \frac{1}{K_m G_{p1} G_{d2}} \frac{G_{d1}}{1-G_{d1}} = \frac{10s+1}{4} \frac{1}{5s} = \frac{1}{2} \frac{10s+1}{10s} = K_c \frac{\tau_I s + 1}{\tau_I s}
\]
Conventional Controller Design

- Process transfer function

\[ G(s) = G_v(s)G_{p1}(s)G_{p2}(s)G_{m1}(s) = \frac{20}{(s + 1)(10s + 1)} \]

- IMC tuning \((\tau_c = 5)\)

\[ K_c = \frac{1}{K} \frac{\tau_1 + \tau_2}{\tau_c} = 0.11 \quad \tau_I = \tau_1 + \tau_2 = 11 \quad \tau_D = \frac{\tau_1 \tau_2}{\tau_1 + \tau_2} = 0.91 \]

- Simulink PID controller parameters

\[ P = K_c = 0.11 \quad I = \frac{K_c}{\tau_I} = 0.01 \quad D = K_c \tau_D = 0.10 \]
Simulink Implementation

Setpoint
Add 3
Outer Controller
PID
Add
Inner Controller
PID
Value
\( \frac{5}{s+1} \)
Add 1
Process
\( \frac{4}{10s+1} \)
Add 2
output

Setpoint
To Workspace

Inner Disturbance

Output

Time
Closed-Loop Performance

Outer Disturbance

Setpoint